Software for Calculating Radiation Patterns for Linear Antenna Arrays

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Abstract: - This paper describes the characteristics of a software developed on MatLab Scripts to obtain the radiation patterns of linear antenna arrays using dipoles as the elements. The software includes, among several other options: the analysis and design of arrays with progressive phase, arrays with non uniform amplitude excitation and arrays with non uniform separation. Some special attributes of the software are the simulation through the straight calculation of the fields, the analysis of sensitivity and the behavior against frequency changes, as well as the use of the current distribution on the dipole calculated by the method of moments.

Key-Words: - Simulation, Antenna Arrays, Radiation Pattern, Dipole, Moment Method, Array Factor, Sensitivity of Antennas

1 Introduction

The antenna arrays (assembly of simple radiators, normally of the same type, distributed according with some geometry and with the proper excitation) represent a good choice to generate a radiation pattern with desired characteristics: major lobe direction, beamwidth, nulls position, side lobe levels, etc.

The arrays are classified according with their geometry and excitation. The linear arrays are those in witch the elements are positioned along a line and could have uniform separation or non uniform separation [1-3].

Fig.1 represents a general linear array whose electric field intensity in far zones is given by [4]

$$E_\theta \approx E_{\theta 1} \cdot AF \qquad (1)$$

where $E_{\theta 1}$ is the electric field due to element 1. AF is known as the array factor and is given by [1-3]

$$AF = \sum_{k=1}^{N} a_k e^{j\left(kd_k \cos\theta + \beta_k\right)} \qquad (2)$$

Since the array factor has 3(N-1) design parameters, large arrays require specialized software for proper design.

In this paper we present a software whose main goal is to design and analyze linear antenna arrays. It includes a program to obtain the pattern of a dipole of any length. The dipole is used as the element of the array.

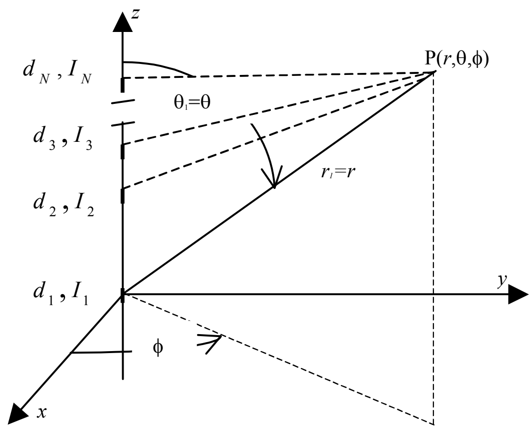


Figure 1. Linear Antenna Array with N elements

2 Characteristics of the Software

The software consists on about 30 programs used by the main program “*arreglo.m*” [5]. This program is menu driven. Fig.2 shows the main menu, used to give data as the number, length and location of dipoles, but also permits to open new menus in order to construct special arrays. This is accomplished by selecting a non uniform separation, excitation of dipoles, an so on. Also from the main menu we are able to choose analysis to get results for the radiation pattern of the dipole, the array factor and the total pattern as well as the sensibility to some changes.

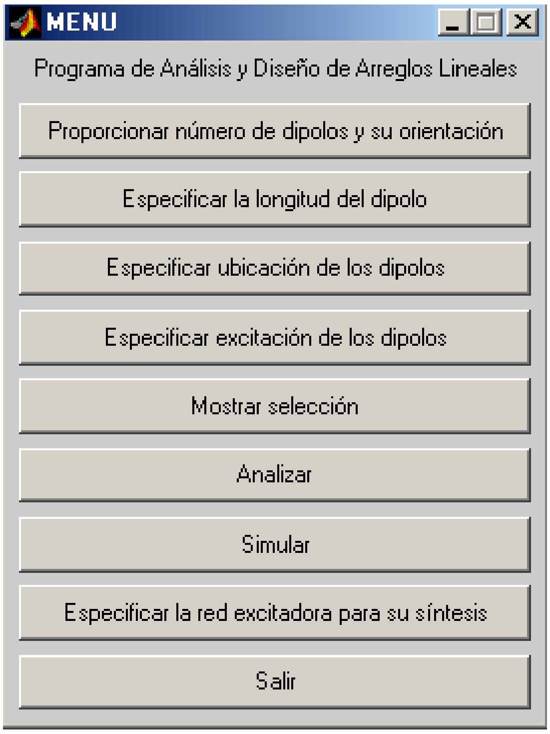


Figure 2. Menu used to Design Linear Arrays

In the analysis option, it is possible to obtain the radiation pattern of the dipole by using a sinusoidal current distribution along the dipole or by the more accuracy current calculated by the method of moments. Also there are options to calculate the array factor and the total radiation pattern that can be presented as a three dimensional figure. Fig.3 shows these options and also the options to observe the frequency response and the sensitivity where the radiation pattern is altered when the length, position and excitation of each element are changed in a systematic way.

3 Radiation Pattern of a Dipole

A dipole consists in two colinear wires separated by a small gap where the excitation is applied. Ideally the gap and the diameter of the wires have to be zero and it can be considered as a continuous filamental wire. In the far zone region, the electric field produced by the dipole considering an ideal sinusoidal current distribution is given by [1]

$$E_\theta \cong j\eta \frac{I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right)-\cos\left(\frac{kl}{2}\right)}{sen\theta}\right] \qquad (3)$$

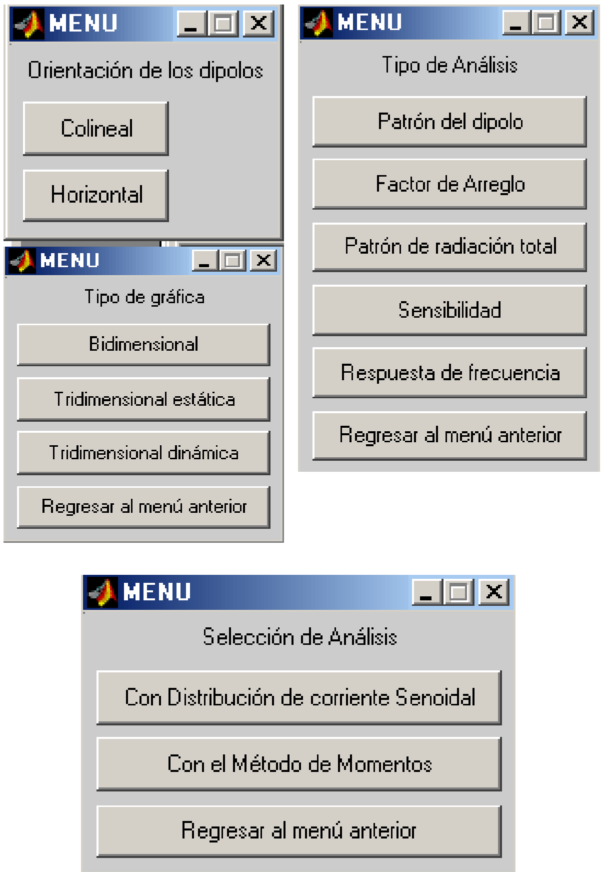


Figure 3. Menus used to do analysis and drawing

In order to calculate the current distribution when the gap and the finite dimension of the wires are considered, we have to solve using the method of moments, the Pocklington equation that relates the current (unknown parameter), and the field on the filament centered in a cylinder with finite radius that represents the dipole. The integral equation that results for this problem can be written as [1]

$$\int_{-l/2}^{l/2} I_z(z')\left[\left(\frac{\partial^2}{\partial z^2}+k^2\right)G(z,z')\right]dz' = -j\omega\varepsilon E_z^i,\ \rho=a \qquad (4a)$$

where

$$G(z,z') = \frac{1}{2\pi}\int_0^{2\pi}\frac{e^{-jkR}}{4\pi R}d\phi' \qquad (4b)$$

and

$$R = \sqrt{4a^2 sen^2\left(\frac{\phi'}{2}\right)+(z-z')^2} \qquad (4c)$$

The method of moments (moment method) is used to solve Ec. (4a) where $I_z(z')$ is the known. In this process we have to choose a family of independent linear functions (basis functions) whose weighted addition gives the solution. It is necessary also to define another set of linear independent functions known as weighted functions, and an inner product between both sets of functions that involves integration.

A program was developed to solve Ec. (4a) where the basis functions were pulse subdomain functions, the weigthed functions chosen were Dirac delta functions. Once the current distribution was known, the vector potential function was calculated and then the electric field was determined for all the points to construct the radiation pattern.

Fig.4 shows the radiation pattern for the electric field of a λ/2 dipole with a gap of *l*/71 and *D*=0.002λ at *R*=40λ using both the sinusoidal current distribution and the one obtained by applying the method of moments. The menus used were: "analizar" → "patrón del dipolo" → "distribución senoidal"; and "analizar" → "patrón del dipolo" → "método de momentos".

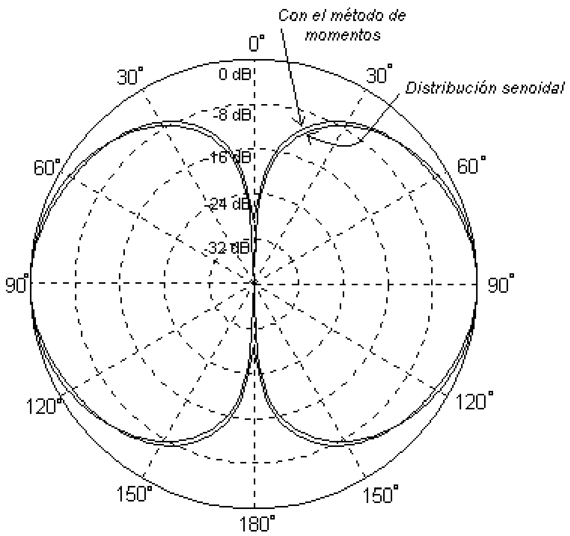


Figure 4. Radiation patterns for a λ/2 dipole

4 AF of Progressive Phase Arrays

The normalized array factor of arrays with uniform amplitude and progressive phase can be obtained from Ec. (2) by setting

$$d_1 = 0,\quad d_2 = d,\quad d_3 = 2d,\quad \cdots, d_N = (N-1)d \tag{5a}$$

$$a_1 = a_2 = \cdots = a_N = 1 \tag{5b}$$

$$\beta_1 = 0,\quad \beta_2 = \beta,\quad \beta_3 = 2\beta,\quad \cdots,\quad \beta_N = (N-1)\beta \tag{5c}$$

$$AF_n = \frac{1}{N}\sum_{n=1}^{N} e^{j(n-1)\psi} \tag{6a}$$

where

$$\psi = kd\cos\theta + \beta \tag{6b}$$

d represents the distance between elements and β is the progressive phase. In this case the design parameters are: the number of elements, their separation, and their progressive phase. It is possible to show that Ec. (6) can be written as [7,8]

$$AF_N = \frac{e^{j(N-1)\psi/2}}{N}\frac{sen\,N\psi/2}{sen\,\psi/2} \approx e^{j(N-1)\psi/2}\frac{sen\,N\psi/2}{N\psi/2} \tag{7}$$

where the approximation is valid when $\psi/2 << 1$.

The main lobe is oriented to

$$\theta_{1,2} = \cos^{-1}\left(\frac{-\beta \pm \frac{2.782}{N}}{kd}\right) \tag{8}$$

and the first side lobe occurs at

$$\theta_3 \approx \cos^{-1}\left(\frac{\frac{3\pi}{N} - \beta}{kd}\right) \tag{9}$$

with amplitude of -13.46 dB from the main lobe.

Fig.5 shows the array factor of an array with 8 elements, *d*=0.5λ and θ_l=60°. The menus used were: "analizar" → "factor de arreglo".

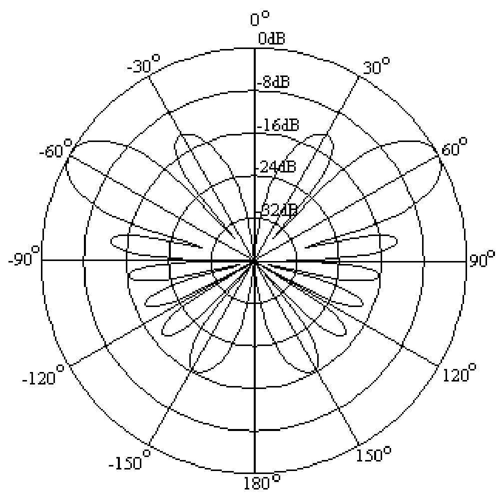


Figure 5. Array factor of a linear array with N=8, *d*=λ/2 and θ_l=60°

5 AF of Nonuniform Amplitud Arrays

In this case the amplitude of excitation of each element varies accordingly to certain distribution. Many distributions can be chosen but the most common in the literature are the binomial distribution and the Dolph-Tschebysheff distribution.

The array factor of an array with a even number of elements N along the *z*-axes, is given by

$$AF_{2M} = \sum_{n=1}^{M} a_n \cos\left[\left(\frac{2n-1}{2}\right)kd\cos\theta\right] \quad (10)$$

where $M = N/2$. It can be seen that the main lobe is directed towards $\theta=\pi/2$. i.e., broadside radiation.

When the number of elements in the array is odd, the array factor can be written as

$$AF_{2M+1} = \sum_{n=1}^{M+1} a_n \cos[(n-1)kd\cos\theta] \quad (11)$$

where $M = (N-1)/2$ and it is also broadside.

5.1 Binomial Distribution

The binomial distribution is so called, because it follows the binomial expansion

$$(1+x)^{m-1} = 1 + P_{m2}x + P_{m3}x^2 + P_{m4}x^3 + \ldots \quad (12)$$

where

$$P_{mr} = \frac{1}{(r-1)!}\prod_{i=1}^{r-1}(m-i) \quad \text{for} \begin{cases} m \geq 2 \\ r \geq 2 \end{cases} \quad (13)$$

$m+1$ is the number of elements in the array and r is the index of the element position. The AF does not present minor lobes for $d \leq \lambda/2$. Fig.6 shows an AF of an array with binomial distribution, 8 elements and d=0.4λ. The menus used were: "Excitación de dipolos: Amplitud variable y Fase cero" → "Tipo de distribución de amplitud: Binomial".

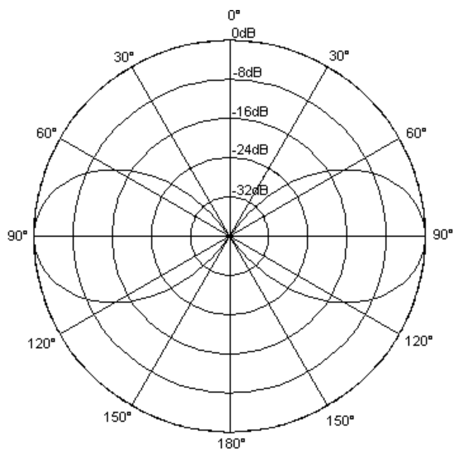


Figure 6. AF of a binomial array with N=8, d=0.4λ

5.2 Tschebyscheff Distribution

The Tschebyscheff distribution is deduced by expanding Ecs. (10) or (11) in terms of the cosine function to an integer power. Normalized expressions for even and odd number of elements, we obtain respectively,

$$(AF_N)_{2M} = a_1T_1(z) + a_2T_3(z) + a_3T_5(z) + \ldots + a_MT_{2M+1}(z) \quad (14)$$

$$(AF_N)_{2M+1} = a_1T_0(z) + a_2T_2(z) + a_3T_4(z) + \ldots + a_MT_{2M}(z) \quad (15)$$

where $z = \cos u$ for $|z| < 1$, $z = \cosh u$ for $|z| > 1$; $u = \frac{\pi d}{\lambda}\cos\theta$ and $T_n(z)$ are known as Tschebyscheff polynomials that can be written as

$$T_m(z) = \cos\left[m\cos^{-1}(z)\right] \quad (16)$$

$$T_m(z) = \cosh\left[m\cosh^{-1}(z)\right] \quad (17)$$

For each value of θ there is a correspondence with u and z. In order to design the AF, a ratio between the minor lobes and the main lobe is established through $R_0 = T_m(z_0) = \cosh\left[m\cosh^{-1} z_0\right]$, then Ec. (10) or (11) is expanded and equated to the Tschebyscheff polynomial of degree m=N-1, in this way the a_n coefficients are found. Fig.7 shows the array factor produced by a Dolph-Tschebyscheff distribution with an array of 8 elements and a lobe ratio of 15 dB. The used menus were: "Excitación de dipolos: Amplitud variable y Fase cero" → "Tipo de distribución de amplitud: Dolph-Tschebyscheff" and then "Tipo de Análisis: Factor de arreglo".

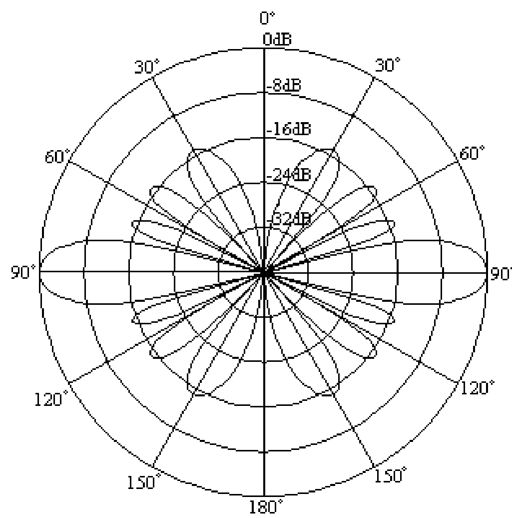


Figure 7. AF of a Dolph-Tschebyscheff array with N=8 and lobes ratio of 15 dB

6 AF of Nonuniform Separation Arrays

The typical linear arrays are characterized by having an uniform separation between elements and have variation in the amplitude and phase of excitation. But, the principle of pattern multiplication presented in Ec. (1) is valid for arrays with identical elements and oriented in the same direction. Here we present some array factors that have been investigated with spacing distributions: binomial, Tschebyshev, geometric, and sinusoidal. The array factors for these arrays are given as:

Asymmetric array (N+1 elements)

$$AF = \sum_{n=1}^{N} e^{jkd_n \cos\theta} \text{ , where } d_1 = 0 \qquad (18)$$

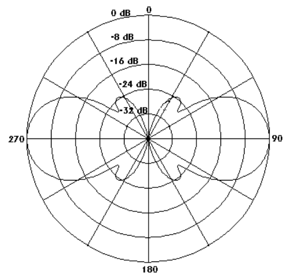
Simmetric array (even number of elements, 2M)

$$AF = \sum_{n=1}^{M} \cos(kd_n \cos\theta) \qquad (19)$$

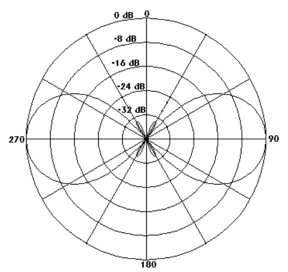
Simmetric array (odd number of elements, 2M+1)

$$AF_N = \sum_{n=1}^{M+1} \cos(kd_n \cos\theta) \text{ , where } d_1 = 0 \qquad (20)$$

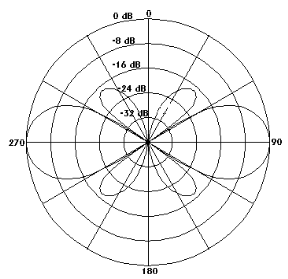
Fig.8 shows some examples of array factors of arrays with diverse separation distributions [9]. The used menus were: “Especificar ubicación de los dipolos: Separación según una distribución específica” → “Tipo de distribución de separación: Binomial, Dolph-Tschebyscheff, Triangular…” and then “Tipo de Análisis: Factor de arreglo”.



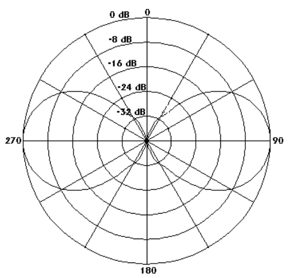
a).- Binomial, 5 el. (l=1λ)



b).- Tsch., 3 el. (l=1λ)



c).- Geom., 5 el. (l=2λ)



d).- Sen., 3 el. (l=1λ)

Figure 8. AF of arrays with different separations

7 Sensitivity Analysis

When position, amplitude and phase for each element of an array has been determined, it is useful to know the effect that every of these parameters has on the radiation pattern, so during the fabrication process, and the design of the excitation or combining network, focus on the important parameters can be made. Sensitivity analysis serves to this purpose.

The software presented in this paper has the option to perform a sensitivity analysis of the angle of the main lobe, main lobe width, directivity and lobe ratio as a function of length of the dipole, and the position, amplitude and phase of every dipole. To accomplish the sensitivity analysis the program calculates and analyses the radiation pattern repeatedly changing systematically the parameter considered. The results are presented as a set of two dimensional plots.

As an example, consider the power pattern of a collinear array of eight dipoles of 0.25λ separated 0.75λ which is shown in Fig. 9. Direction of the main lobe is 60°, lobe width is 13°, directivity is 4.84 and lobe ratio is 3.684. The plots in Fig. 10 shows the sensitivity factors of directivity and lobe ratio as a function of change in position of every dipole. In this case the position of the second and seventh elements tend to modify more importantly the directivity. The options selected in the menus to perform this analysis are: “Analizar” → “Sensibilidad” → “Posicion de los dipolos”.

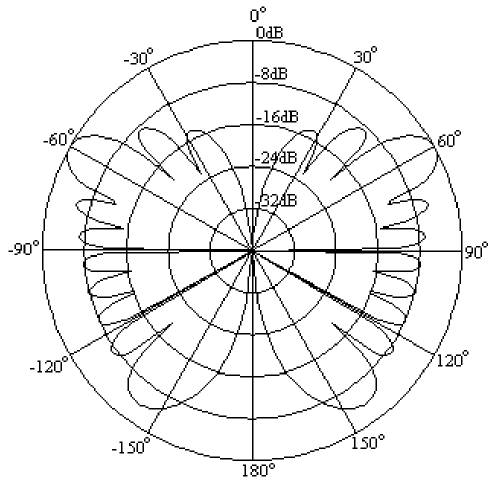


Figure 9. Total pattern obtained by multiplication of patterns principle

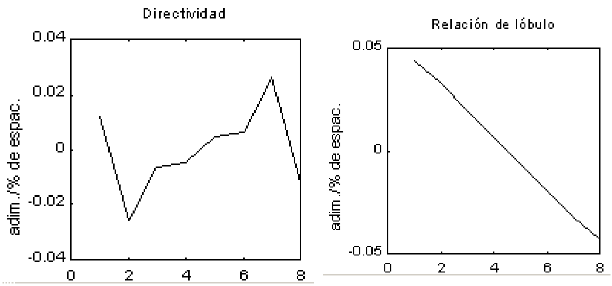


Figure 10. Sensitivity curves of the pattern

8 Frequency response

Frequency response is useful in determining the bandwidth of an antenna array, specially when signals considered have several channels or the bandwidth is not very small.

The software permits to visualize the frequency response of the angle of the main lobe, main lobe width, directivity and lobe ratio, from $0.8f_0$ to $1.2f_0$, where f_0 is the center frequency. As in the case of the sensitivity analysis, the program calculates and analyses the radiation pattern repeatedly changing in each iteration the frequency. Changing the frequency is equivalent to change the wavelength for all dimensions in the array, this is the technique used by the software. Fig. 11 shows the frequency response of the array with the radiation pattern displayed in Fig. 9. The main lobe changes direction from 66.5° to 55° approximately for the considered range.

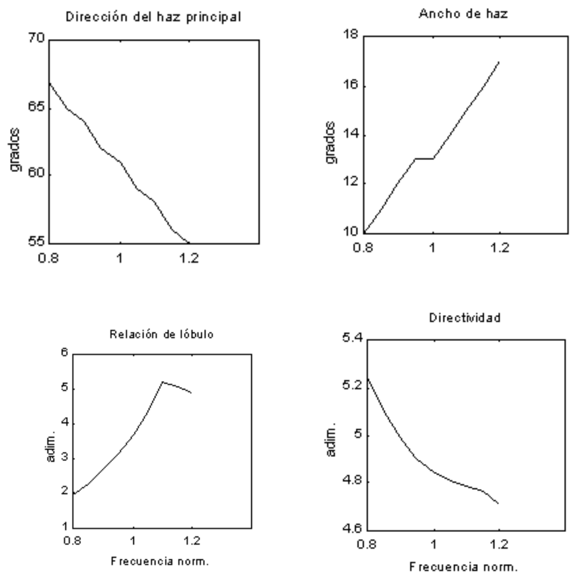


Figura 11. Frequency response of the pattern

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