Proposal for a New Sub-Optimal Decoding Algorithm for Block Codes in 8-PSK and 16-PSK Constellations

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Abstract: - This is an extension algorithm previously presented in [1] for decoded modulation systems with block codes for 8-PSK and 16-PSK constellations. The proposed algorithm presents a search mechanism of the least reliable points from the first code array and generate vectors for the Modified BCM, ZNA decoder, described in [1]. It is also presented a mapping of the 8-PSK and 16-PSK constellations in BPSK so the Modified ZNA could get the relevant results. Finally computer simulated results are compared for the 8-PSK and 16-PSK constellations, acquired from the sub-optimal algorithm proposed (Viterbi for BCM and MLD).

Key Words: - Block Codes; Data Communication; Decoding; Error-Correcting Codes; Communication Systems; ZNA.

1 Introduction

One of the top technologies today in communication systems for error correcting in a data transmission channel are turbo codes. These codes provide a good signal to noise ratio, but they might increase delay in processing (due to the interleaver size, for example). This could harm synchronized communications such as voice and videoconferences [7]. This may not be an ideal solution for users with a low-data-rate voice transmissions in a simple channel like compact radios. The performance of communication systems can be optimized taking a modulation with a higher number of points, keeping the information transmission rate and average energy of the system by combining the modulation and decoding operations, thus called decoded modulation. According to the previous explanation, the decoded modulation concept is of great importance for the development of reliable data transmission systems, since it allows reaching the desired performance without increasing the required transmission power or bandwidth. This assures a more efficient use of the resources. Since 1982, the research involving decoded modulation has been a respected issue from the revolutionary method presented by Ungerboeck [3] to enhance digital transmitting efficiency. This technique basically allows getting significant decoding gain on top of a conventional digital modulation set up (not decoded), without the expansion of the spectral occupation or the decreasing of the original data information to an intermediate level transmitting power. The technique presented in [3] used a modulation method with convolutional codes, known as group partition mapping. The group partition mapping was later used in [4] to present a modulation design using a block code class in space of signals for QAM constellations (*Quadrature Amplitude Modulation*). The M-PSK scheme was done in [2], showing the capability and flexibility potentials that the BCM (*Block Coded Modulation*) design can offer. The designs in [4] and [2] show performances akin to the ones obtained from the technique in [3]. Moreover, within determined limits, the block coding avoids decoding delays and unrestrained propagation risks from errors associated to convolutional codes. The traditional way for BCM decoding design is based in the Maximum likelihood algorithms (MLD) [5]. The TCM (*Trellis Coded Modulation*) designs, opposite of BCMs, attained great progress through the technique demonstrated in [3] or even similar techniques, also became more popular, apparently to its relatively low decoding complexity. For this reason, techniques for BCM designs have been studied with the purpose to reduce complexity. A method to reduce decoding complexity for the BPSK design was presented in [1]. This method is based in the adding rule (an operator defined in [1]) and tries to identify when a vector is in the *Voronoi* Region of the zero codeword with the help of a limiter proposed in [1].

Sayegh in [2] proposed a decoding system based on the reliability of the transmitted symbol in a constellation, where the first bit of the transmitted symbol is more susceptible to noise interference than the others. Because of that, a more resistant code was suggested for the first code of the array.

The purpose of this effort is to present a soft decoding algorithm for block codes in 8-PSK and 16-PSK constellations that provides a reduction in the decoding processing time, and a correction performance next to ideal (MLD or Viterbi [6] for BCM) for digital data transmission in a communication channel. Also, results are shown with according to its implementation in the study of cases.

2 Coding

Sayegh in [2] proposed the construction of arrays made by block codes with the intention to obtain analog results to the ones acquired with convolutional codes [3].

In the design presented in [2] a matrix made from several binary codes (code array) is shown in Fig. 1.

The code array is transmitted column by column. Each column is represented by a point in the constellation in a way that the first digit in the first row corresponds to the digit more to the left in the constellation. The decoding rate in the array is the sum of all the code rates in the array. Here is the mathematical representation:

$$R_c = \frac{\sum_{i=1}^{L} k_i}{L \cdot n} \qquad (1)$$

where $0 < k_i \leq n$, i = 1, ..., L, n is the number of columns and L is the number of rows.

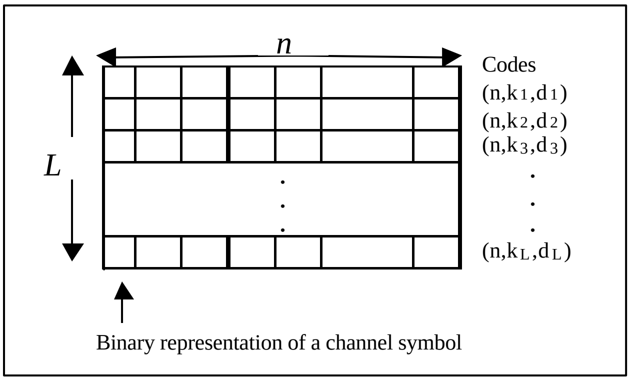

Fig. 1 – Binary array of L rows and n columns.

3 Decoding Specification

The sub-optimal decoding process proposed was developed for 8-PSK and 16-PSK modulations using the method shown in [1].

The proposed algorithm tries to select the least reliable points using hard decision decoding. Since the first array in the code is more consistent than the others, it has more correction power than the other codes [2]. We consider that the corrected point in this first code to be the least reliable and the candidate vectors are generated from it, this way the Modified ZNA (*Zero Neighbors Algorithm*) decoder is able to find the codeword corresponding to the second code in the array. In order to take a broad view of the Modified ZNA decoder for other constellations, it's necessary to convert the received points to the BPSK modulation. This is because this algorithm (Modified ZNA) was developed for BPSK modulation and uses a one-dimensional vector as an input parameter to analyze the received points. To perform the conversion, the following equation is used in the proposed algorithm:

$$coord = \begin{cases} \max\{\text{abs}(X), \text{abs}(Y)\} & \text{if symbol } 2^{nd} \text{ bit} = 1; \\ -\max\{\text{abs}(X), \text{abs}(Y)\} & \text{if symbol } 2^{nd} \text{ bit} = 0; \end{cases} \qquad (2)$$

where *coord* is the coordinate value that represents the symbol in the one-dimensional vector, *abs* is the absolute value and X and Y are the received point coordinates.

Since the mapping for BPSK is done like this: 0 $\rightarrow$ -1 and 1 $\rightarrow$ 1, thus the received vector is confined between {-1,+1}.

The equation 2 uses the received coordinates, taking as a result the nearest value of –1 for bit 0 and 1 for bit 1 (absolute value). If the second bit of the symbol is 1 then this value has to be positive, if not (zero) is negative. This conversion is only for the second bit of the symbol for 8-PSK as well as for 16-PSK. That's because the second bit of the symbol is part of the second code of the array and this is where the Modified ZNA is going to act.

Example: Consider the s_0 (8-PSK) point that represents the 1**0**1 symbol and with received coordinates (-0.9,0.2). The bit that interests us in the symbol is the central one, which is zero (0), and that in BPSK is modulated by –1, therefore the equation 2 is used to obtain the value that represents the one-dimensional symbol.

coord = - max {abs(-0.9), abs(0.2)}
coord = - max {0.9, 0.3}
coord = - 0.9

Once we have the one-dimensional vector, the candidate vectors are generated from the combination of the least reliable points and their opposite. The other points that are not part of the least reliable points are copied to the candidate vector without change (the points already must be mapped for the BPSK modulation). The number of vectors varies according to the amount of corrected errors in the first code (Table 1). After the entire candidate vector had been generated, each one of them is decoded by the Modified ZNA decoder [1]. Then a candidate arrangement code vector is generated from the corrected points (Hard Decision and Modified ZNA) and the Minimum Euclidean distance is used between the received array code and the candidates of the array code to select the array that will be codeword corrected.

3.1 Least Reliable Points

The least reliable points vary according the correction capability of the first code. From these points are generated candidate vectors that consist of a combination of the least reliable points and their opposites. The number of combinations can be represented by the following equation:

$$\text{Combinations} = 2^t \qquad (3)$$

where t is the number of errors detected and corrected by the first code. The combinations for the code with correction capability of up to three errors are shown on Table 1.

Table 1 – Combinations by number of errors found in the first code array.

T = 1	*T=2*	*T=3*
+ Cood	*+ Cood1, + Cood2*	*+ Cood1, + Cood2, + Cood3*
- Cood	*- Cood1, + Cood2*	*- Cood1, + Cood2, + Cood3*
	+ Cood1, - Cood2	*+ Cood1, - Cood2, + Cood3*
	- Cood1, - Cood2	*+ Cood1, + Cood2, - Cood3*
		+ Cood1, - Cood2, - Cood3
		- Cood1, + Cood2, - Cood3
		- Cood1, - Cood2, + Cood3
		- Cood1, - Cood2, - Cood3

Cood is the coordinate generated by 2. The number in the table indicates the least reliable point used.

3.2 Algorithm Steps

1. Apply abrupt decoding in the first code;
2. For the following code of the array you must:
   a. Map the received points for the BPSK modulation. The mapping is based in the constellation used;
   b. Choose the least reliable points. The selection of the points is done from the corrected points in the first code of the array;
   c. Generate the candidate vectors that consist of the combination of the least reliable points and their opposites;
   d. Apply the Modified ZNA decoder for each of the generated vectors to compose candidate codewords and vector points supposedly corrected.
3. After composing all the valid candidate codewords, generate the array candidates. The other codes belonging to the array are formed from the new points corrected by the two decoders (Hard Decision and Modified ZNA);
4. Apply the Euclidean Distance to select the correct codeword array.

The valid candidate words are the ones that the ZNA decoder corrected and only the least reliable points.

3.3 Decoding Example

This section presents a decoding example using the proposed algorithm for the 8-PSK modulations. For the example a rate $r = 2/3$ was used. The arrangement is made up by the codes described in Table 2.

Table 3 presents the distribution of points in the 8-PSK constellations.

Table 2 - Codeword array.

Codes
C(7,1,7)
C(7,6,2)
C(7,7,1)

Table 3 – Symbols and their equivalent points.

Bits	Coordinates
101	(-1, 0)
001	(-0.707, 0.707)
111	(0, 1)
011	(0.707, 0.707)
100	(1, 0)
000	(0.707, -0.707)
110	(0, -1)
010	(-0.707, -0.707)

Codeword transmitted:

111111110001000001001

Coordinates transmitted:

{(0, -1), (1, 0), (1, 0), (-1, 0), (0, -1), (1, 0), (-1, 0)}

Coordinates received:

{(0.27, -0.8), (0.98, -0.1), (0.62, -0.31), (-0.35, 0.09), (0.02, -1.23), (0.56, 0.25), (-1.04, 0.03)}

Codeword received:

1101101 1000110 0001011

The algorithm is started applying the abrupt decoding (hard decision decoding) for the first code in the arrangement and obtaining the codeword:

1111111

The mapping for the received points is done for the BPSK modulation for the second code of the array. Table 3 and equation 2 are used to convert the coordinates of the received vector, e.g. (0.27,-0.8) ⇒ 1**1**0. The second bit is the one to be mapped (1), as verified according to its representation in the constellation (according to Table 3); in the BPSK modulation, bit 1 is modulated as 1 according to equation 2: coord = max {abs(0.27), abs(-0.8)} = 0.8.

Mapping the entire received vector, results in:

{0.8, -0.98, -0.62, -0.35, 1.23, 0.56, -1.04}

To find the least reliable points, we must choose the points that diverge from the word received of the first code with the corrected word by the abrupt decoding in the example below:

- Received codeword of the first code: (1 1 **0** 1 1 **0** 1)
- Corrected codeword of the first code: (1 1 **1** 1 1 **1** 1)

Since there are two diverging points in the example (3^{rd} and 6^{th}), a combination between the least reliable points must be done (3^{rd} and 6^{th}) and their opposites (Table 1, t = 2, Combinations = 2^2 = 4). Cood1 = - 0,62 and Cood2 = 0,56.

The results of all combinations are described in Table 4.

Table 4 – Resulting vectors from the combination.

Candidate Vectors
*{0.8, -0.98, **-0.62**, -0.35, 1.23, **0.56**, -1.04}*
*{0.8, -0.98, **0.62**, -0.35, 1.23, **0.56**, -1.04}*
*{0.8, -0.98, **-0.62**, -0.35, 1.23, **-0.56**, -1.04}*
*{0.8, -0.98, **0.62**, -0.35, 1.23, **-0.56**, -1.04}*

Now with all the candidate vectors, the Modified ZNA algorithm must be applied for each of the generated vectors (Table 4). The resulting candidate codewords are shown in Table 5.

Table 5 – Candidate codewords generated by the Modified ZNA algorithm.

Candidate Codewords
1001110
1000100
1000100
1011100

The third code of the array (not coded) is made up from new corrected points by the decoders (Hard Decision and Modified ZNA). Next, the grouping between the generated codes is done. The generated groupings are shown in Table 6. In this example four array codewords are formed because two errors were detected in the first code (Table 1).

Table 6 – Candidate words.

Candidate Arrays
1111111 1000111 0111100
1111111 1000100 0001001
1111111 1000100 0001001
1111111 1011100 1101001

Finally, we select the codeword array with the smaller Euclidean Distance between generated point vectors and the received vector.

Corrected codeword array:

111111110001000001001

4 Results

The specified decoding system through the demonstrated algorithm was simulated in a computer to obtain the performance curves. The computer used was a K6-II 350 MHZ, 64 MB of memory and a 20 GB HD.

Fig. 2 shows the performance curve of the 8-PSK modulation for the proposed system and coding rate *r* = 2/3, compared with the curves for the 4-PSK modulation (not coded) and the MLD algorithm for 8-PSK. The curve for the proposed algorithm follows the 4-PSK curve close to 4 dB. When it's simulated from 6 to 8 dB, presents a higher gain over the 4-PSK modulation (not coded), getting close to the curve generated by the MLD algorithm.

Tables 7 and 8 represent the 8-PSK computer simulated results for 10017 transmission bits for each signal to noise connection. The first (Eb/No) column from the tables represents the signal to noise connections, the second column (BER) represents the error rate (BER), the third column (Tp (/s)) represents the time spent in the decoding process for the 10017 bits. The fourth (Non-corr. bits) and the fifth (Non-corr. arr.) columns represent the number of bits and codeword array non-corrected respectively. On Tables 7 and 8 a signal to noise ratio is observed equal to 3 dB, from the 10017 received bits. The algorithm proposed couldn't correctly decode only 7-code array that the MLD is able to decode. For a noise/rate of 6 dB this number drops only to 1 code array below the MLD. It is also observed that the processing time for the 10017 decreases when the signal to noise ratio increases. That's because its adaptability to the presence of noise left by the Modified ZNA Algorithm and the search system in the hard decision decoding of the least reliable points in the first code.

In Fig. 3 the 16-PSK modulation performance curve can be observed for the proposed system with a decoding rate r = 0.625, which is compared to the 8-PSK modulation curves (not coded) and the Viterbi algorithm [6] for BCM (Sayegh coding), with coding and rate r = 1/2. The curve of the proposed algorithm follows the 8-PSK curve close to 12 dB. When it's simulated to 13 and 14 dB presents a higher gain over the 8-PSK modulation (not coded), getting close to the curve generated by the Viterbi algorithm.

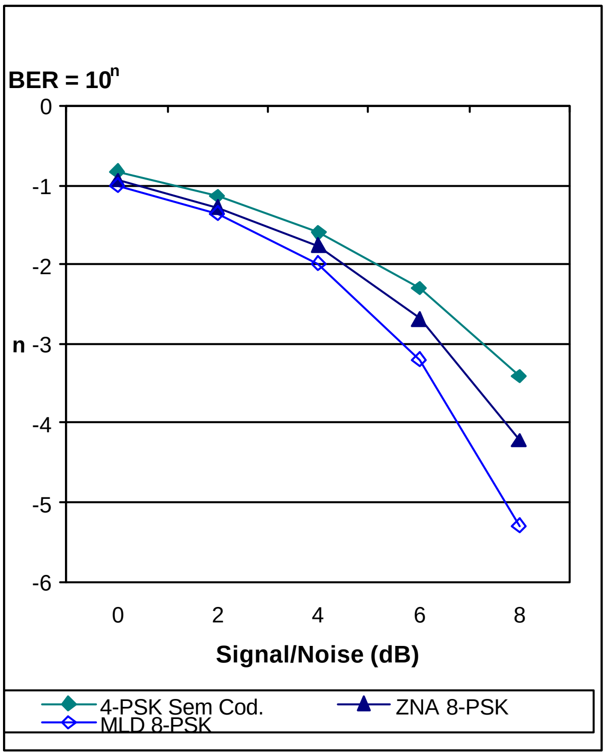

Fig. 2 – Performance of the decoding system for the 8-PSK decoding modulation.

Table 7 – MLD algorithm performance for the 8-PSK modulation.

Eb/No	BER	Tp (/s)	Non-corr. bits	Non-corr. arr.
-----	------------	----------	---------------	--------------
0	0.104023	72	1042	273
1	0.071478	73	716	200
2	0.044724	72	448	25
3	0.026755	73	268	69
4	0.011780	72	118	29
5	0.004792	73	48	10
6	0.001597	72	16	**3**

Table 8 – Proposed algorithm performance for the 8-PSK modulation.

Eb/No	BER	Tp (/s)	Non-corr. bits	Non-corr. arr.
-----	------------	----------	------------	--------------
0	0.117001	4	1172	289
1	0.085754	3	859	215
2	0.055705	3	558	140
3	0.030848	3	309	76
4	0.017271	1	173	40
5	0.007288	1	73	13
6	0.002496	0	25	**4**

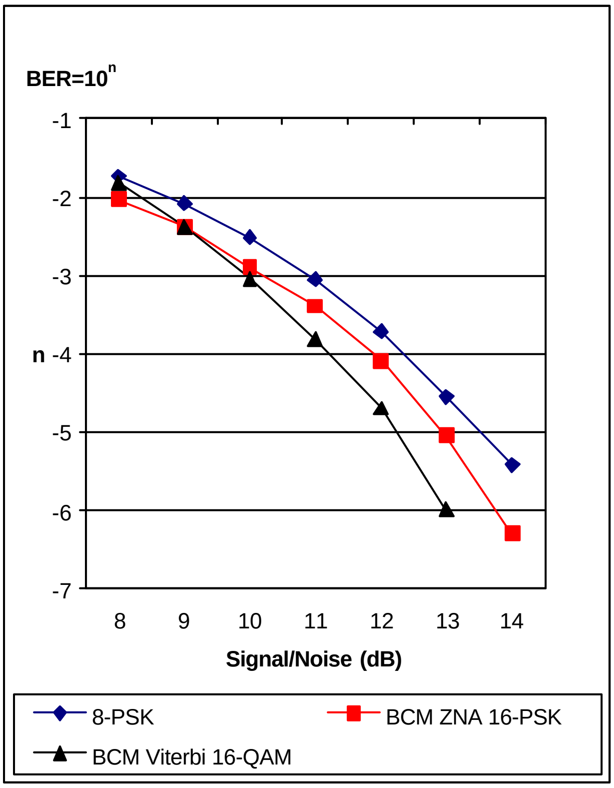

Fig. 3 – Decoding System Performance for the 16-PSK Modulation.

Analyzing both performances, it was observed that the gains in dB for the uncoded modulations are similar to 8-PSK and 16-PSK, this way the algorithm proposed makes formal any generic algorithm presented in [1].

5 Conclusion

This work proposed a new sub-optimal decoding algorithm for block codes in 8-PSK and 16-PSK constellations. This algorithm made use of the coding concept proposed in [2] and used the Modified ZNA Algorithm for the decoding [1]. The performance curves were obtained by computer simulation. We showed that the proposed algorithm has sub-optimal near to MLD (8-PSK) and Viterbi [6] for BCM (16-PSK). The advantage is in its decoding velocity. This feature is obtained with the selection of the least reliable points and the use of the Modified ZNA Algorithm [1].

Also, the algorithm presented is an asynchronous decoding algorithm, because the most distorted words by noise need a larger processing time to be decoded compared to the words less affected by the noise.

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