

Application of control techniques to the management of the maintenance of repairable devices

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Abstract: – The management of the maintenance of a set of filters for the dust refinement has been analysed by means of techniques based on dependability ([4]-[5]), thus obtaining a model of the lifecycle of such devices. Three approaches have been considered: the first one consisting in not applying any particular management policies besides those suggested by the event succession; the second one consisting in planning the recurrence of checks, by following temporal laws obtained by means of dynamic systems control techniques; the third one, finally, consisting in applying redundant constructions. As shown by a numerical example, the second approach, that is an evolution of the preventive maintenance methods, can result also the most economically favourable one. In fact the expected repetition rates of the control checks are quite economical parameters to be used to change reliability and the checks can be conducted also through sensors.

Key-Words: – Reliability, Control Systems, Stochastic Systems, Decision Support and Management Systems, Maintenance.

1 Introduction

The probability distributions of the characteristic events of a set of filters for dust refinement have been analysed, to decide eventual changes in the relative maintenance management system. The techniques, that have been used, are derived from the control of dynamic systems (to change the reliability and the MTTF) and are compared with usual reliability techniques based on redundancy.

The devices considered are repairable in the sense that, when damaged, can be repaired and reused or stored in a warehouse waiting to be used. The time distributions of the events that interest such devices can be considered exponential ones, with constant transition rates from one state to another ([6]).

Given the randomness of failure instants, the logistic strategy to manage such devices consists in doing periodic state checks ([4]), with eventual substitution when the failure is upcoming (preventive maintenance) or after the damage has happened (corrective maintenance).

To compare different methods of maintenance management, in such a way that also economic advantages and disadvantages are analysed, three cases have been considered:

1. absence of particular checks far from the happening instants of events related to the filters;
2. presence of checks planned with the help of techniques from the control of dynamic systems;
3. presence of redundancy.

In the first case considered, with absence of particular checks besides the interventions that are strictly necessary to avoid the plant stop, the number of interventions is unbalanced toward the corrective maintenance. Consequently, the provisions in the

warehouse must be dimensioned for any eventuality (passive management).

In the second case, instead, where the check times are planned in such a way that the number of preventive checks is increased to overcome the corrective ones, the steady state reliability, the expected time to failure of the filters and the number of devices stored for provision are changed (active management).

Finally, in the third case, where redundancy is used, it is possible, again, to have changes in the MTTF and in the provisions (management of redundant devices), but being the devices more expensive than in the previous cases, to adopt such a management strategy can reveal itself a not winning choice.

In particular, for each case, the cost indexes of every device and the provision amount that is necessary to avoid stops in the production cycle have been considered.

2 Assumptions

A model of the lifecycle of a repairable device can be obtained with the following hypothesis:

- only one state for maintenance and checks;
- there are no stored or functioning devices whose failures are not detectable (failure coverage);
- the transition probability from the state S_i , at time instant t , to S_j , at time instant $t+\Delta t$, is proportional to the time interval Δt , for Δt small with respect to the transition times; the proportionality constant, called transition (or failure, reparation, check, etc.) rate does not depend on t and Δt ;
- eventual reparations do not modify in time the failure rates;
- independent devices of the same kind have the same probabilistic behaviour.

3 Filter modelling

The filters considered are constituted by a frame, that simplifies substitutions, by a membrane, that is the actual filter, and by two flux sensors, placed before and after the membrane.

The membrane is mounted on an inner frame. Differential flux changes, greater than the expected ones for the work that is running, must be ascribed to the following causes

- the membrane is obstructed;
- the membrane is damaged;
- the inner frame is damaged.

In the first eventuality, given the nature of the dust, the drawback can be solved with the immersion of the membrane in a suitable solvent.

In the second and in the third eventuality it is necessary to operate a repairation, with the substitution of the membrane or of the inner frame.

The immersion in the solvent is done in the warehouse, that is de-localised in suitable tanks in the neighborhood of the operative location of the filter units, exerted in a FIFO way (there is also a localised warehouse with further provisions, but is not considered in the modelling procedure)

Reparations, decided after an optical check carried out by the operator, are done in a single laboratory.

It may happen that since the filter is obstructed, the optical check cannot detect that the membrane is actually damaged, in such case the repairation must be done after the permanence in the warehouse.

3.1 Model 1 for filter lifecycle

The lifecycle of a repairable device, with the maximum allowed simplification, can be depicted as in Fig. 1, where infant mortality and obsolescence due to age are not considered.

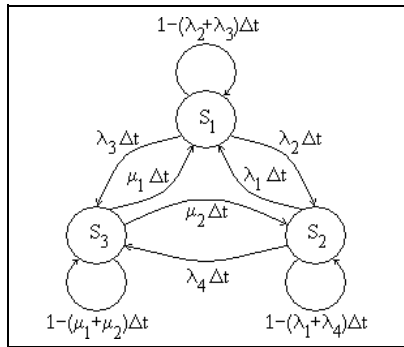


Fig. 1: Lifecycle of a repairable filter

Symbols have the following meaning:

- S_1 : state that represents the warehouse,
- S_2 : state that represents a functioning device
- S_3 : state that represents reparation interventions,

$\lambda_i \Delta t$: transition probability in the interval $[t, t + \Delta t)$,

- λ_1 : rate of storage of correctly functioning devices,
- λ_2 : rate of going in use of a stored device,
- λ_3 : failure rate of a stored device,
- λ_4 : failure rate of a functioning device,

μ_1 : rate of storage after a maintenance intervention,
 μ_2 : rate of going in use after a maintenance intervention.

The physical meanings of the transition rates, assure their boundedness and non-negativity:

$$0 \leq \lambda_1 \leq \lambda_{1,\max}, \quad 0 \leq \lambda_3 \leq \lambda_{3,\max}, \quad 0 \leq \mu_1 \leq \mu_{1,\max},$$

$$0 \leq \lambda_2 \leq \lambda_{2,\max}, \quad 0 \leq \lambda_4 \leq \lambda_{4,\max}, \quad 0 \leq \mu_2 \leq \mu_{2,\max}.$$

The following transition (Kolmogoroff's) equations can be obtained from the graph of Fig. 1,

$$dx(t) / dt = Ax(t) + \mu \quad (1)$$

with

$$A = \begin{pmatrix} -d_1 & a \\ b & -d_2 \end{pmatrix}; \quad x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}; \quad \mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} \quad (2)$$

$$d_1 = \lambda_2 + \lambda_3 + \mu_1; \quad a = \lambda_1 - \mu_1;$$

$$d_2 = \lambda_1 + \lambda_4 + \mu_2; \quad b = \lambda_2 - \mu_2;$$

where:

$x_1(t)$ = probability of being in S_1 at t ,

$x_2(t)$ = probability of being in S_2 at t ,

$x_3(t) = 1 - x_1(t) - x_2(t)$ = probability of being in S_3 at t .

Reliability, $R_1(t)$, can be defined as the probability of not being in S_3 at t ,

$$R_1(t) := 1 - x_3(t) = C x(t), \quad \text{with} \quad C = \begin{pmatrix} 1 & 1 \end{pmatrix}.$$

For $t \rightarrow \infty$ one has:

$$R_1(\infty) = x_1(\infty) + x_2(\infty),$$

$$x_1(\infty) = \frac{d_2 \mu_1 + a \mu_2}{d_1 d_2 - ab}, \quad x_2(\infty) = \frac{b \mu_1 + d_1 \mu_2}{d_1 d_2 - ab}.$$

For what concerns the MTFE, considering that $R'_1(t) := R_1(t) - R_1(\infty)$ goes to zero for $t \rightarrow \infty$, if $x_1(t) \geq x_1(\infty)$ and $x_2(t) \geq x_2(\infty)$, the following probability density can be defined:

$$f(t) := -\frac{dR'_1(t)}{dt} = -\frac{dR_1(t)}{dt}.$$

Then the following definition can be used for the expected time to failure:

$$MTFE_1 := \int_0^\infty t f(t) dt = \int_0^\infty R'_1(t) dt = \tau_{11} + \tau_{21},$$

where

$$\tau_{11} = \frac{d_2(x_1(0) - x_1(\infty)) + a(x_2(0) - x_2(\infty))}{d_1 d_2 - ab}$$

$$\tau_{21} = \frac{b(x_1(0) - x_1(\infty)) + d_1(x_2(0) - x_2(\infty))}{d_1 d_2 - ab}$$

are, for each device, respectively the expected time spent waiting to enter in the warehouse, $\tau_{11} = -E_{x_1}\{t\}$, and the expected time spent waiting to enter in function, $\tau_{21} = -E_{x_2}\{t\}$.

An evaluation of expected steady state costs versus time, for each device, can be done using the following parameters:

- C_{A1} is the cost of each device,
- $C_{M1} \times \tau_{M1}$ is the expected cost of the permanence in the warehouse,
- $C_{U1} \times \tau_{U1}$ is the expected gain earned during functioning,

- $C_{R1} \times \tau_{R1}$ is the expected cost in waiting to be repaired,
- $C_{N1} \times N_1$ is the expected cost of repairs,
- $N_1 = t / (\tau_{M1} + \tau_{U1} + \tau_{R1})$ is the expected number of repairs in $[0, t]$, it can be identified with the expected number of lifecycles in the same interval,
- τ_{M1} is the steady-state expected time spent in the warehouse, whose estimate is $x_2(\infty)\tau_{21} + x_3(\infty)MTTF_1$,
- τ_{U1} is the steady-state expected time spent in operation, whose estimate is $x_1(\infty)\tau_{11} + x_3(\infty)MTTF_1$,
- τ_{R1} is the steady-state expected time spent in repair, whose estimate is $x_1(\infty)\tau_{11} + x_2(\infty)\tau_{21}$.

The expected cost is:

$$C_1 = C_{A1} + \frac{(C_{M1} \cdot \tau_{M1} - C_{U1} \cdot \tau_{U1} + C_{R1} \cdot \tau_{R1} + C_{N1}) \cdot t}{\tau_{M1} + \tau_{U1} + \tau_{R1}}$$

For what concerns provision dimensioning, if n is the number of devices that must be operating during steady state, then it is necessary to have at least

$$n_1 = \frac{n}{x_2(\infty)} = n + \frac{x_1(\infty)}{x_2(\infty)}n + \frac{x_3(\infty)}{x_2(\infty)}n$$

devices.

3.2 Model 2 for filter lifecycle

By analysing model 1 it has been possible to find, in the lifecycle of such devices, suitable control variables to be used in a feedback chain to modify the mean time to failure of the filters, thus realizing, at the same time, the economic target of reducing the number of substitutions due to corrective maintenance.

In such a case, to have new equations as comparable as possible with the others, it is necessary to do the following change in Fig. 1:

S_3 : becomes a state that represents both checks (preventive maintenance) and eventual repair,

and two transition rates must be replaced as follows:

$$\begin{aligned} \lambda_3 &\leftarrow \lambda_3 + u_1; & \lambda_3: & \text{actual failure rate of a device in the} \\ & & & \text{warehouse,} \\ u_1 &: & \text{check rate on a device in the} \\ & & & \text{warehouse} \\ \lambda_4 &\leftarrow \lambda_4 + u_2; & \lambda_4: & \text{actual failure rate of a device in} \\ & & & \text{operation,} \\ u_2 &: & \text{check rate on a device in} \\ & & & \text{operation,} \end{aligned}$$

The new rates are subject to suitable physical limits too:

$$\begin{aligned} 0 \leq u_1(t) \leq u_{1,\max}, & \quad \forall t \geq 0, \\ 0 \leq u_2(t) \leq u_{2,\max}, & \quad \forall t \geq 0. \end{aligned}$$

If u_1 and u_2 are decided a priori from the management system (for example according to legislation), the model can be used for analytic purposes. Instead, the main target here considered has been the design of a scheduling strategy for the maintenance checks.

Note that (1) is not a normal physical system, with physical inputs that are random variables, rather it is an implicit method to describe the probability distributions of certain temporal events. Then, it has been possible to develop a method for modifying such probabilities, by making some events happen in a controlled way.

The new equations are:

$$dx(t)/dt = Ax(t) - X(t)u(t) + \mu \quad (3)$$

with A , x e μ as in (2), and

$$u(t) = \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix}, \quad X(t) = \begin{pmatrix} x_1(t) & 0 \\ 0 & x_2(t) \end{pmatrix}.$$

Note that, being system (3) bilinear, the targets that must be reached and the solution that is thus obtained, are not so trivial.

The control law adopted is:

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} \frac{\mu_1 + a\bar{x}_2}{\bar{x}_1} - d_1 \\ \frac{\mu_2 + b\bar{x}_1}{\bar{x}_2} - d_2 \end{pmatrix} \quad (4)$$

where \bar{x} represents the desired steady state value, obtained through (4). The acceptable values of \bar{x} are those inside the following set:

$$\begin{aligned} 0 \leq \bar{x}_1 &\leq 1, \\ 0 \leq \bar{x}_2 &\leq 1, \\ \bar{x}_1 + \bar{x}_2 &\leq 1, \end{aligned} \quad (5)$$

$$\mu_1 - d_1 \bar{x}_1 + a \bar{x}_2 \geq 0,$$

$$\mu_2 + b \bar{x}_1 - d_2 \bar{x}_2 \geq 0.$$

The meaning of (4) is as follows: checks must be done when it strikes the smaller between the two time instants t_{λ_i} , in which the device can't be used since it needs repairs, and t_i , defined by the random sequence

$$t_i(k) = -\frac{1}{u_i} \log(1 - \delta_i(k)),$$

where $\delta_i(k)$ is the k -th sample of a random sequence with uniform distribution in $[0, 1]$ (obtainable with the **random** function of any programming language).

Defining reliability $R_2(t)$, as in the previous case,

$$R_2(t) := 1 - x_3(t) = C x(t),$$

this time one has $x(\infty) = \bar{x}$, and hence:

$$R_2(\infty) = \bar{x}_1 + \bar{x}_2.$$

For what concerns the MTTF one has:

$$MTTF_2 := \tau_{12} + \tau_{22},$$

where

$$\tau_{12} = \frac{(d_2 + u_2)(x_1(0) - \bar{x}_1) + a(x_2(0) - \bar{x}_2)}{(d_1 + u_1)(d_2 + u_2) - ab}$$

$$\tau_{22} = \frac{b(x_1(0) - \bar{x}_1) + (d_1 + u_1)(x_2(0) - \bar{x}_2)}{(d_1 + u_1)(d_2 + u_2) - ab}$$

are respectively the expected time spent waiting to enter in the warehouse, and the expected time spent waiting to enter in function, in this case.

The expected cost, with obvious meaning of symbols, is:

$$C_2 = C_{A2} + \frac{(C_{M2} \cdot \tau_{M2} - C_{U2} \cdot \tau_{U2} + C_{R2} \cdot \tau_{R2} + C_{N2}) \cdot t}{\tau_{M2} + \tau_{U2} + \tau_{R2}}$$

For what concerns provision dimensioning, if n is the number of devices that must be operating during steady state, then it is necessary to have at least

$$n_2 = \frac{n}{\bar{x}_2} = n + \frac{\bar{x}_1}{\bar{x}_2}n + \frac{\bar{x}_3}{\bar{x}_2}n$$

devices.

3.3 Model 3 for filter lifecycle

Near to standard filters, there are also redundant ones. The redundant device has two membranes, supported by two inner frames that are able to rotate around a pivot, in such a way to be changed. During operations the upper membrane is used before of the lower one. However, the membrane substitution is done in pair.

To reduce death time, after an optical check and thanks to his knowledge of the following workings for that production line, the operator decides, in case of flux variation through the upper membrane, if it is necessary to pass to the lower membrane, or to immerse in solvent the membrane pair, or, finally, to send the pair in repair for damage of the upper membrane. Otherwise, if the flux variation happens through the lower membrane, thing that implies at least an obstruction of the upper membrane, the operator can decide to send the membrane pair to the warehouse to do the cleaning, or to send them to the repairing laboratory if a damage has occurred.

What follows in the management of the redundant device is done as for model 1, included the eventuality that one or both the components of a membrane pair can be detected as damaged only after the solvent bath is finished.

The graph depicting the lifecycle of such a device is shown in Fig. 2.

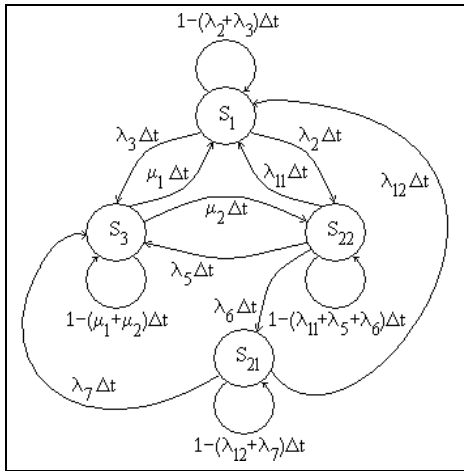


Fig. 2: Lifecycle of a redundant filter

With respect to Fig. 1 there are some changes. Instead of a single state, S_2 , there are:

S_{22} : state representing the device with the upper membrane in operation, its corresponding probability is $x_{22}(t)$,

S_{21} : state representing the device with the lower membrane in operation, its corresponding probability is $x_{21}(t)$.

Moreover, the following transition rates, different from the others, complete the model:

λ_{11} : rate of storage, starting from a functioning upper membrane, of a redundant filter,

λ_{12} : rate of storage, starting from a functioning lower membrane, of a redundant filter,

λ_5 : failure rate, starting from a functioning upper membrane, of redundant filter,

λ_6 : transition rate from upper to lower functioning membrane,

λ_7 : failure rate, starting from a functioning lower membrane, of redundant filter.

The (Kolmogoroff's) equations are:

$$d\tilde{x}(t)/dt = \tilde{A}\tilde{x}(t) + \tilde{\mu} \quad (6)$$

with

$$\tilde{A} = \begin{pmatrix} \bar{A} & -M \\ h & -d_3 \end{pmatrix}; \quad \tilde{\mu} = \begin{pmatrix} \mu \\ 0 \end{pmatrix}; \quad \tilde{x}(t) = \begin{pmatrix} x_1(t) \\ \tilde{x}_2(t) \end{pmatrix}; \quad \tilde{x}_2(t) = \begin{pmatrix} x_{22}(t) \\ x_{21}(t) \end{pmatrix}$$

$$\bar{A} = \begin{pmatrix} -d_1 & \tilde{a} \\ b & -\tilde{d}_2 \end{pmatrix}; \quad M = \begin{pmatrix} \mu_1 - \lambda_{12} \\ \mu_2 \end{pmatrix}; \quad h = (0 \quad \lambda_6); \quad d_3 = \lambda_{12} + \lambda_7$$

$$\tilde{d}_2 = \lambda_{11} + \lambda_5 + \lambda_6 + \mu_2; \quad \tilde{a} = \lambda_{11} - \mu_1.$$

Defining reliability $R_3(t)$, as in the previous case, one has:

$$R_3(t) := 1 - x_3(t) = \tilde{C} x(t), \quad \text{with } \tilde{C} = (1 \quad 1 \quad 1)$$

(in fact, $x_3(t) = 1 - x_1(t) - x_{22}(t) - x_{21}(t)$ in such case), then:

$$R_3(\infty) = x_1(\infty) + x_{22}(\infty) + x_{21}(\infty),$$

$$x_1(\infty) = \frac{\bar{d}_2 \mu_1 + \bar{a} \mu_2}{d_1 \bar{d}_2 - \bar{a} b},$$

$$x_{22}(\infty) = \frac{b \mu_1 + d_1 \mu_2}{d_1 \bar{d}_2 - \bar{a} b},$$

$$x_{21}(\infty) = \frac{\lambda_6}{d_3} \frac{b \mu_1 + d_1 \mu_2}{d_1 \bar{d}_2 - \bar{a} b},$$

$$\bar{d}_2 = \tilde{d}_2 + \frac{\lambda_6}{d_3} \mu_2, \quad \bar{a} = \tilde{a} + \frac{\lambda_6}{d_3} (\lambda_{12} - \mu_1).$$

For what concerns the MTTF one has:

$$MTTF_3 := \tau_{13} + \tau_{23},$$

where

$$\tau_{13} = \frac{\bar{d}_2 d_3 (x_1(0) - x_1(\infty)) + \bar{a} d_3 (x_{22}(0) - x_{22}(\infty)) + d (x_{21}(0) - x_{21}(\infty))}{d_3 (d_1 \bar{d}_2 - \bar{a} b)}$$

$$\tau_{23} = \frac{\bar{b} d_3 (x_1(0) - x_1(\infty)) + d_1 d_3 (x_{22}(0) - x_{22}(\infty)) + \bar{b} (x_{21}(0) - x_{21}(\infty))}{d_3 (d_1 \bar{d}_2 - \bar{a} b)}$$

$$d = \tilde{d}_2 (\mu_1 - \lambda_{12}) + \tilde{a} \mu_2,$$

$$\bar{d}_3 = d_3 + \lambda_6,$$

$$\bar{b} = d_1 (\tilde{d}_2 - \mu_2) - (\tilde{a} + \mu_1 - \lambda_{12}) b$$

are respectively the expected time spent waiting to enter in the warehouse, and the expected time spent waiting to enter in function, in this case.

The expected cost, with obvious meaning of symbols, is:

$$C_3 = C_{A3} + \frac{(C_{M3} \cdot \tau_{M3} - C_{U3} \cdot \tau_{U3} + C_{R3} \cdot \tau_{R3} + C_{N3}) \cdot t}{\tau_{M3} + \tau_{U3} + \tau_{R3}}$$

For what concerns provision dimensioning, if n is the number of devices that must be operating during steady state, then it is necessary to have at least

$$n_3 = \frac{n}{x_{22}(\infty) + x_{21}(\infty)} = n + \frac{x_1(\infty)}{x_{22}(\infty) + x_{21}(\infty)} n + \frac{x_3(\infty)}{x_{22}(\infty) + x_{21}(\infty)} n$$

membranes.

4 Numerical cost evaluation

A numerical simulation has been done with values derived from a real plant, to compare the cost indexes of three management methods.

For what concerns model 1 (passive management) one has the following parameter values:

Parameter	Transitions/month
λ_1	1.56250
λ_2	1.87500
λ_3	0.93750
λ_4	0.93750
μ_1	1.25000
μ_2	0.62500

The cost index is shown in Fig. 3 (a negative value of the index means that there is a profit: when the index is zero it means that one has repaid the initial investment due to the device purchase). Note that the amortization of the single device is reached after 4 months. At steady state it is advisable to have the disposal of $n_1=1500$ devices, to face up the operation of 500 devices.

For what concerns model 2 (active management), besides the values listed for model 1 one has the following further values, obtained from (4):

Parameter	Transitions/month
u_1	1.25000
u_2	0.62500

The cost index is shown in Fig. 3. Note that the amortization of the single device is reached after 10 days. At steady state it is advisable to have the disposal of $n_2=2000$, to face up the operation of 500 devices.

For what concerns model 3 (management of redundant devices), besides the values listed for model 1 one has the following further values:

Parameter	Transitions/month
λ_{11}	0.78125
λ_{12}	0.78125
λ_5	0.46875
λ_6	1.25000
λ_7	0.46875

The cost index is shown in Fig. 3. Si noti che l'indice è sempre in perdita. At steady state it is advisable to have the disposal of $n_3=1000$, to face up the operation of 500 devices.

The cost indexes of Fig. 3 are normalised with respect to C_A , the cost of a single membrane, and are based on the following table:

Costs of model 1	Costs of model 2	Costs of model 3
$C_{A1} = C_A$	$C_{A2} = C_A$	$C_{A3} = 2 C_A$
$C_{M1} = 0.01 C_A$	$C_{M2} = 0.01 C_A$	$C_{M3} = 0.01 C_A$
$C_{U1} = 10 C_A$	$C_{U2} = 10 C_A$	$C_{U3} = 10 C_A$
$C_{R1} = 0.1 C_A$	$C_{R2} = 0.1 C_A$	$C_{R3} = 0.1 C_A$
$C_{N1} = 0.5 C_A$	$C_{N2} = 0.05 C_A$	$C_{N3} = 0.5 C_A$

Note that the difference between model 1 and model 2 lies on the cost of the manutentive interventions, that in the second case is reduced since it relates to preventive interventions. While the difference between model 1

and model 3 lies in the cost of the filter, that is double for the last one.

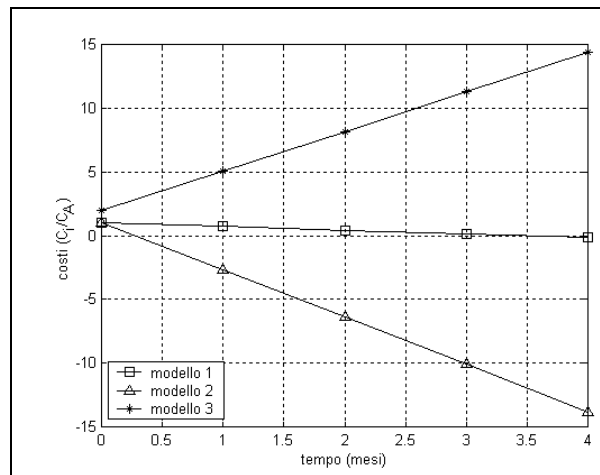


Fig. 3: Cost indexes for the three models

Conclusion

One of the more important logistic problems regards the provision management of devices. The planning of the control checks is settled by safety standards in such a way to be correlated to the expected mean time to failure (MTTF).

Thanks to a probabilistic model ([5]-[7]) of the lifecycle of a repairable device, it is possible to implement a control strategy that modifies the MTTF, to reduce the rate of the corrective maintenance interventions.

The control law proposed modifies some probabilities by means of the random repetition of checks with established distribution.

The increment of the number of checks to be done in the time unit characterises an increment of expenses, but reduces the necessity of corrective maintenance, whose interventions are more expensive of the preventive ones, thus obtaining in a long period an effective thrift. Moreover, organisational problems that could rise by increasing the reparation rates are thus avoided.

The meaning of check rate is that of inverse of mean time between checks ([6]-[7]), thing that permits to schedule a plan of the checks.

What just stated can be proved by the comparison of the cost indexes concerning the three management methods described in section 3, as shown in section 4.

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