State-space based Modern Control: Application for Predictive Control Design

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Abstract: This paper presents basic formulation of predictive control design from input/output relationship of a linear system. With an observable canonical form representation of the system under identification, a deadbeat predictive controller is developed and presented in this paper. The method combines together the concepts of system identification and predictive controller design. A control force is also calculated in term of input/output time histories. The formulation satisfies simultaneously system identification and predictive controller design. Numerical example is used to illustrate the deadbeat predictive controller performance.

Key-Words: predictive control, system identification

1 Introduction

There are many literatures on the subject of adaptive control[1]-[2]. Most of them use a linear input/output model that describes the current output prediction as a linear combination of past input and output measurements. The finite difference model, which is commonly called the Auto-Regressive moving average with eXogenous input (ARX) is the one used most by researchers for adaptive control design. General Predictive Control (GPC), as an example, uses the ARX model with the absence of the direct transmission term and builds a multi-step ahead output predictor by solving the Diophantine equation recursively [3]. In contrast with those traditional predictive controllers design, the new approach integrates a state-space based modern control into its corresponding ARX model in the form of an observer-based full-state feedback controller. This formulation exploits the use of the relationship between the state-space model and the ARX model.

State space representation of dynamical model is very useful in conducting controller design particularly in modern control theory. The output variables are related to the input variables via an intermediate quantity called state vector. The state information is needed to build a state feedback controller in modern control. The traditional approach for control of dynamic system involved two steps as system modeling and controller design. System identification is a technique to build mathematical model of a dynamical system within an acceptable level of accuracy from input/output time history[4]-[5].The controller can be designed and it's efficiency based on the accuracy of the model.

The new approach starts with the use of system identification technique to determine the ARX Markov parameters and forms the system matrices. The predictive control gain is computed from the suitable form of an observer-canonical form representation[6]-[7]. This approach has "two" design parameters as a control horizon and a ARX model order which are related to the order of the system. By appropriate adjusting these two parameters, the predictive controller can be achieved and implemented in real time. A numerical example is also presented to illustrate the performance of this new controller.

2 Mathematical Formulation

A linear finite difference model for the $rx1$ output $y(k)$ and the $sx1$ input $u(k)$ is described by

$$\begin{aligned} y(k) &= a_1 y(k-1) + a_2 y(k-2) + \cdots + a_p y(k-p) \\ &+ b_0 u(k) + b_1 u(k-1) + b_2 u(k-2) + \cdots b_p u(k-p) \end{aligned} \quad (1)$$

This represents the relationship between the input, output and also means that the current output can be computed by the time series of the past inputs and past outputs. The finite difference model is also referred to as the Auto-Regressive moving average with eXogenous input (ARX) model. The coefficient matrices, $a_i\,(i=1,2,\dots,p)$ of rxr and $b_i\,(i=0,1,2,\dots,p)$ of rxs, are referred to the observer Markov parameters (OMP) or ARX parameters. The matrix b_0 is the direct transmission term. By shifting one time step, one obtains

$$\begin{aligned} y(k+1) &= a_1 y(k) + a_2 y(k-1) + \cdots + a_p y(k-p+1) \\ &+ b_0 u(k+1) + b_1 u(k) + b_2 u(k-1) + \cdots b_p u(k-p+1) \end{aligned} \quad (2)$$

Define the following relation

$$\begin{aligned} a_1^{(1)} &= a_1 a_1 + a_2 & b_0^{(1)} &= a_1 b_0 + b_1 \\ a_2^{(1)} &= a_1 a_2 + a_3 & b_1^{(1)} &= a_1 b_1 + b_2 \\ &\vdots & b_2^{(1)} &= a_1 b_2 + b_3 \\ a_{p-1}^{(1)} &= a_1 a_{p-1} + a_p & &\vdots \\ a_p^{(1)} &= a_1 a_p & b_{p-1}^{(1)} &= a_1 b_{p-1} + b_p \\ & & b_p^{(1)} &= a_1 b_p \end{aligned} \quad (3)$$

From (1) and (3) equation (2) becomes

$$\begin{aligned} y(k+1) &= a_1^{(1)} y(k-1) + a_2^{(1)} y(k-2) + \cdots + a_p^{(1)} y(k-p) \\ &+ b_0 u(k+1) + b_0^{(1)} u(k) \\ &+ b_1^{(1)} u(k-1) + b_2^{(1)} u(k-2) + \cdots b_p^{(1)} u(k-p) \end{aligned} \quad (4)$$

express the output at the time step $k+j$

$$\begin{aligned} y(k+j) &= a_1^{(j)} y(k-1) + a_2^{(j)} y(k-2) + \cdots + a_p^{(j)} y(k-p) \\ &+ b_0 u(k+j) + b_0^{(1)} u(k+j-1) + \ldots + b_0^{(j)} u(k) \\ &+ b_1^{(j)} u(k-1) + b_2^{(j)} u(k-2) + \cdots b_p^{(j)} u(k-p) \end{aligned} \quad (5)$$

where

$$\begin{aligned} a_1^{(j)} &= a_1^{(j-1)} a_1 + a_2^{(j-1)} & b_0^{(j)} &= a_1^{(j-1)} b_0 + b_1^{(j-1)} \\ a_2^{(j)} &= a_1^{(j-1)} a_2 + a_3^{(j-1)} & b_1^{(j)} &= a_1^{(j-1)} b_1 + b_2^{(j-1)} \\ &\vdots & &\vdots \\ a_{p-1}^{(j)} &= a_1^{(j-1)} a_{p-1} + a_p^{(j-1)} & b_{p-1}^{(j)} &= a_1^{(j-1)} b_{p-1} + b_p^{(j-1)} \\ a_p^{(j)} &= a_1^{(j-1)} a_p & b_p^{(1)} &= a_1^{(j-1)} b_p \end{aligned}$$

Chose the state variables as follows

$$\begin{aligned} x_1(k) &= y(k) - b_o u(k) \\ x_2(k) &= y(k+1) - b_o u(k+1) - b_o^{(1)} u(k) \\ x_3(k) &= y(k+2) - b_o u(k+2) - b_o^{(1)} u(k+1) - b_o^{(2)} u(k) \\ &\vdots \\ x_p(k) &= y(k+p-1) - b_o u(k+p-1) - b_o^{(1)} u(k+p-2) \\ &\quad -,\ldots,- b_o^{(p-1)} u(k) \end{aligned} \quad (6)$$

where each vector $x_i(k), i=1,2,\dots,p$ has length as the number of output r

$$\begin{aligned} x_1(k+1) &= x_2(k) + b_o^{(1)} u(k) \\ x_2(k+1) &= x_3(k) + b_o^{(2)} u(k) \\ x_3(k+1) &= x_4(k) + b_o^{(3)} u(k) \\ &\vdots \\ x_p(k+1) &= y(k+p) - b_o u(k+p) \\ &\quad - b_o^{(1)} u(k+p-1) -,\ldots,- b_o^{(p-1)} u(k+1) \\ &= a_1 x_p(k) + a_2 x_{p-1}(k) + \ldots \\ &\quad + a_p x_1(k) + b_o^{(p)} u(k) \end{aligned} \quad (7)$$

The above equations can be arranged in the state space matrix canonical form as

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned} \quad (8)$$

where

$$A=\begin{bmatrix} 0 & I & 0 & \cdots & 0 & 0 \\ 0 & 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & I \\ a_p & a_{p-1} & a_{p-2} & \cdots & a_2 & a_1 \end{bmatrix} \quad , \quad B=\begin{bmatrix} b_0^{(1)} \\ b_0^{(2)} \\ \vdots \\ b_0^{(p-1)} \\ b_0^{(p)} \end{bmatrix}$$

$$C=[I \quad 0 \quad \cdots \quad 0 \quad 0], \; D=b_o$$

The state vector x becomes a $rp\times 1$ vector, the state matrix A is a $rp\times rp$ matrix, the input matrix B is a $rp\times s$ matrix, and the output matrix C is a $r\times rp$ matrix.

The observability matrix of the canonical form is

$$M=\begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{p-1} \end{bmatrix} = \begin{bmatrix} I & 0 & 0 & \cdots & 0 \\ 0 & I & 0 & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & I \end{bmatrix} \qquad (9)$$

which is nonsingular or means all the state vector x are observable.

Defines the observer gain matrix

$$G=\begin{bmatrix} a_1^{(0)} \\ a_1^{(1)} \\ \vdots \\ a_1^{(p-2)} \\ a_1^{(p-1)} \end{bmatrix} \qquad (10)$$

which will makes the observer state matrix $A+GC$ to zero into p steps

$$(A+GC)^p=\begin{bmatrix} a_1^{(0)} & I & 0 & \cdots & 0 & 0 \\ a_1^{(1)} & 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_1^{(p-2)} & 0 & 0 & \cdots & 0 & I \\ a_1^{(p-1)}+a_p & a_{p-1} & a_{p-2} & \cdots & a_2 & a_1 \end{bmatrix}^p = 0 \qquad (11)$$

3 Deadbeat Predictive Control

By the state space representation, one may produce a deadbeat feedback predictive control law. From equation (8) can produce

$$\begin{aligned} x(k+1) &= Ax(k)+Bu(k) \\ x(k+2) &= A^2x(k)+[AB \quad B]\begin{bmatrix} u(k) \\ u(k+1) \end{bmatrix} \\ &\vdots \\ x(k+q) &= A^q x(k)+\Gamma u_q(k) \end{aligned} \qquad (12)$$

where

$$u_q(k)=\begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+q-1) \end{bmatrix}$$

and $\Gamma=[A^{q-1}B \quad A^{q-2}B \quad \cdots \quad B]$

$$=\begin{bmatrix} b_0^{(q)} & b_0^{(q-1)} & \cdots & b_0^{(1)} \\ b_0^{(q+1)} & b_0^{(q)} & \cdots & b_0^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ b_0^{(q+p-1)} & b_0^{(q+p-2)} & \cdots & b_0^{(p)} \end{bmatrix}$$

The matrix Γ is the controllability matrix. The integer q must be chosen so that $qs\geq n$, where n is the order of the system and s is the number of inputs, to make The matrix Γ has rank n.

Equation (12) shows that state $x(k+q)$ will become zero when the input

$$u_q(k)=-[\Gamma]^{\nabla} A^q x(k) \qquad (13)$$

where $^{\nabla}$ means pseudo-inverse

By the receding control horizon technique, the control force will be

$$\begin{aligned} u_c(k) &= G_c x(k) \\ &= -\{first..s..rows..of..[\Gamma]^{\nabla}\}A^q x(k) \end{aligned} \qquad (14)$$

Computational Steps

1. From input/output data, determine the open-loop observer Markov parameters (ARX) parameters by either batch or recursive least square technique.
2. Realize system matrices A,B,C,D and form the state space model in equation (8) and corresponding observer gain matrix in equation (10)
3. Compute control gain G_c defined in equation (14) using the controllability matrix with the given integer q. The integer q must be large enough so that $ps>n$.
4. Do the feedback control

4 Numerical Example

The simple two-degree-of freedom, mass-spring-damper system is used to illustrate the performance of the predictive controller. Let *m1*=*m2*=4 kg, *c1*=*c2*=1 N-s/m, and *k1*=*k2*=4 N/m. The input to the system is force at *m1* while the observed output is at *m2*. The sampling period is 0.2 s. The equation of motion is given as

$$M\ddot{x} + C\dot{x} + Kx = Fu$$

where

$$M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}, C = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}$$

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}, F = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

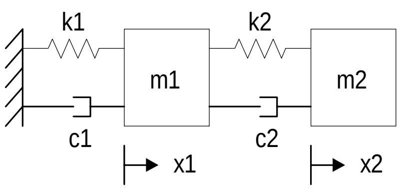

Figure 1 Mass-spring-damper system

By first start exciting the system at *m1* to identify the system parameters at 500 data point. Figure 2 shows the input and output response of the system. Since the system has two degree of freedom, the smallest order of the ARX model *p* is 4. The smallest value *q* for predictive controller is therefore 4. However, the predictive controller to make system to the rest at 4 steps is not practical because it will need excessive control. Instead, one should consider the case where the controller is computed with *q*=20 that make the control force more applicable. Figure 3 shows the input and output with *p*=4 and *q*=20 without the noise. And figure 4 shows the input and output with the same number of *p* and *q* with the presence of the noise.

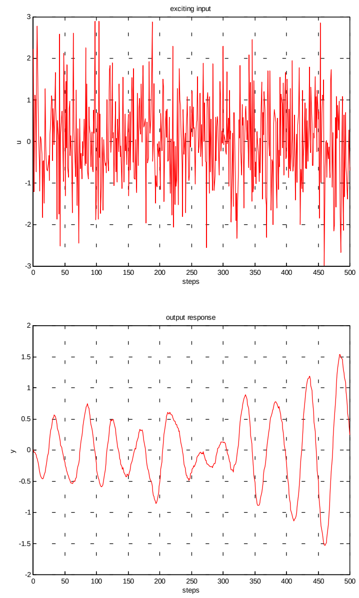

Figure 2 Input and output response of open-loop system

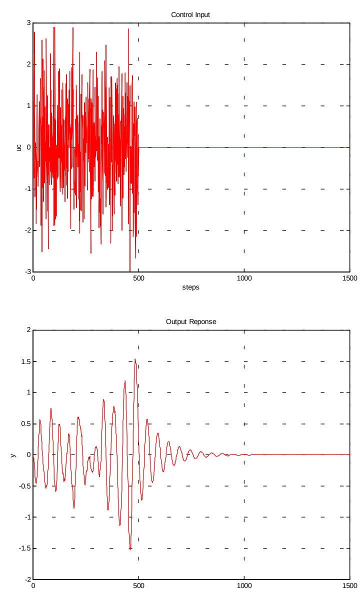

Figure 3 Input and output response of closed-loop system, controller is turn on after 200 data point by using p=4, q=20

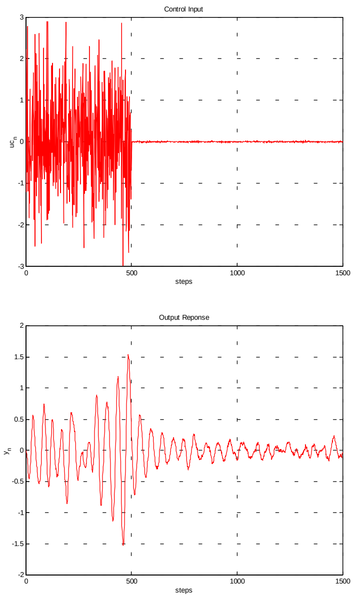

Figure 4 Input and output response of closed-loop system, controller is turn on after 200 data point by using p=4, q=20, with the presence of noise

5 Conclusion

The conventional procedure for any controller designs includes two steps, i.e., first perform system identification within an acceptable level of accuracy and follow by controller design. This paper presents the new approach of predictive control by of implementing system identification into the algorithm. The algorithm provides the state-space representation of the deadbeat predictive control law. It computes the deadbeat gain for observer-based full-state feedback then is converted into the input/output gain used in the classical predictive control design. The connection between the classical state-space control law and predictive control law is clearly defined.

References:

[1] Richalet J., Rault A., Testud J.L. and Papon J.,
"Model Predictive Heuristic Control : Applications to Industrial Processes", Automatica, Vol. 14, No. 5, pp. 413-428, 1978.

[2] Clarke, D.W., Mohtadi, C., and Tuffs, P.S., 'Generalized Predictive Control: Part i and ii,' Automatica, vol.23, pp.137-160, 1987.

[3] Garcia, C. E., Prett, D. M., and Morari M., "Model Predictive Control," Automatica, Vol. 25, 1989, pp 335-348.

[4] Juang, J.-N., Applied System Identification, PRT Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1994.

[5] Huang, J.-K., Hsiao, M.-H., and Cox, D.E., "Indirect Identification of Linear Dynamics," Journal of Guidance, Control, and Dynamics, Vol. 19, No. 4, July-August 1996, pp. 836-841.

[6] Juang, J.-N. and Phan, M.G., "Deadbeat Predictive Controller ," NASA Technical Memorandum.

[7] Juang, J.-N. and Phan, M.G., "Recursive Deadbeat Controller Design," NASA Technical Memorandum.

[8] Chinvorarat, S., Schoen, M., Lu, B., and Huang, J-K. " Set Point Tracking Predictive Control by System Identification Approach," 1999 American Control Conference, 2-4 June 1999, San Diego, CA, pp. 331-335.