Novel Approach to the Time-Optimal System Design Methodology

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Abstract: **-** The formulation of the process of analog system design has been done on the basis of the control theory application. This approach produces many different design strategies inside the same optimization procedure and allows determine the problem of the optimal design strategy existence from the computer time point of view. Different kinds of system design strategies have been evaluated from the operations number. This analysis shows that the traditional approach is not time-optimal at least for the electronic circuit design. General methodology for any system design was formulated by means of optimum control theory. This approach generates practically infinite number of the different design strategies. The problem of the time-optimal design algorithm construction is defined as the problem of functional minimization of the optimal control theory. Numerical results of some nonlinear passive and active electronic circuit design demonstrate the efficiency of the proposed methodology and prove the non-optimality of the traditional design strategy. These examples show that the potential computer time gain of the optimal design strategy with respect to the traditional design strategy increases when the size and complexity of the system increase.

Key-Words: - Time-optimal design algorithm, control theory formulation.

1 Introduction

The problem of the computer time reduction of a large system design is one of the essential problems of the total quality design improvement. This problem has a special significance for the VLSI electronic circuit design. Any system design methodology includes two main parts as a rule: the model of the system, which can be simulated as algebraic equations or differential-integral equations and a parametric optimization procedure that achieves the objective function optimal point. The traditional design strategy for the system design has two fixed determined parts. The first part is the mathematical model of the physical system and the second one is the optimization procedure. In limits of this conception it is possible to change optimization strategy and use different models and different analysis methods. However, the time of the large-scale circuit analysis and the time of optimization procedure increase when the network scale increases.

There are some powerful methods that reduce the necessary time for the circuit analysis. Because a matrix of the large-scale circuit is a very sparse, the special sparse matrix techniques are used successfully for this purpose [1]-[2]. Other approach to reduce the amount of computational required for the linear and nonlinear equations is based on the decomposition techniques. The partitioning of a circuit matrix into bordered-block diagonal form can be done by branches tearing as in [3], or by nodes tearing as in [4] and jointly with direct solution algorithms gives the solution of the problem. The extension of the direct solution methods can be obtained by hierarchical decomposition and macromodel representation [5]. Other approach for achieving decomposition at the nonlinear level consists on a special iteration techniques and has been realized in [6] for the iterated timing analysis and circuit simulation. Optimization technique that is used for the circuit optimization and design, exert a very strong influence on the total necessary computer time too. The numerical methods are developed both for the unconstrained and for the constrained optimization [7] and will be improved later on. Meanwhile, it is possible to reformulate the total design problem and generalize it to obtain a set of different design strategies inside the same optimization procedure. It is clear that a finite but a large number of different strategies include more possibilities for the selection of one or several design strategies that are time-optimal or quasi-time-optimal ones. This is especially right if we have infinite number of the different design strategies. On the contrary of the traditional design strategy, the modified traditional design strategy has only one part, because all system parameters are determined as independent and the objective function of the optimization procedure includes additional penalty functions that describe the model of the physical system. In this case the equations of the model of the physical system disappear. On the other hand, it is possible to re-determine the total design problem, to generalize it, to obtain a set of the different design strategies. First of all, we define the time-optimal design strategy as the algorithm that achieves the optimum point of the objective function of the design process at the minimal computer time. The main problem of this formulation is the search of the special conditions, which need to be satisfied for the optimal algorithm construction.

The idea of the control theory use, which was introduced in [8] is developed now for the design of the systems that are described by the non-linear algebraic equation model. This methodology generalizes the design problem and can reduce the total necessary computer design time.

2 Problem Formulation

The design process for any analog system design can be defined as the problem of the objective function $C(X)$ minimization for $X \in R^N$ with the system of constraints. It is supposed that the minimum of the objective function $C(X)$ achieves all design objects and the constraint system is the mathematical model of the physical system. It is supposed also, that the system model can be described as the system of nonlinear equations:

$$g_j(X) = 0 \qquad j = 1,2,\ldots,M \qquad (1)$$

The vector X can be separated in two parts: $X = (X', X'')$. The vector $X' \in R^K$ can be named as the vector of independent variables, where K is the number of independent variables, and the vector $X'' \in E^M$ is the vector of dependent variables, where $N = K + M$. This separation is very conditional, because any variable can be defined as independent or dependent parameter. If the electronic system is described, it is more traditional and natural to define the system elements as independent variables and the physical parameters (voltages, currents, and so on) as dependent variables, but it is not obligatory.

The optimization process for the objective function $C(X)$ minimization for two-step procedure can be defined as following vector equation:

$$X^{s+1} = X^s + t_s \cdot H^s \qquad (2)$$

with constraints (1), where s is the iterations number, t_s is the iteration parameter, $t_s \in R^1$, H is the direction of the objective function $C(X)$ decreasing. The vector H is the function of $C(X)$. This is a typical formulation for the constrained optimization problem. This problem can be transformed to the unconstrained optimization problem for *K*=*N*-*M* variables. In this case the design problem is defined in more traditional form as an unconstrained optimization process in the space R^K:

$$X'^{s+1} = X'^s + t_s \cdot H^s \qquad (3)$$

with the system (1) which is solved at each step of the optimization procedure.

The specific character of the design process at least for the electronic systems consists in the fact that it is not necessary to fulfill the condition (1) for all steps of the optimization process. It is quite enough to fulfill these conditions for the final point of the design process.

The problem (1), (3) can be redefined in form when there is no difference between independent and dependent variables. All components of the vector X can be defined as independent. This is the main idea for the penalty function method application. In this case the vector function H is the function of the objective function $C(X)$ and the additional penalty function $j(X)$: $H^s = f\left(C(X^s), j(X^s)\right)$. The penalty function structure includes all equations of the system (1) and can be defined for example as:

$$j(X^s) = \frac{1}{e}\sum_{i=1}^{M} g_i^2(X^s) \qquad (4)$$

In this case we define the design problem as the unconstrained optimization (2) in the space R^N without any additional system but for the other type of the objective function $F(X)$. This function can be defined for example as an additive function: $F(X) = C(X) + j(X)$. In this case we achieve the minimum of the initial objective function $C(X)$ and comply with the system (1) in the final point of the optimization process. This method can be named as modified traditional design method and it produces another design strategy and another trajectory line in the space R^N. On the other hand, it is possible to generalize the idea of the additional penalty function application if to make up the penalty function as one part of the system (1) only, and the other part of this system is defined as constraints. In this case the penalty function includes first Z items only, $j(X^s) = \frac{1}{e}\sum_{i=1}^{Z} g_i^2(X^s)$ where $Z \in [0, M]$ and M-Z equations make up one modification of the system (1):

$$g_j(X) = 0 \qquad j = Z+1, Z+2, \ldots, M \qquad (1')$$

It is clear, that each new value of the parameter Z produces a new design strategy and a new trajectory line. This idea can be generalized more in case when the penalty function $j(X)$ includes Z arbitrary equations from the system (1). The total number of different design strategies is equal to 2^M, if $Z \in [0, M]$. All these strategies exist inside the same optimization procedure. The optimization procedure is realized in the space R^{K+Z}. The number of the dependent variables M increases rapidly with the system complexity increasing. In this case the number of different design strategies increases exponentially. These different strategies have various computer times because they have different operations number. It is appropriate in this case to define the problem of the search of an optimal design strategy that has the minimal computer time. The most general approach can be constructed on the basis of the design problem formulation as the problem of optimal control [8]-[9]. It is possible to define a design strategy by equations (1'), (2) with the variable value of the parameter Z during the all optimization process. It means that we can change the number of independent variables and the number of the terms of the penalty function at each point of the optimization procedure. It is convenient to introduce in consideration a vector of the special control functions $U = (u_1, u_2, \ldots, u_M)$ for this aim, where $u_j \in \Omega$; $\Omega = \{0;1\}$. These control variables are introduced artificially to generalize the design process. The sense of the control function u_j is next: the equation number j is present in the system (1') and the term $g_j^2(X)$ is removed from the right part of the formula (4) when $u_j = 0$, and on the contrary, the equation number j is removed from the system (1') and is present in the right part of the formula (4) when $u_j = 1$. In this case we have the following formulas for the model of the system and for the penalty function:

$$(1 - u_j)\, g_j(X) = 0 \qquad j = 1, 2, \ldots, M \qquad (5)$$

$$j(X^s) = \frac{1}{e}\sum_{j=1}^{M} u_j \cdot g_j^2(X^s) \qquad (6)$$

All control variables u_j are the functions of the current point of the optimization process. The vector of the directional movement H is the function of the vectors X and U in this case: $H = f(X, U)$. The total number of the different design strategies, which are produced inside the same optimization procedure, is practically infinite. Among all of these strategies exist one or few optimal strategies that achieve the design objects for the minimum computer time. So, the problem of the time-optimal design strategy finding is formulated as the typical minimal-time problem of the control theory. The main problem of this definition is unknown optimal dependencies of all control functions. The solution of this problem may be finding by some approximate methods of the optimal control theory.

3 Time-Optimal System Design Problem Formulation

3.1 Continuous form

It is possible to determine the problem of any analog system design as the problem of the optimal control. The principal system of equations in this case consists of two parts and can be determined by two systems of equations:

$$\frac{dx_i}{dt} = f_i(X, U), \qquad i = 0, 1, \ldots, N \qquad (7)$$

$$(1 - u_j)\, g_j(X) = 0, \qquad j = 1, 2, \ldots, M \qquad (8)$$

where N=K+M, x_0 is the additional variable, U is the vector of control variables, $U = (u_1, u_2, \ldots, u_M)$, $u_j \in \Omega$; $\Omega = \{0;1\}$.

The functions of the right hand part of the system (7) are determined as:

$$f_i(X,U) = -b \frac{d}{dx_i} \left\{ C(X) + \frac{1}{e} \sum_{j=1}^{M} u_j g_j^2(X) \right\} \qquad (9)$$

for $i = 1,2,\ldots,K$,

$$f_i(X,U) = -b \cdot u_{i-K} \frac{d}{dx_i} \left\{ C(X) + \frac{1}{e} \sum_{j=1}^{M} u_j g_j^2(X) \right\} + \frac{(1-u_{i-K})}{dt} \{ -x'_i + h_i(X) \} \qquad (9')$$

for $i = K+1, K+2, \ldots, N$,

where b is the iteration parameter; the operator $\frac{d}{dx_i}$ hear and below means $\frac{d}{dx_i} j(X) = \frac{¶ j(X)}{¶ x_i} + \sum_{p=K+1}^{K+M} \frac{¶ j(X)}{¶ x_p} \frac{¶ x_p}{¶ x_i}$, x'_i is equal to $x_i(t-dt)$; $h_i(X)$ is the implicit function ($x_i = h_i(X)$) that is determined by the system (8).

The sense of the control variables u_j is provided in section 2. These variables have the time dependency in general case. The equation number j is removed from (8) and the dependent variable x_{K+j} is transformed to the independent when u_j=1. This independent parameter is defined by the formulas (7), (9'). In this case there is no difference between formulas (9) and (9'), because the parameter x_{K+j} is an ordinary independent parameter. On the other hand, the equation (7) with the right part (9') is transformed to the identity $\frac{dx_i}{dt} = \frac{dx_i}{dt}$, when u_j=0, because $h_i(X) - x'_i = x_i(t) - x_i(t-dt) = dx_i$. It means that at this time moment the parameter x_i is dependent one and the current value of this parameter can be obtained from the system (8) directly. This transformation of the vectors X' and X'' can be done at any time moment. The function $f_0(X,U)$ is determined as the necessary calculation time for one step of the system (7) integration. This function depends on the concrete design strategy. The additional variable x_0 is determined as the total computer time T for the system design. In this case we determine the problem of the time-optimal system design as the classical problem of the functional minimization of the optimal control theory. In this context the aim of the optimal control is to result each function $f_i(X,U)$ to zero for the final time t_{fin} , to minimize the objective function, and to minimize the total computer time x_0. By this formulation the general design strategy of the previous section is the particular case only. It is possible to re-determine this general design strategy as method with the fixed values of all control functions u_j . The total number of the different design strategies, which is produced by the general design strategy, is equal to 2^M. On the contrary, the idea that defined the design process by means of equations (7)-(9) generates practically an infinite number of the different design strategies. Each design strategy has its own trajectory in space R^N. It is clear, that the time comparison of the different trajectories is adequate only in case when the final trajectory point is the same. On the other hand, the objective function *C(X)* has a set of local minimal points, because the design problem is a non-linear problem in general. It is necessary to put the additional simple conditions to achieve the same point of the objective function for the different design strategies. However, the non-simple problem is not a specific feature of new design problem formulation. We have this type of problem always when we begin the design process from the different start points. It is supposed below that the simple conditions are provided.

To minimize the total design computer time it is necessary to find the optimal behavior of the control functions u_j during the design process. The functions $f_i(X,U)$ are piecewise continued as the temporal functions because the control functions u_j have discontinuities. The problem for the system (7) with the non-continued or non-smoothed functions (9), (9') can be solved most adequately by means of Pontryagin's maximum principle [10].

The idea of the system design problem formulation as the functional minimization problem of the control theory can be embedded into different optimization procedures. In this paper three optimization algorithms were selected as the typical representatives of three main groups of the optimization procedures. There are the gradient method, the Newton's method and the Davidon-Fletcher-Powell (DFP) method. These optimization algorithms serve well both independent methods and as the basis for the different others optimization methods.

3.2 Discrete form

The continuous form of the problem is transformed in next subsections to the discrete form for three optimization methods.

3.2.1 Gradient method

The discrete form of this method for each component of the vector X is determined by the following equations:

$$x_i^{s+1} = x_i^s + t_s \cdot f_i(X,U), \quad i = 1,2,\ldots,K,K+1,\ldots N \tag{10}$$

$$\left(1-u_j\right) g_j(X) = 0, \quad j = 1,2,\ldots,M \tag{11}$$

where the components $f_i(X,U)$ are given by:

$$f_i(X,U) = -\frac{d}{dx_i} F(X,U) \tag{12}$$

for $i = 1,2,\ldots,K$,

$$f_i(X,U) = -u_{i-K} \frac{d}{dx_i} F(X,U) + \frac{\left(1-u_{i-K}\right)}{t_s}\left\{-x_i^s + h_i(X)\right\} \tag{12´}$$

for $i = K+1, K+2,\ldots,N$,

where $F(X,U) = C(X) + \frac{1}{e}\sum_{j=1}^{M} u_j g_j^2(X)$.

The control variables u_j have dependency from the step number s in general case. The dependent variable x_{K+j} is transformed to the independent when u_j=1. This independent parameter is defined by the formulas (10), (12'). In this case there is no difference between formulas (12) and (12'), because the parameter x_{K+j} is an ordinary independent variable. On the other hand, the equation (10) with the right part (12') is transformed to the identity $x_{K+j}^{s+1} = x_{K+j}^{s+1}$, when u_j = 0. It means that at this step of the design process the equation number $K+j$ disappear from the system (10), parameter x_{K+j} is a dependent one and the current value of this parameter can be obtained from the system (11) directly. This transformation can be done at any step.

3.2.2 Newton's method and DFP method

Two main systems are (10) and (11) as for gradient method, but the functions $f_i(X,U)$ are given by:

$$f_i(X,U) = -\sum_{k=1}^{N} b_{ik} \frac{d}{dx_k} F(X,U)$$

for $i = 1,2,\ldots,K$,

$$f_i(X,U) = -u_{i-K}\sum_{k=1}^{N} b_{ik} \frac{d}{dx_k} F(X,U) + \frac{\left(1-u_{i-K}\right)}{t_s}\left\{-x_i^s + h_i(X)\right\}$$

for $i = K+1, K+2,\ldots, N$,

where b_{ik} is the element of the matrix $\left\{F''(X,U)\right\}^{-1}$ for the Newton's method and the element of the matrix *B(X,U)* for the DFP method. In the last case this element is defined by following expression $B_{s+1} = B_s + \frac{R^s\left(R^s\right)^T}{\left(R^s\right)^T Q^s} - \frac{\left(B_s Q^s\right)\left(B_s Q^s\right)^T}{\left(Q^s\right)^T B_s Q^s}$,

where B_0 is the unit matrix, $s = 0,1,\ldots$ and $R^s = X^{s+1} - X^s$, $Q^s = F'\left(X^{s+1},U^{s+1}\right) - F'\left(X^s,U^s\right)$.

4 Examples

Some passive and active non-linear electronic circuits have been analyzed to demonstrate this system design approach based on the optimal control theory. The passive circuits have various nodal numbers from 1 to 5, $(M \in [1,5])$. Three examples of the transistor circuits have three, five and seven nodes respectively. The design process has been realized on DC mode for all circuits. The detailed analysis of the passive electronic circuit for $M = 5$ is presented below in sections 4.1. The active circuit analysis is presented in sections 4.2 for $M = 7$. The objective function $C(X)$ has been determined as the sum of the squared differences between beforehand-defined values and current values of the nodal voltages for some nodes with additional inequalities for some circuit elements. It is supposed also that the additional physical constraints for the passive element are provided. All these elements are positive. To obtain this property it is convenient to change all admittance values y_i to x_i^2. The iteration parameter t_s was adapted on the basis of well-known idea to minimize the objective function at each point of the optimization process as one variable function.

4.1 Passive nonlinear circuits

In Fig. 1 there is a circuit that has six independent variables as admittance $y_1, y_2, y_3, y_4, y_5, y_6$ ($K=6$) and five dependent variables as nodal voltages V_1, V_2, V_3, V_4, V_5 ($M=5$) at the nodes 1, 2, 3, 4, 5. Non-linear circuit elements have dependencies: $y_{n1} = a_{n1} + b_{n1} \cdot (V_3 - V_2)^2$, $y_{n2} = a_{n2} + b_{n2} \cdot (V_4 - V_2)^2$. Non-linearity parameters b_{n1}, b_{n2} are equal to 1.0. The system of the optimization procedure equations and the system of the model's equations have eleven and five equations respectively.

The results of the analysis of the complete set of the design strategies with the fixed value of the control functions are given in Table 1. There are 32 different strategies in this case. The first line of the table corresponds to the traditional design strategy. The last line corresponds to the modified traditional strategy. The other lines correspond to the intermediate strategies. The optimal strategies from this table (number 32, 32, 15 for three optimization methods respectively) nevertheless are not optimal in general, as for the previous examples. The optimal trajectories were found by the additional optimization procedure and the data of these time-optimal design strategies are given in Table 2. The optimal strategy for the gradient method and for the Newton method has two

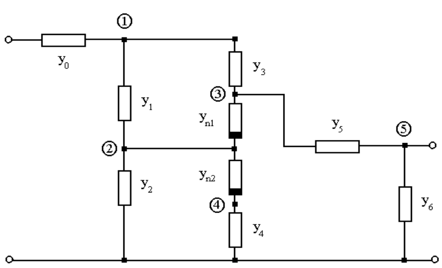

Fig. 1. Circuit topology for K=6 and M=5.

Table 1. Complete set of the design strategies for five-nodes passive circuit.

N	Control functions vector U (u1,u2,u3,u4,u5)	Gradient method		Newton method		DFP method	
		Iterations number	Total design time (sec)	Iterations number	Total design time (sec)	Iterations number	Total design time (sec)
1	(00000)	214	4.179	15	1.839	29	1.012
2	(00001)	529	7.627	15	1.775	20	0.521
3	(00010)	378	6.986	7	1.041	31	1.031
4	(00011)	78	1.036	11	1.485	27	0.654
5	(00100)	1172	21.711	11	1.638	18	0.601
6	(00101)	1548	20.576	9	1.216	32	0.775
7	(00110)	995	13.198	12	1.617	19	0.459
8	(00111)	118	1.107	15	1.751	44	0.771
9	(01000)	251	5.666	13	2.331	30	1.211
10	(01001)	371	4.921	10	1.351	54	1.305
11	(01010)	158	2.093	12	1.618	13	0.314
12	(01011)	549	5.141	15	1.751	77	1.341
13	(01100)	268	4.307	10	1.616	34	0.991
14	(01101)	207	1.944	15	1.754	76	1.327
15	(01110)	198	0.779	13	0.681	18	0.141
16	(01111)	204	0.659	19	0.935	233	1.561
17	(10000)	255	3.674	9	1.064	15	0.391
18	(10001)	180	1.887	15	1.627	18	0.347
19	(10010)	133	1.762	9	1.214	24	0.581
20	(10011)	685	6.423	22	2.571	46	0.804
21	(10100)	254	3.372	10	1.351	33	0.801
22	(10101)	1144	10.726	11	1.284	31	0.542
23	(10110)	526	4.926	10	1.167	17	0.296
24	(10111)	1349	8.832	23	2.226	55	0.692
25	(11000)	402	6.443	14	2.257	33	0.957
26	(11001)	1849	17.295	9	1.049	69	1.201
27	(11010)	148	1.385	11	1.283	48	0.836
28	(11011)	1052	6.887	16	1.551	127	1.598
29	(11100)	156	1.775	13	1.795	28	0.578
30	(11101)	263	1.722	19	1.842	138	1.736
31	(11110)	135	0.436	14	0.689	29	0.194
32	(11111)	161	0.199	24	0.485	185	0.611

Table 2. Data of the optimal design strategies.

N	Method	Optimal control functions vector U (u1, u2, u3, u4, u5)	Iterations number	Switching points	Total design time (sec)
1	Gradient method	(11111); (00000); (11111)	116	59; 60	0.161
2	Newton method	(11111); (10000); (11111)	12	3; 4	0.341
3	DFP method	(01110); (11111)	22	7	0.104

swithing points and for DFP method has one switching point. The time gain of the optimal design strategy is equal to 25.95, 5.09 and 9.73 for the gradient, Newton and DFP methods respectively. The optimum behavior of the control functions u_1, u_2, u_3, u_4, u_5 during the design process for the DFP method are shown in Fig. 2.

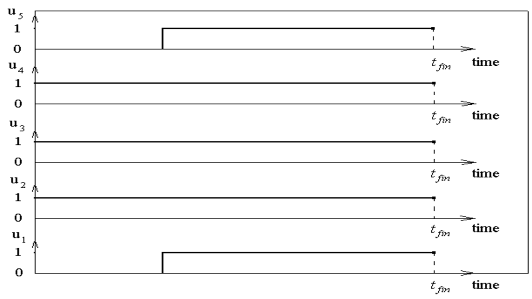

Fig. 2. Optimum dependencies of the control functions u_1, u_2, u_3, u_4, u_5 for the DFP optimization method.

The optimal time dependencies of the control functions u_j for all examples don't have any definite law and have been obtained by the additional optimization procedure. The results of all analyzed passive circuits for *M=1,2,3,4,5* are presented in Fig. 3 for three different optimization procedures. This is the computer time gain of the optimum design strategy with respect to the traditional design strategy as the function of the dependent parameters' number *M*. The traditional design approach is not time-optimal and the time gain increases very fast with the *M* increasing.

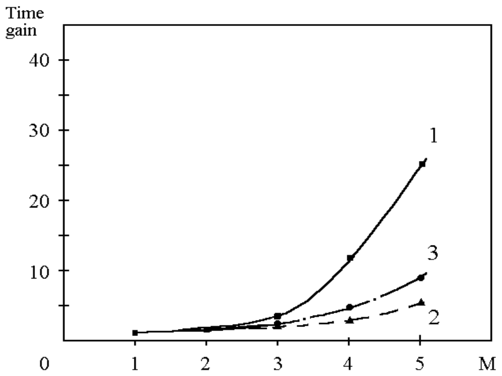

Fig. 3. Computer time gain of the optimal design strategy.

4.2 Active nonlinear circuits

In Fig. 4 there is a circuit of the transistor amplifier that consists of three transistor cells. The one, two and three transistor cell circuits were analyzed separately. The one transistor cell circuit was analyzed as the first example. In this case the circuit includes three nodes only. The second circuit includes two transistor cells and the five-node circuit was analyzed. The last situation includes the full circuit of the Fig. 4 with three transistors and seven nodes and was analyzed detailed below.

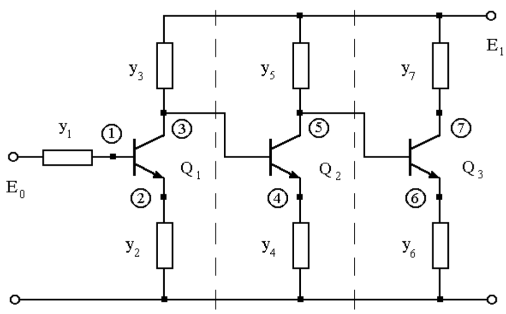

Fig. 4. Circuit topology for three-cell transistor amplifier.

The Ebers-Moll static model of the transistor has been used. The analyzed circuit has seven independent variables $y_1, y_2, y_3, y_4, y_5, y_6, y_7$ as admittance (*K=7*) and seven dependent variables $V_1, V_2, V_3, V_4, V_5, V_6, V_7$ as nodal voltages (*M=7*). The results of the analysis of the traditional design strategy and 24 other strategies that have the computer time less than the traditional strategy with the fixed value of the control functions are given in Table 3.

Table 3. Data of the three-transistor cell circuit analysis.

N	Control functions vector U (u1,u2,u3,u4,u5,u6,u7)	Gradient method		DFP method	
		Iterations number	Total design time (sec)	Iterations number	Total design time (sec)
1	(0000000)	6379	321.09	854	64.47
2	(0010101)	922	54.53	764	52.29
3	(0010110)	1667	80.71	650	46.13
4	(0010111)	767	35.35	426	22.68
5	(0011100)	3024	159.67	940	52.71
6	(0011101)	823	37.73	177	7.71
7	(0011110)	3068	86.87	450	14.56
8	(0011111)	553	15.75	170	6.93
9	(0110101)	465	10.01	101	2.66
10	(0110110)	1157	31.92	111	3.85
11	(0110111)	501	8.82	124	2.66
12	(0111100)	2643	72.66	314	9.24
13	(0111101)	507	9.24	170	4.62
14	(0111110)	3070	67.27	423	12.25
15	(1010101)	1345	28.07	397	16.94
16	(1010111)	615	10.01	191	4.62
17	(1011101)	699	10.71	197	4.97
18	(1011111)	366	4.97	103	1.96
19	(1110101)	789	10.43	201	4.97
20	(1110110)	3893	61.53	1158	18.06
21	(1110111)	749	7.71	148	2.11
22	(1111100)	4325	90.72	945	19.18
23	(1111101)	796	8.47	133	2.31
24	(1111110)	2149	29.26	1104	13.44
25	(1111111)	2031	5.67	180	0.77

The optimal strategies from this table (number 18 and 25 for two optimization procedures respectively) are not optimal in general and the data for the time-optimal strategies are given in Table 4. The time gain of the optimal design strategy with respect to the traditional strategy is equal to 285 for the gradient method and 200 for the DFP method. The potential computer time gain of the time-optimal design strategy with respect to the traditional design strategy as the function of the transistor cell number N_{TR} is presented in Fig. 5.

Table 4. Data of the optimal design strategies.

N	Method	Optimal control functions vector U (u1,u2,u3,u4,u5,u6,u7)	Iterations number	Switching points	Total design time (sec)	Computer time gain
1	Gradient method	(1111111); (1111101)	363	350	1.127	285
2	DFP method	(1111111); (1110111)	69	66	0.322	200

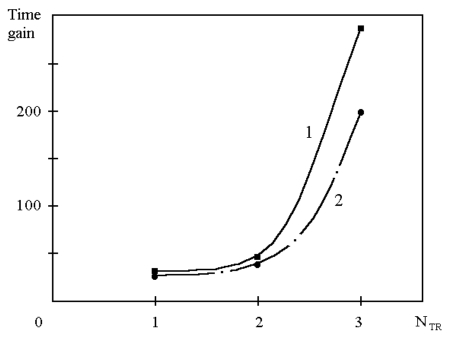

Fig. 5. Optimal strategy computer time gain for the active circuits. 1-Gradient method, 2-DFP method.

This result confirms the rule that the total computer time gain of the time-optimal design strategy increases when the complexity of the circuit increases. The comparison of the results for passive and active circuits shows that the computer time gain is larger for the active circuits because of more complexity in this last case. The potential time gain is realized only in case when we found the algorithm for the optimal trajectories systematically construction. This problem can be solved on the basis of the approximate methods of the optimal control theory [11]. The formulation of the intrinsic properties and restrictions of the optimal design trajectory can be defined as the first problem that needs to be solved on this way.

The above-described approach serves as the theoretic foundation for the time-optimal design algorithm searching and promises to improve the design process characteristics when the optimal design algorithm will be constructed.

5 Conclusion

The traditional approach for the analog circuit design is not time-optimal. The problem of the optimum algorithm construction can be solved more adequately on the basis of the optimal control theory application. The time-optimal design algorithm is formulated as the problem of the functional optimization of the optimal control theory. In this case it is necessary to elect one optimal trajectory from the quasi-infinite number of the different design strategies, which are produced. The maximum principle can serves in this case as the basis for the election of the optimal dependencies of the control functions. This approach reduces considerably the total computer time for the system design. Analysis of the different electronic systems gives the possibility to conclude that the potential computer time gain of the time-optimal strategy increases when the size and complexity of the system increase. The above-described approach gives the possibility to find the time-optimal algorithm as the approximate solution of the typical problem of the optimal control theory.

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