Digital ICD Control with Applications to Submarine Depth Control

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Abstract: Based on the ICD (Individual channel Design) approach, the design of a multivariable digital submarine depth control system is presented. The model of the submarine is the standard 80 meters British submarine. As expected, the frequency rate of sampling introduces limitations to achieve the required responses. Furthermore, the discrete model presents zeros close to imaginary axis resulting in a control system with bandwidths under the specified requirements. Nevertheless, it is found that the digital ICD, similar to it's continuous counterpart, is a useful approach to analyse and design multivariable digital control systems in a transparent way, that is, in a way in which the limitations and possibilities to meet specifications can be elucidated prior to the design of any controller. In particular, the controller designed hereby was designed following a classical control approach.

Key-Words: multivariable control, digital control, robustness, submarine applications

1 Introduction

The design of multivariable control systems from an engineering point of view -in a way in which the possibilities and limitations of the plant can be elucidated prior to control design and corrections or adjustments to the controller is transparent- is a problem which can be tackled by the ICD approach. The cornerstone of the ICD is the so called Multivariable Structure Function γ(s). By means of γ(s) the robustness structure of the system: number of RHPP's and RHPZ's, and the level of coupling can be determined. In previous papers [1,2,3] was found that ICD is more a framework of analysis than design for multivariable control systems. It is an approach that also defines the necessary conditions for robustness based on the classical phase and gain margins but for highly couple multivariable systems. Also, under the ICD approach it has been proved that it is no always necessary the use of full matrix controllers, moreover if the Multivariable Structure Function γ(s) of the process is close to the point (1,0) it may result in a non robust control system. Due to the advantages digital control has it is necessary to explore how ICD can be used in the design of multivariable digital control systems. In [2] the design of a continuous control system for the submarine based on the ICD approach which meets robustness and performance gives an insight to the possibilities to design a discrete version.

2 Submarine Model

The model of the submarine, taken from [2], is given by the following matrix transfer function G(s):

$$\begin{bmatrix} z(s) \\ \theta \end{bmatrix} = G(s) \begin{bmatrix} \delta_B(S) \\ \delta_S(S) \end{bmatrix} \qquad (1)$$

where:

Z(s): represents the depth of the submarine,
θ: the heading angle,
$\delta_B(S)$: angle of the prow hydroplane,
$\delta_S(S)$: angle of the stern hydroplane, and

the matrix transfer function $G(s)$ is given by:

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \qquad (2)$$

where the individual transfer functions are given by:

$$
\begin{aligned}
g_{11}(s) &= \frac{-0.0076(s+0.5461)(s+0.0494)}{s(s+0.0629)(s+0.0336\pm 0.0472j)} \\
g_{12}(s) &= \frac{-0.0229(s+0.0604)(s-0.1815)}{s(s+0.0629)(s+0.0336\pm 0.0472j)} \\
g_{21}(s) &= \frac{0.00017s(s+0.0306)}{s(s+0.0629)(s+0.0336\pm 0.0472j)} \\
g_{22}(s) &= \frac{-0.0022s(s+0.0556)}{s(s+0.0629)(s+0.0336\pm 0.0472j)}
\end{aligned} \tag{3}
$$

The bandwidth of the individual transfer functions, $g_{ij}(s), i = j = 1,2$, are:

$$
\begin{aligned}
w_{Bg11} &= 0.1660\, rad/\sec \quad ; \quad w_{Bg12} = 0.1816\, rad/\sec \\
w_{Bg21} &= 0.0781\, rad/\sec \quad ; \quad w_{Bg22} = 0.0512\, rad/\sec
\end{aligned}
$$

To calculate the digital model of the submarine, it is necessary to define an appropriate sampling frequency W_M. The sampling frequency W_M can be obtained following the criterion $W_m = [5w_B, 20w_B]$, [4]. Due to the fact that the maximum bandwidth is given by $w_{Bg12} = 0.1816\, rad/\sec$, and because all the individual transfer functions of $G(s)$, in equation (3), have roll off's with slops of –40dB's/dec, an appropriate W_M which includes all the significant dynamics is given by $W_m = 1\, rad/\sec$. Therefore, the resulting sampling period is:

$$
T = \frac{2\pi}{W_m} = 2\pi = 6.28\sec \approx 6\sec.
$$

The discrete matrix transfer function $G(z) = Z\{G(s)\}$ including the ZOH (zero order hold) with sampling period $T = 6\sec$. is given by:

$$
G(z) = \begin{bmatrix} g_{11}(z) & g_{12}(z) \\ g_{21}(z) & g_{22}(z) \end{bmatrix} \tag{4}
$$

where:

$$
\begin{aligned}
g_{11}(z) &= \frac{-0.248(z+1.8025)(z-0.7435)(z+0.0133)}{(z-1)(z-0.6856)(z-0.7849\pm 0.2284j)} \\
g_{12}(z) &= \frac{-0.2223(z-3.0081)(z-0.6960)(z+0.6269)}{(z-1)(z-0.6856)(z-0.7849\pm 0.2284j)} \\
g_{21}(z) &= \frac{0.025(z-1)(z-0.8323)(z+0.8195)}{(z-1)(z-0.6856)(z-0.7849\pm 0.2284j)} \\
g_{22}(z) &= \frac{-0.0340(z-1)(z-0.7163)(z+0.8614)}{(z-1)(z-0.6856)(z-0.7849\pm 0.2284j)}
\end{aligned} \tag{5}
$$

3 ICD Review

The review of the ICD will be limited to the case of a 2x2 MIMO control system. The block diagram of a 2x2 control system with a diagonal matrix control

$$
K = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \tag{6}
$$

is given by the following Fig. 1

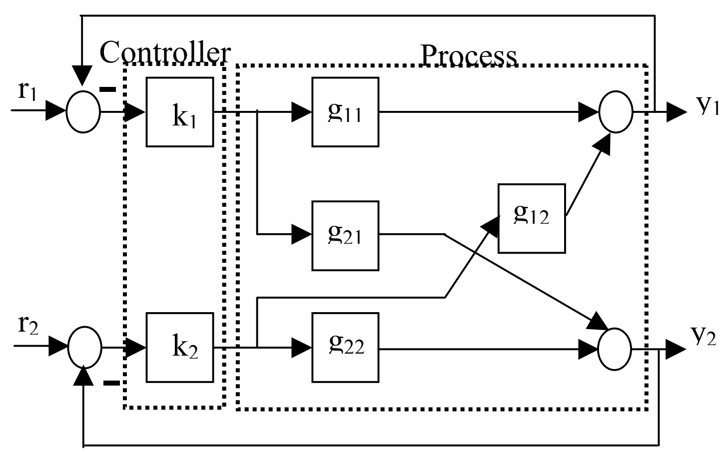

Fig. 1 2x2 control system with diagonal controller

The individual open loop channels, $\frac{y_1(z)}{r_1(z)} = C_1(z)$ and $\frac{y_2(z)}{r_2(z)} = C_2(z)$, are given by:

$$C_1(z) = k_1\, g_{11}\,(1-\gamma\, h_2) \qquad (7)$$

$$C_2(z) = k_2\, g_{22}\,(1-\gamma\, h_1) \qquad (8)$$

where the Multivariable Structure Function $\gamma(z)$ is given by:

$$\gamma(z) = \frac{g_{12}g_{21}}{g_{11}g_{22}} \qquad (9)$$

and subsystems $h_1(z)$ and $h_2(z)$ are given by:

$$h_i = \frac{k_i g_{ii}}{1+k_i g_{ii}}; \quad i = 1,2 \qquad (10)$$

The cross coupling relations ($\frac{y_1(z)}{r_2(z)}$ and $\frac{y_2(z)}{r_1(z)}$) are depicted in Fig.2

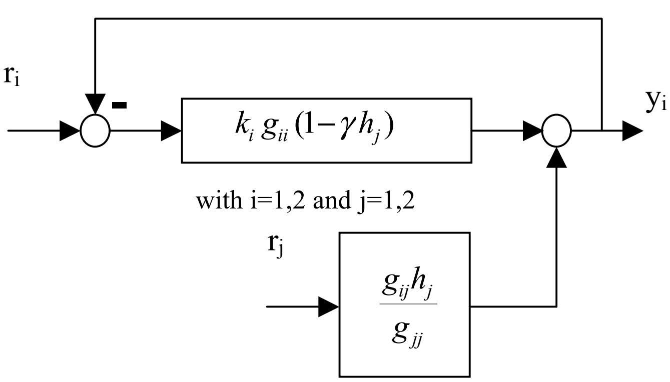

Fig. 2 Cross coupling relations in a 2x2 control system with diagonal controller

The structure of the poles and zeros of the individual open loop channels, equations (7-8), are described in Table 1

CHANNEL	ZEROS OF...	POLES OF...
$C_1(z)$	$g_{11}\,(1-\gamma\, h_2)$	$g_{11}, g_{12}, g_{21}, h_2$
$C_2(z)$	$g_{22}\,(1-\gamma\, h_1)$	$g_{22}, g_{12}, g_{21}, h_1$

Table 1 Channels open loop poles and zeros

Also, the transmission zeros of the process $G(z)$ are the zeros of $(1-\gamma)$. Therefore, the Nyquist stability criteria can be used to determine the RHPZ of $G(z)$, $C_1(z)$ and $C_2(z)$, that is, by the number of encirclements to the point (1,0) of the Nyquist trajectories of $\gamma(z)$, $\gamma h_2(z)$ and $\gamma h_1(z)$ respectively.

Finally, the conditions required for robustness are defined in the following set of gain and phase margins of Table 2

MEASURE OF…	GAIN AND PHASE MARGINS OF…
Subsystems h_i structural robustness	gain and phase margins of $k_i g_{ii}$, $i = 1,2$
Structural robustness of channels C_i, $i = 1,2$	Gain and phase margins of γh_1 *and* γh_2
Robustness of C_i, $i = 1,2$	Gain and Phase margins of C_i, $i = 1,2$

Table 2 Set of phase and gain margins for Robustness of a 2x2 MIMO control system

It necessary to recall that in the case of digital control systems the Nyquist stability criteria is given by [5]:

Let $G(z)H(z)$ be an open loop transfer function. The system will be close loop stable if :

$$\Phi_{11} = -(P_{-1} + 0.5P_w)180^0 \qquad (11)$$

where:

Φ_{11} = The total angle of $G(z)H(z)$, when $z = \exp^{jwT}$ and $w = [\frac{W_M}{2}, W_M]$, around the point (-1,0).

Φ_{11} = P_{-1} = Number of RHPP's of $G(z)H(z)$.

$\Phi_{11} = P_w$ = Number of poles of $G(z)H(z)$ on the unit circle.

4 Controller Design

From equations (3) and (9) the Multivariable Structure Function $\gamma(z)$ of the submarine is given by:

$$\gamma(z) = \frac{-0.6595(z-3.0081)(z-0.8323)(z-0.6960)(z+0.8195)(z+0.6269}{(z-0.7435)(z-0.7163)(z+1.8025)(z+0.8614)(z+0.0113)} \quad (12)$$

From equation (12) is possible to see that $\gamma(z)$ has a pole and a zero out of the unit circle, therefore $\gamma(z)$ is unstable and non-minimum phase. The Nyquist plot of $\gamma(z)$ is shown in Fig. 3

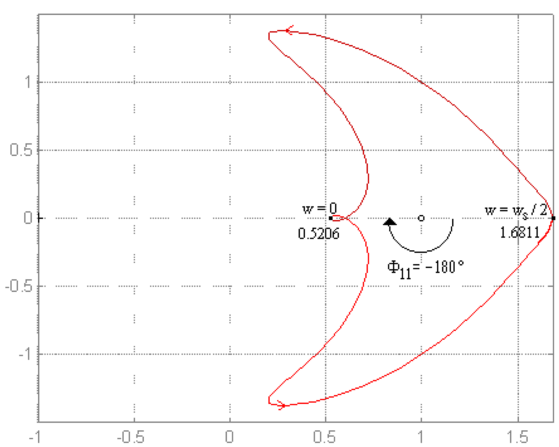

Fig. 3 Nyquist plot of γ(z)

By equation (11) and because Φ_{11} of *γ(z)* is equal to –180°, as shown in Fig. 3, the process $G(z)$ is minimum phase. Nevertheless, to maintain this condition in a close loop condition is necessary that $\gamma h_1(z)$ and $\gamma h_2(z)$ also have Φ_{11} equal to –180°.

The gain a phase margins of *γ(z)* are 4.43dB's and 55.87° respectively (similar to it's continuous counterpart) as shown in Fig. 4. Despite the small gain margin, at low frequency, the process is assumed structurally robust because at low frequencies the uncertainties can be neglected.

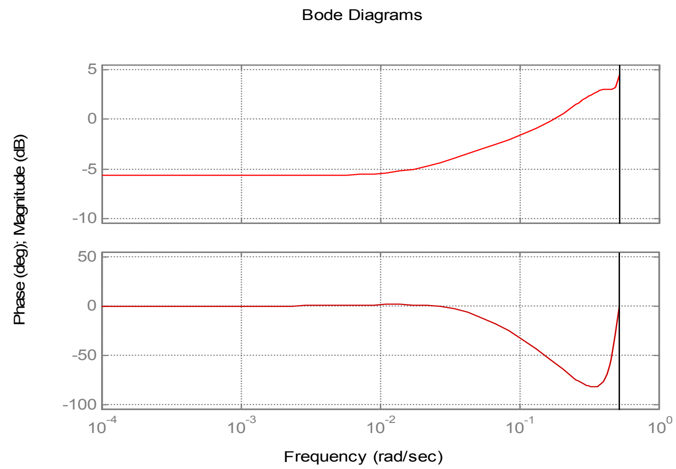

Fig. 4 Bode plots of γ(z)

The control problem consist of designing a controller such that the submarine maintains a constant depth Z(z) relative to a mean sea level with a head angle θ = 0°. To meet these responses the required bandwidths for channels $C_1(z)$ and $C_2(z)$ should be 0.5*rad/sec* and 0.05*rad/sec* respectively.

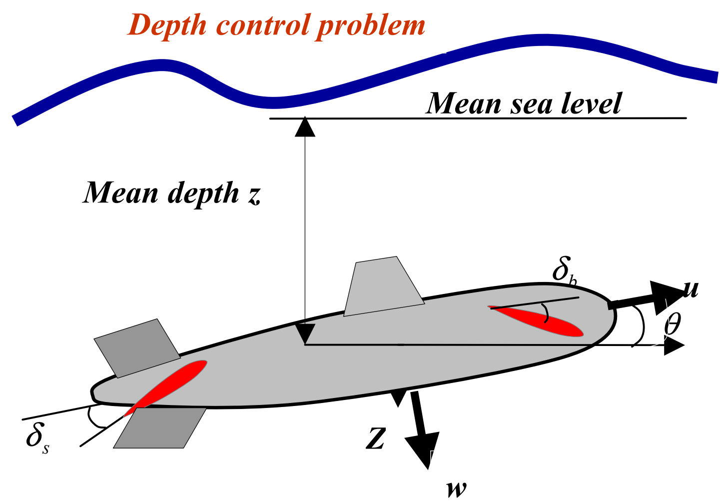

Fig. 5 Submarine depth control problem

A matrix control $K(z)$ which satisfies the required robustness conditions indicated in Table 2 is given by:

$$K(z) = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} = \begin{bmatrix} -0.01403 & 0 \\ 0 & \dfrac{-1.72(z-0.9)^2}{(z-1)(z-0.98)} \end{bmatrix} \quad (13)$$

With the controller $K(z)$, equation (13), the Nyquist plots of $\gamma h_1(z)$ and $\gamma h_2(z)$ are far from the point (1,0), therefore the process remains structurally robust. Unfortunately the Φ_{11} of $\gamma h_1(z)$ and $\gamma h_2(z)$ no longer encircle the point (1,0) resulting in the introduction of non-minimum phase zeros in the open loop channels $C_1(z)$ and $C_2(z)$. The reason of this problem are the required low bandwidths which provokes roll offs in $\gamma h_1(z)$ and $\gamma h_2(z)$ before the Nyquist trajectories encircle the point (1,0) as shown in Fig. 6. Hence, the bandwidths of channels $C_1(z)$ and $C_2(z)$ will not longer match the specified requirements.

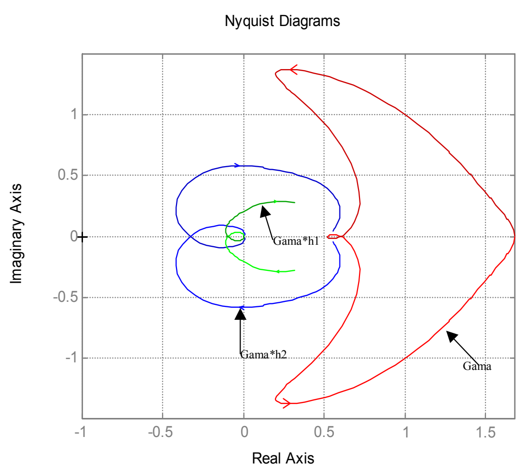

Fig. 6 Nyquist plots of $\gamma(z)$, $\gamma h_1(z)$ and $\gamma h_2(z)$

The Bode plots of $K_2 g_{22}$ and $k_1 g_{11}$ are shown in Fig. 7 y Fig. 8. From these figures is possible to see that $h_1(s)$ and $h_2(s)$ are structurally robust.

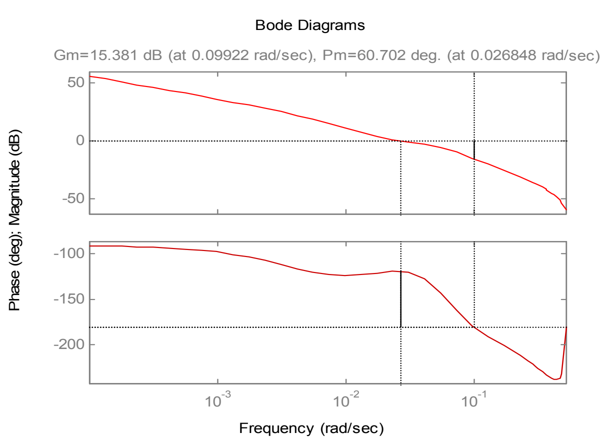

Fig. 7 Bode plots of $K_2 g_{22}$

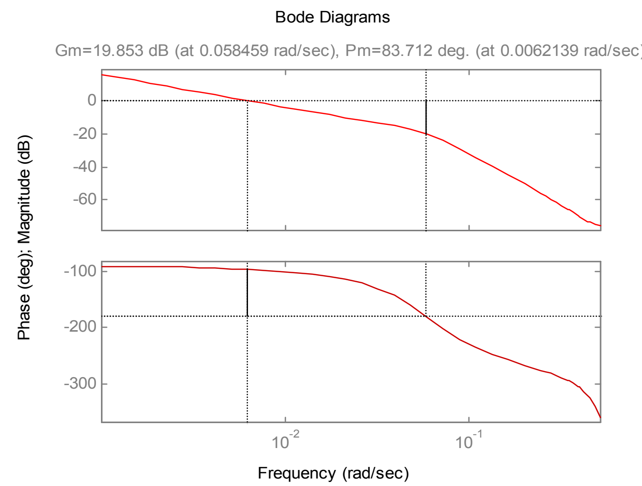

Fig.8 Bode plots of $k_1 g_{11}$

The Bode plots of the Individual Channels $C_1(z)$ and $C_2(z)$ are shown in Fig. 9 y Fig.10.

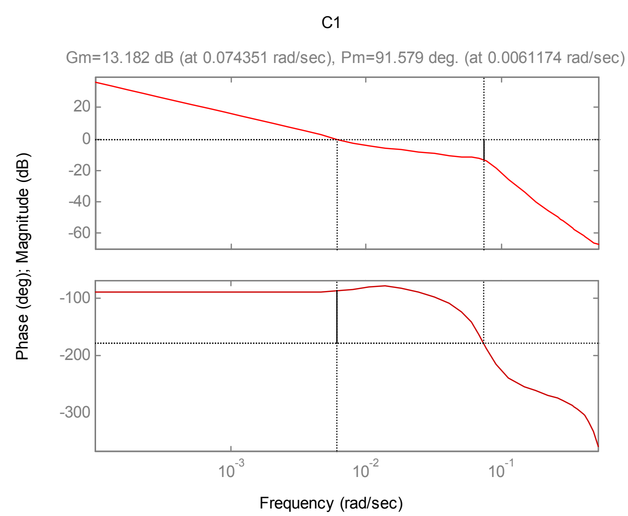

Fig. 9 Bode plots of $C_1(z)$

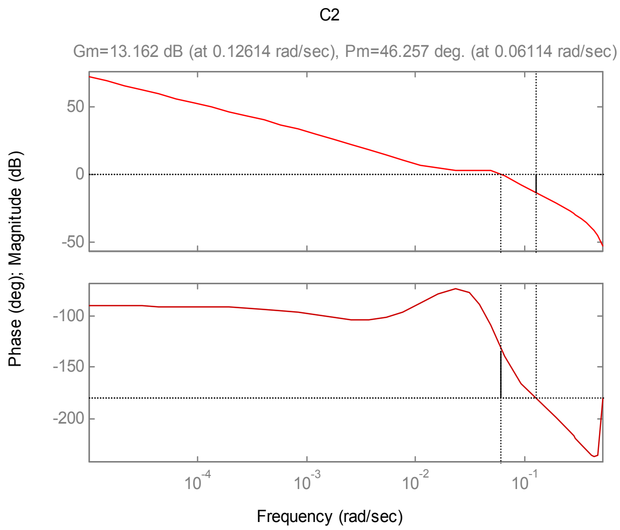

Fig. 10 Bode plots of $C_2(z)$

Finally, the steps responses of the close loop system are shown in Fig. 11 and Fig. 12.

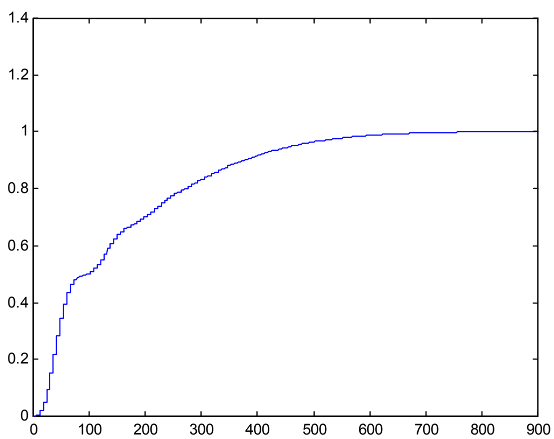

Fig. 11 Step response of channel C_1

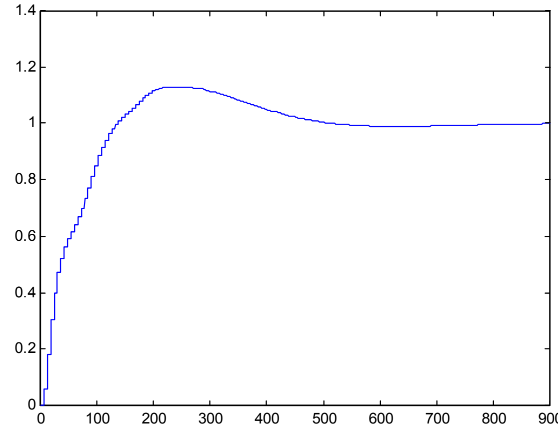

Fig. 12 Step response of channel C_2

5 Conclusion

Using the Individual Channel Design approach a digital version of a submarine depth control system was designed. Additional problems, with respect to the continuous version raised when the discrete model was calculated, that is, the individual transfers functions $g_{11}(z)$ is no minimum phase (it's continuous counter parts is minimum phase), resulting in a unstable Multivariable Structure Function $\gamma(z)$. Also, due to the conditions of design the individual channel are no minimum phase with the consequences that no longer the required bandwidth can be satisfied. Nonetheless, a robust sub optimal (with bandwidths below the specifications) control system was designed. On the other hand it was shown that ICD is also an appropriate approach to design multivariable digital control systems. For the particular case of the submarine depth control problem is recommended to investigate how the frequency sampling rate can be modified to obtain a better discrete model.

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