H_∞ Control Design of PWM Voltage-Controlled DC-DC Ćuk Converter

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Abstract: - In the present paper an H_∞ control design is presented for DC-DC Ćuk converter using its average model. Lyapunov equation is used to derivate the controller and to estimate the L_2 gain too. Theoretical result is supported by numerical simulations.

Key-Words: - H_∞ control, PWM, Lyapunov equation, Ćuk converter.

1 Introduction

The Ćuk converter is usually controlled by means of finite sampling frequency pulse-width-modulation (PWM) control schemes. In the present paper we develop a PWM control scheme using an H_∞ control law to regulate the Ćuk converter affected by an ac disturbance (ripple) in its DC source. This ripple disturbance is considered in the average model of the Ćuk converter, where this model was used to obtain the H_∞ control law. Our propose differs from the propose in [3] from the point of view that we use a Lyapunov equation to derivate our controller and to estimate the L_2 gain too. Different controllers are obtained for different Lyapunov equations given us more flexibility in the design procedure. A computer program for simulating the Ćuk converter was developed to support our theoretical result.

2 Problem Formulation

The PWM Ćuk convert system is shown in Fig. 1 where r_1 and r_2 are internal resistances of inductors L_1 and L_2 respectively, E is the DC source voltage and $w(t)$ is the disturbance on the system. This disturbance can be some ac variation (ripple) presented in the DC source E.

The switching operation is described in Fig. 2 where Ts is the switching period divided in two time intervals, one is uTs where the transistor (TR) is ON and the Diode (D) is OFF, the second one is $(1-u)Ts$ where the operations is reversed, that is, Tr is OFF and D is ON. It is assumed that $0<u<1$.

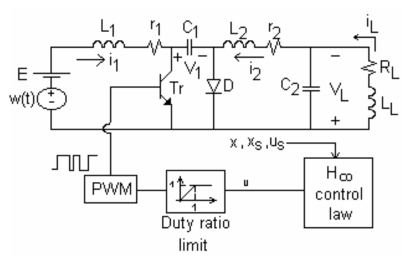

Fig.1 PWM Ćuk converter system.

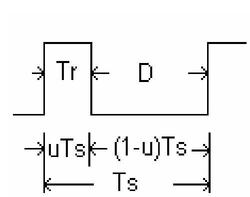

Fig. 2 Time ratio of switching operation.

The state space average model of the Ćuk converter system is described by the following equations [1]

$$\dot{x} = a_x(x) + b_2(x)u + b_1 w \qquad (1)$$

with $a_x(x) = A_0 x + g$, $b_2(x) = A_1 x$, where A_0, A_1, b_1 and g are the circuit parameters given by

$$A_0 = \begin{bmatrix} -r_1/L_1 & -1/L_1 & 0 & 0 & 0 \\ 1/C_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -r_2/L_2 & 0 & -1/L_2 \\ 0 & 0 & 0 & -R_L/L_L & 1/L_L \\ 0 & 0 & 1/C_2 & -1/C_2 & 0 \end{bmatrix} \qquad (2)$$

$$A_1 = \begin{bmatrix} 0 & 1/L_1 & 0 & 0 & 0 \\ -1/C_1 & 0 & -1/C_1 & 0 & 0 \\ 0 & 1/L_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (3)$$

$$b_1 = \begin{bmatrix} 1/L_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad g = \begin{bmatrix} E/L_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad (4)$$

and the state vector is $x^T = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5] = [i_1 \quad v_1 \quad i_2 \quad i_L \quad v_L]$.

Assume that $x=x_s$, $u=u_s$, and $w=0$ denotes the operating point of the Ćuk converter. Then, via change of variables $z=x-x_s$ and $v=u-u_s$, the system (1) in the new variables z and v is given by

$$\dot{z} = a_z(z) + b_2(z)v + b_1 w \qquad (5)$$

with $a_z(z) = (A_0 + A_1 u_s)z$ and $b_2(z) = A_1(z + x_s)$.

The system

$$\dot{z} = (A_o + A_1 u_s)z \qquad (6)$$

is asymptotically stable for any constant $u=u_s$ [1], this means that next Lyapunov equation

$$PA_z + A_z^T P = -Q \qquad (7)$$

where $A_z = A_0 + A_1 u_s$ presents a unique positive definite symmetric solution P for any given positive definite symmetric matrix Q (see theorem 3.6 in [2]), and the next

$$V = z^T P z \qquad (8)$$

is a Lyapunov function for the system (6). In resume, the next equality is obtained

$$\dot{V} = \frac{\partial V}{\partial z} a_z(z) = -z^T Q z \quad . \qquad (9)$$

To postulate the H$_\infty$ control problem it is required to define the virtual output to be controlled, we propose this as follows:

$$k(t) = \begin{pmatrix} (\delta Q)^{1/2} z \\ v \end{pmatrix} \qquad (10)$$

where δ is a positive constant less than one.

The H$_\infty$ control problem is as follows. Given a scalar $\gamma>0$, design a smooth state feedback control law $v=v_1(x)$ for (5) and (10) with $v_1(0)=0$, and such that:
1) With $w=0$ the origin is asymptotic sable equilibrium of the closed loop system,
2) The L_2 gain from w to k of the closed-loop system is not longer than γ, i.e., there exist a function $\beta: R^5 \to R$, with $\beta(0)=0$ and such that for any initial condition z_o of (5), the inequality

$$\int_0^t \|k(t)\|^2 dt \le \gamma^2 \int_0^t \|w(t)\|^2 dt + \beta(z_0) \qquad (11)$$

is satisfied for all $t>0$ and all piecewise continuous functions $w(t)$.

3 Problem Solution

Next is our main result.

Theorem 1.- Given a positive definite symmetric matrix Q , the control law

$$v = -b_2^T(z) P z \qquad (12)$$

where P is the unique positive definite symmetric solution to the Lyapunov equation (7), is a solution to the H$_\infty$ control problem with

$$\gamma > \sqrt{\frac{\lambda_{\max}(P b_1 b_1^T P)}{(1-\delta)\lambda_{\min}(Q)}} \qquad (13)$$

where $\lambda_{max}(*)$ and $\lambda_{min}(*)$ denote the maximum and minimum eigenvalues respectively of the matrix *.

proof.- Using equation (8), its time derivative along of the trajectory of the system (5) yields,

$$\dot{V} = \frac{\partial V}{\partial z}[a_z(z) + b_2(z)v + b_1 w] \qquad (14)$$

Define

$$\begin{aligned} H(z,w,v) &= \dot{V} + \|k\|^2 - \gamma^2\|w\|^2 \\ &= \dot{V} + \|(\delta Q)^{1/2} z\|^2 + \|v\|^2 - \gamma^2 \|w\|^2 \end{aligned} \qquad (15)$$

Then, using (14), we have

$$H = \frac{\partial V}{\partial z}[a_z(z) + b_2(z)v + b_1 w]$$
$$+\left\|(\delta Q)^{1/2} z\right\|^2 + \|v\|^2 - \gamma^2\|w\|^2 \qquad (16)$$

Solving $\frac{\partial H}{\partial w} = 0$ and $\frac{\partial H}{\partial v} = 0$ for w and v, respectively, we obtain

$$w_1 = \frac{1}{2\gamma^2} b_1^T (\frac{\partial V}{\partial z})^T, \qquad (17)$$

and

$$v_1 = -\frac{1}{2} b_2^T(z)(\frac{\partial V}{\partial z})^T. \qquad (18)$$

Next considerer

$$H^* = H(z, w_1, v_1)$$
$$= \frac{\partial V}{\partial z} a_z(z) + \frac{1}{4}\frac{\partial V}{\partial z}[\frac{1}{\gamma^2} b_1 b_1^T - b_2(z) b_2^T(z)](\frac{\partial V}{\partial z})^T$$
$$+\left\|(\delta Q)^{1/2} z\right\|^2 \qquad (19)$$

Using (9) the above equation yields

$$H^* = -z^T Q z + \frac{1}{4}\frac{\partial V}{\partial z}[\frac{1}{\gamma^2} b_1 b_1^T - b_2(z) b_2^T(z)](\frac{\partial V}{\partial z})^T$$
$$+\left\|(\delta Q)^{1/2} z\right\|^2 \qquad (20)$$

utilizing (8), we get

$$H^* = -(1-\delta) z^T Q z + z^T[\frac{1}{\gamma^2} P b_1 b_1^T P - P b_2(z) b_2^T(z) P] z$$
$$\le -(1-\delta) z^T Q z + \frac{1}{\gamma^2} z^T P b_1 b_1^T P z$$
$$\le -(1-\delta)\lambda_{\min}(Q)\|z\|^2 + \frac{1}{\gamma^2}\lambda_{\max}(P b_1 b_1^T P)\|z\|^2$$
$$= -[(1-\delta)\lambda_{\min}(Q) - \frac{1}{\gamma^2}\lambda_{\max}(P b_1 b_1^T P)]\|z\|^2 \qquad (21)$$

Since w_1 is the maximizing w for $H(z,w,v)$, then $H(z,w,v) \le H^*(z,w_1,v_1)$ for any w. Hence, we have

$$\dot{V} \le -[(1-\delta)\lambda_{\min}(Q) - \frac{1}{\gamma^2}\lambda_{\max}(P b_1 b_1^T P)]\|z\|^2$$
$$-\left\|(\delta Q)^{1/2} z\right\|^2 - \|v\|^2 + \gamma^2\|w\|$$
$$= -[(1-\delta)\lambda_{\min}(Q) - \frac{1}{\gamma^2}\lambda_{\max}(P b_1 b_1^T P)]\|z\|^2$$
$$-\|k\|^2 + \gamma^2\|w\| \qquad (22)$$

Now, we show that the origin of the closed-loop system is globally asymptotic stable. When $w=0$, it follows from (22) that

$$\dot{V} \le -[(1-\delta)\lambda_{\min}(Q) - \frac{1}{\gamma^2}\lambda_{\max}(P b_1 b_1^T P)]\|z\|^2$$

where the condition in (13) secures that $\dot{V}$ is negative definite. This proves asymptotic stability. Next, taking integration on both sides of the equation (22), from *0* to *t>0*, it results that

$$V(z(t)) - V(z_o) \le -\int_0^t \|k\|^2 dt + \gamma^2 \int_o^t \|w\|^2 dt$$

where z_o is the initial condition of the system (5). Since $V(z)>0$ for any $z\neq 0$, the above implies that

$$\int_0^t \|k\|^2 dt \le \gamma^2 \int_o^t \|w\|^2 dt + \beta(z_0)$$

for any $w \in L_2$, where $\beta(z_0) = V(z_0)$. This conclude proof theorem 1. ♦

4 Simulation results

Our theoretical result is supported by the next simulation example. A computer program for simulating the Ćuk converter system of Fig. 1 was developed with parameters (taken from [1]) $L_1=L_2=1mH$, $C_1=100\mu F$, $C_2=10\mu F$, $R_l=15\Omega$, $r_1=1\Omega$, $r_2=0.5\Omega$, $L_L=10mH$, $E=30V$, and with operating point

$$x_s^T = [11.0204 \quad 75.9184 \quad 3.6735 \quad 3.6735 \quad 55.1020]$$

and

$$u_s = 0.75.$$

The modulation frequency is chosen as *50kHZ*. The control law in theorem 1 was developed solving the Lyapunov equation (7) with $Q=I_{5x5}$. Fig.3 shows the simulation results with $w(t)=0$ and Fig.4 for the case when $w(t)=1\sin(2\pi ft)$ with $f=60Hz$.

5 Conclusion

In the present paper we developed an H_∞ controller for the DC-DC Ćuk converter using a Lyapunov equation. The estimation of the L_2 gain is given in function to the solution to this Lyapunov equation. Simulation results are presented to support our theoretical result. The procedure presented here can be easily extended for the Buck-Boost and Boost converter systems.

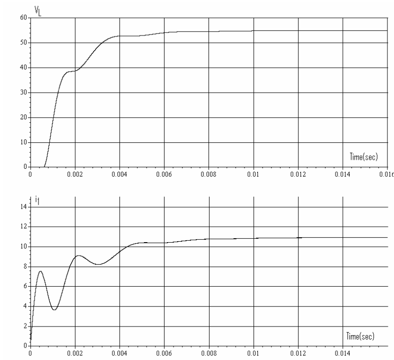

Fig. 3 Simulation results for the unperturbed case: Upper picture is V_L vs time (in seconds) and bottom picture is i_1 vs time (in seconds).

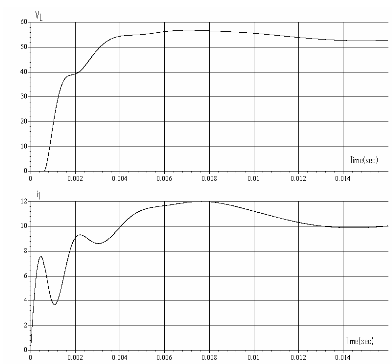

Fig. 4 Simulation results for the perturbed case: Upper picture is V_L vs time (in seconds) and bottom picture is i_1 vs time (in seconds).

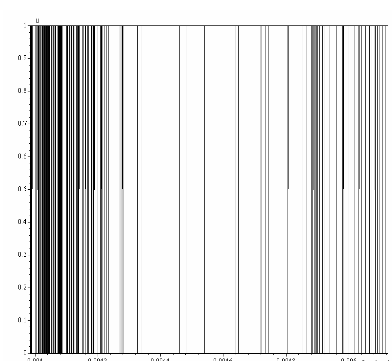

Fig.5 Plot of the control law u vs time (in seconds) for the perturbed case.

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