Design of a Hysteresis Chaotic Circuit

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Abstract: - A new design of a hysteresis chaotic circuit is proposed. The simple circuit consists of one capacitor, one inductor, one resistor (negative) and one hysteresis VCVS. Computer simulation results, showing the existence of the double scroll attractor, are reported.

Key-Words: - Chaos, chaotic circuit, hysteresis, attractor, bifurcation, oscillator

1. Introduction

The design of simple chaotic circuits is very important not only to understand the nature of chaos but also to develop sophisticated applications that actually use this behavior. For example, the Chua circuit [1], the Colpitts oscillator [2] and the hysteresis chaos generator [3] are among the circuits in which a variety of dynamical behaviors including chaos have been reported.

In this paper, we propose a new and simple design of a hysteresis chaotic circuit. The circuit is designed by using only one capacitor, one inductor, one negative resistor and one hysteresis voltage controlled voltage source VCVS. It is well known that at least three states are required to generate chaos in a autonomous system. In the proposed circuit the capacitor and the inductor provide two states and the hysteresis corresponds to the third state. The stretching process is achieved by the presence of the negative resistance while the folding mechanism is realized by the hysteresis VCVS. As in the case of the Chua's circuit chip [4], the implementation of our circuit in a single chip is realizable.

In the following sections, the design procedure of the proposed circuit is reported. Both simulation results using Matlab Simulink for the dynamical equations, and Electronics Workbench for the circuit implementation, are presented. The existence of the double scroll strange attractor is shown.

2. Circuit Design

Fig. 1 shows a simple design of the hysteresis chaotic circuit.

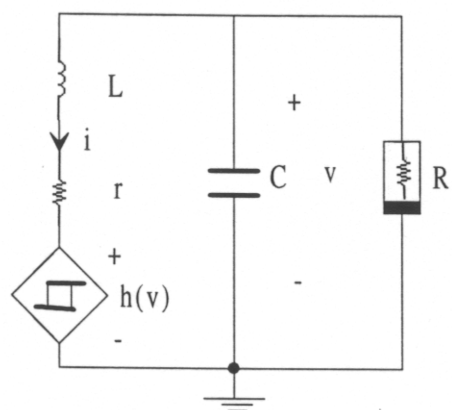

Fig.1. Hysteresis chaotic circuit

We focus on the case where the resistance R is negative. Applying KCL and KVL, the state equations of the autonomous circuit are:

$$\begin{bmatrix} C\dot{v} \\ L\dot{i} \end{bmatrix} = \begin{bmatrix} -1/R & -1 \\ 1 & -r \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ h(v) \end{bmatrix} \tag{1}$$

Where

$$h(v) = \begin{cases} V_0 & for\; v \geq -E \\ -V_0 & for\; v \leq \;\; E \end{cases} \tag{2}$$

The hysteresis function $h(v)$ is shown in Fig. 2.

[1] This work was supported by the Louisiana BORSF contract No. LEQSF(1999-2001)-RD-A-47

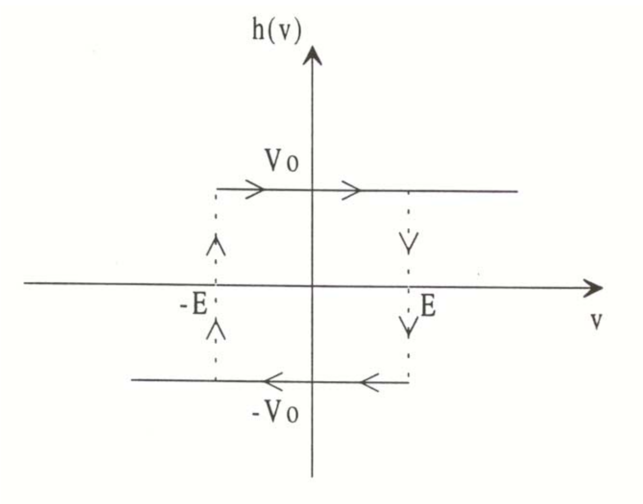

Fig.2. Hysteresis function.

When v hits the right threshold value, $h(v)$ switches to V_o and when v hits the left threshold value, $h(v)$ switches to $-V_o$. The resistance R is negative within a range of values of v and supplies energy to the rLC part of the circuit. This will lead to the stretching mechanism (i.e. the voltage v and current i amplitudes will increase). Then, the hysteresis jump provides the folding mechanism if V_o is within the range of v in which R is negative.

For simplicity, to determine the parameters values to generate chaos, we use the following dynamical system. These equations are not the dimensionless system of (1).

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} a & -1 \\ 1 & -b \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ f(x) \end{bmatrix} \tag{3}$$

Where

$$f(x) = \begin{cases} 1 & for\ x \geq -1 \\ -1 & for\ x \leq \ 1 \end{cases} \tag{4}$$

a, and b are positive, $f(x)$ is a hysteresis function which switches from 1 to -1 if x hits -1 and vice versa. In order for the stretching mechanism to occur, the eigenvalues λ = σ+jω of system (3) must be complex with positive real part if the following holds.

$$\begin{cases} b < a \\ and \\ -2-a < b < 2-a \end{cases} \tag{5}$$

In this case, $\sigma = \frac{a-b}{2}$ and $\omega = \frac{\sqrt{4-(a+b)^2}}{2}$.

There are two unstable equilibrium points located at $Q^+ = \left(\frac{1}{1-ab}, \frac{a}{1-a}\right)$ and $Q^+ = \left(\frac{-1}{1-ab}, \frac{-a}{1-a}\right)$.

Fig.3 shows the block diagram of the hysteresis chaotic system described by (3).

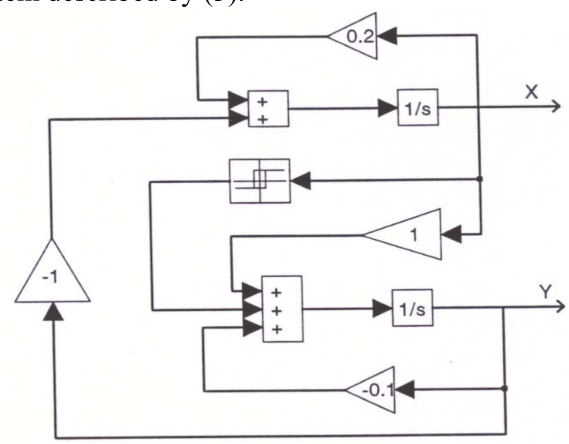

Fig.3. Block diagram of the hysteresis chaotic system

The simulation of the dynamical system (3) using Matlab Simulink [5] displays a chaotic attractor as shown by the phase portrait of Fig. 4. The following parameters are selected a=0.2, b=0.1 and the initial conditions are $x(0) = 0.5$ and $y(0) = 0$. The reader is referred to [3, 6] for more details about the sufficient condition for chaos generation. In the next section, we describe the implementation of system (1).

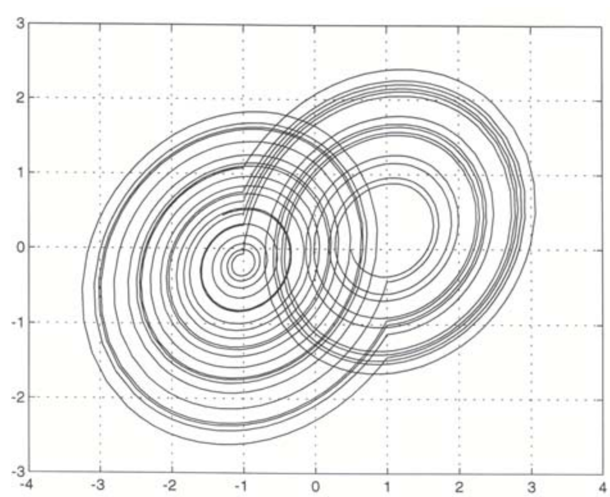

Fig.4. Chaotic attractor (a=0.2 and b=0.1), ($horizonal\ axis$: x; $vertical\ axis$: y).

3. Circuit Implementation

The proposed hysteresis chaotic circuit is shown in Fig. 5. It consists of three blocks: The block B1 is the rLC part of the circuit. Given an initial condition, the rLC circuit will oscillate. However, due to the loss of energy in r these oscillations will not sustain. For the capacitor voltage and the inductor current to oscillate divergently, energy must be provided to this part of the circuit. This is achieved through the use block B2 implementing a negative resistor and consisting of R_1, R_2,R_3 and opamp.

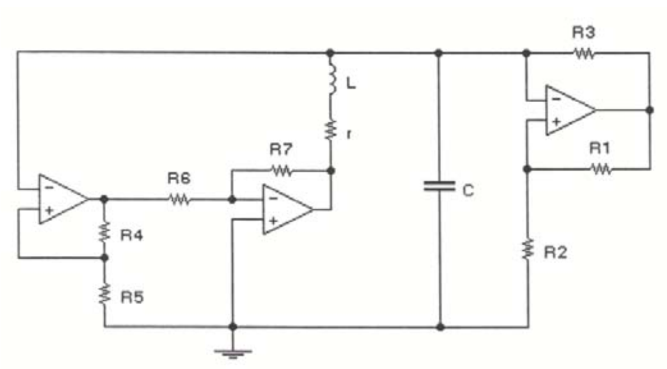

Fig. 5. Implementation of the hysteresis chaotic circuit

Block B2 can described by the following equations [7].

$$i_R = \begin{cases} \frac{1}{R_3}(v - V_{sat}) & if \quad v < \beta V_{sat} \\ -\frac{R_1}{R_2 R_3} v & if \quad -\beta V_{sat} < v < \beta V_{sat} \\ \frac{1}{R_3}(v + V_{sat}) & if \quad v > -\beta V_{sat} \end{cases} \quad (6)$$

Where $\beta = R_2 / (R_1 + R_2)$. Block B2 is used only in the case of a negative resistance where $R = \left(-R_1 / R_2 R_3\right)^{-1}$ $for\ -\beta V_{sat} < v < \beta V_{sat}$. V_{sat} is the operational amplifier saturation voltage. Block B3, consisting of R_4, R_5, R_6, R_7 and two opamps, implements the nonlinear hysteresis function. The output V_h is described as:

$$V_h = \begin{cases} +V_{sat} & if \quad V_{in} \geq V_{LT} \\ -V_{sat} & if \quad V_{in} \leq V_{UT} \end{cases} \quad (7)$$

Where the lower threshold voltage V_{LT} and the upper threshold voltage V_{UT} are related to the operational amplifier saturation voltage V_{sat} by the following equations

$$V_{LT} = \frac{R_5}{R_4 + R_5}(V_{sat}) \ and \ V_{UT} = \frac{R_5}{R_4 + R_5}(-V_{sat}) \quad (8)$$

V_{in} is the input threshold voltage which is set to 1V in our case. V_{sat} is determined by the power supplies and internal structure of the opamps. The inverting amplifier is used to adjust the voltage V_h and change the hysteresis direction.

$$h(v) = -\frac{R_7}{R_6} V_h \quad (9)$$

In order to implement equations (1) and (2) using the proposed circuit, the component parameters must be set appropriately using only the given value of a and b. Since the voltages in the proposed circuit are measured in Volts and currents in milliAmperes, it is important to rescale all currents by a factor k_1. The effect is to reduce the capacitance by a factor k_1 and to increase resistances by the same factor. Since capacitors in the order of nF are easier to use and more available, then a time rescaling by a factor k_2 is needed. The effect is to reduce the capacitance by a factor k_2; resistances are not affected by a time scaling. In our case we have selected $k_1 = 10^3 ; k_2 = 2 \times 10^6$. In accordance with previous design consideration, suitable values for the circuit components are :

C = 0.5 nF, L = 0.5 mH, $R_1 = R_2$ = 1 kΩ, R3 = 5 kΩ, R_4= 14 kΩ, R_5 = 1kΩ, R_6 = 100 kΩ,, R_7 = 10 kΩ

The results of the Electronic Workbench [8] simulation, the closest to designing real circuits, are shown in Fig.6 (r=100 Ω), Fig.7 (r=180 Ω), and Fig.8 (r=220 Ω, a sink equilibrium point). In these entire figures, the horizontal axis represents the voltage across the capacitor v_c and the vertical axis represents the voltage $v_y = ri + h(v)$.

4. Conclusion

In this paper, a new and simple realization of a hysteresis chaotic circuit was shown. The proposed design uses only one capacitor, one inductor, one negative resistor and a hysteresis VCVS. The circuit was simulated using Electronics Workbench, and it shows an existing attractor. Next we will try to study higher dimensional chaos.

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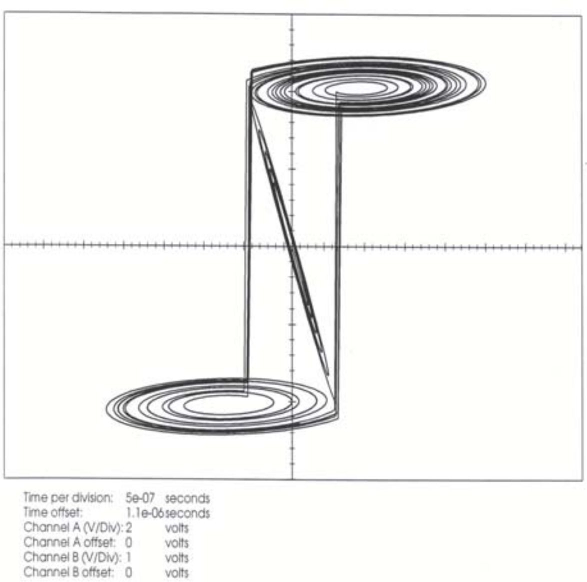

Fig. 6. The simulated attractor of the hysteresis circuit for r = 100Ω.

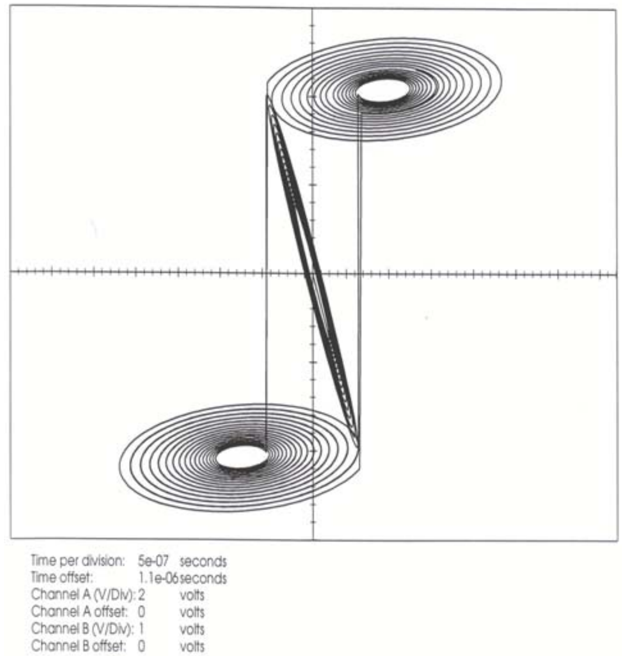

Fig. 7. The simulated attractor of the hysteresis circuit for r = 180Ω.

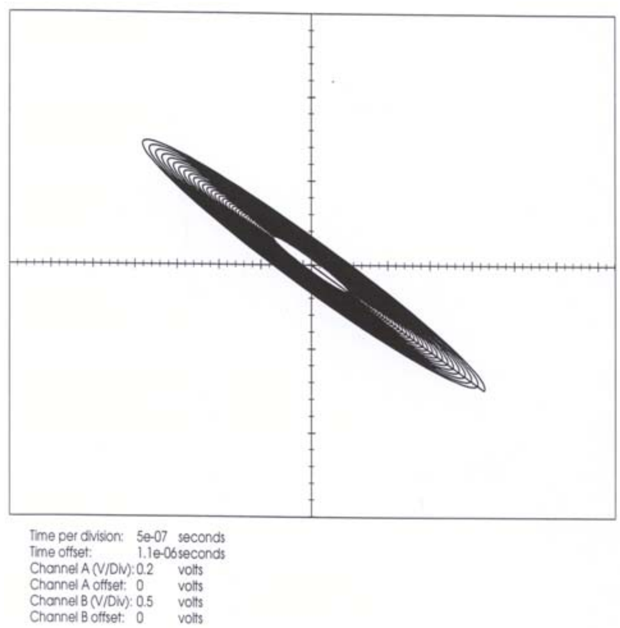

Fig. 8. The simulated attractor of the hysteresis circuit for r = 220Ω.