MMDS and scattering from multilayer cylindrical structure

A. P. ANYUTIN(1) and V.I. STASEVICH(1)

(1) Russian New University, 22, Radio Street, Moscow 1005007, Russia
tel: (095)182-2624

Abstract: - A new modification of the method of discrete sources are applied for solving a 2D scattering problems by metal or dielectric cylindrical structure covered with dielectric layer. The developed method. The problems of accuracy, choosing auxiliary contours and stable results are discussed.

Key-Words: - the modification of the method discrete sources, multiplayer scattering problems.

1 Introduction

It is known that scattering from 2D multilayer cylindrical structure could be reduced to multiple scattering problems. The last problem is a classical problem of the diffraction theory, which has been under wide discussing since the middle of 20th century. The main contribution in theoretical investigation of this problem was made in works [1-4].

This paper is concerned some numerical aspects of the method MMDS for solving a 2D multilayer problem deal with extending a new modification of the method of discrete sources (MMDS) for solving a scattering problem by metal or dielectric cylindrical structures covered by dielectric layer (- or layers).

2 2D multilayer scattering problem by covered metal cylindrical structure

In the beginning let consider the 2D scattering problem of E polarized wave \( u_0(\vec{r}) \) by perfect conducting cylinder with cross section contour as \( r_2(\varphi) \) function in cylindrical system of coordinates \((z,r,\varphi)\) which is placed into dielectric cylinder with with cross section contour as \( r_1(\varphi) \) function (see Fig.1). So we have two surfaces \( S_1, S_2 \) with different centers location. The diffracted field \( u^1(\vec{r}) \) outside of \( S_1 \) and refracted field \( u_2(\vec{r}) = u_{12}(\vec{r}) + u_{21}(\vec{r}) \) between \( S_1 \) and \( S_2 \) have to satisfy next boundary condition for E polarized incident wave on each \( S_1, S_2 \):

\[
[u_{21}(\vec{r}) + u_{12}(\vec{r})]_{S_1} = [u_0(\vec{r}) + u^1(\vec{r})]_{S_1} \quad (1)
\]

\[
\frac{d}{dn}[u_{21}(\vec{r}) + u_{12}(\vec{r})]_{S_1} = \frac{d}{dn}[u_0(\vec{r}) + u^1(\vec{r})]_{S_1} \quad (2)
\]

\[
[u_{21}(\vec{r}) + u_{12}(\vec{r})]_{S_2} = 0, \quad (3)
\]

In accordance with the method MMDS we could present each field \( u^1(\vec{r}), u_{12}(\vec{r}), u_{21}(\vec{r}) \) as follows:

\[
u^1(r,\varphi) = \sum_{m=1}^{M} A_m H_0^{(2)}(k | \vec{r} - \vec{r}_m |), \quad (4)
\]

\[
u_{12}(r,\varphi) = \sum_{m=1}^{M} A_{ml} H_0^{(2)}(k_1 | \vec{r} - \vec{r}_{m1} |), \quad (5)
\]

\[
u_{21}(r,\varphi) = \sum_{m=2}^{M} A_{m2} H_0^{(2)}(k_1 | \vec{r} - \vec{r}_{m2} |).
\]

Here, \( H_0^{(2)}(k | \vec{r} - \vec{r}_{m1,m2} |) \) are fundamental solutions to the Helmholtz equation (sources); \( k, k_1 = k\sqrt{\varepsilon_1} \) - are wave numbers of the free space and dielectric respectively ; \( \varepsilon_1 \) - is the relative dielectric penetrability; \( A_{m1,m2} \) are the coefficients to be determined; \( | \vec{r} - \vec{r}_{m1,m2} | = \sqrt{r^2 + r_{m1,m2}^2 - 2rr_{m1,m2} \cos(\varphi - \varphi_m)} \) are the distance between points given by the radius vectors \( \vec{r} \) and \( \vec{r}_{m1}, \vec{r}_{m2} \) in polar coordinates; \( r_{m1}, r_{m2} \) are the radius vectors positions of the sources on auxiliary contour \( \Sigma 11 \) within the boundary \( S1; \vec{r}_{m1} \) are the radius vectors positions of the sources on auxiliary contour \( \Sigma 12 \) outside of the boundary \( S1; \vec{r}_{m2} \) are the radius vectors positions of the sources on auxiliary contour \( \Sigma 21 \) inside of the boundary \( S2; \) \( M \) is the total number of sources on auxiliary contour \( \Sigma 11 \) (or on \( \Sigma 12 \) and \( \Sigma 21 \)). Placing \( \vec{r} \) on \( S1, S2 \) and satisfying (1)-(3) in M points we obtain a system of algebraic equations for coefficients \( A_m, A_{ml}, A_{m2} \).

We had constructed these auxiliary contours \( \Sigma 11, \Sigma 12, \Sigma 21 \) (or in other words we had constructed each functions \( r_{\Sigma 11}(\theta) \), \( r_{\Sigma 12}(\theta) \), \( r_{\Sigma 21}(\theta) \)) as a result of the analytical transformation of the original contours \( S1, S2 \) [5-7]:

\[
\zeta_{11} = r_1(\varphi) \exp\{i\varphi^*\}; \quad \varphi = \varphi' + i\varphi^*; \quad (6)
\]

\[
r_{\Sigma 11} = |\zeta_{11}|; \quad \theta = \arg \zeta_{11},
\]

\[
\zeta_{12} = r_1(\varphi) \exp\{i\varphi^*\}; \quad \varphi = \varphi' + i\varphi^*;
\]
the dielectrical elliptical cylinder $S_1$ (with parameters: $k_1a = 4; k_2b = 12; \varepsilon_r = 4$, $\delta = 10^{-6}$) is shown at Fig. 2.

![Fig.2 Scattering pattern by metal cylinders covered by dielectric layer: $1 - \varphi_0 = 0$, $2 - \varphi_0 = \pi / 4$, $3 - \varphi_0 = \pi / 2$.](image)

### 3 2D multilayer scattering problem by covered dielectric cylindrical structure

Let us now consider the 2D multilayer scattering problem by dielectric cylinder covered by dielectric layer. In this situation we have to construct one more auxiliary contour $\Sigma 22$:

$$\zeta_{22} = r_2 (\varphi) \exp{\{i\varphi\}}; \varphi = \varphi' + i\varphi'';$$

$$r_{22} = |\zeta_{22}|; \theta = \arg \zeta_{22}.$$  

and to take into account the refractive field $u_{22}(\vec{r})$ inside of $S2$:

$$u_{22}(r, \varphi) = \sum_{m=1}^{M} A_m H_0^{(2)}(k_2 |r - \vec{r}_m|),$$  

where $|r - \vec{r}_m| = [r^2 + r_m^2 - 2rr_m\cos(\varphi - \varphi_m)]^{1/2}$, $k_2 = k\sqrt{\varepsilon_2}$, $\vec{r}_m$ are the radius vectors positions of the sources on auxiliary contour $\Sigma 22$ inside of the boundary $S2$. So we have a boundary condition on $S1$, $S2$:

$$[u_{21}(\vec{r}) + u_{12}(\vec{r})]|_{S1} = [u_0(\vec{r}) + u^l(\vec{r})]|_{S1}$$  

$$\frac{d}{dn}[u_{21}(\vec{r}) + u_{12}(\vec{r})]|_{S1} = \frac{d}{dn}[u_0(\vec{r}) + u^l(\vec{r})]|_{S1}$$  

$$[u_{21}(\vec{r}) + u_{12}(\vec{r})]|_{S2} = u_{22}(\vec{r}) |_{S2}$$  

$$\frac{d}{dn}[u_{21}(\vec{r}) + u_{12}(\vec{r})]|_{S2} = \frac{d}{dn}[u_{22}(\vec{r})]|_{S2}$$
Example of calculated relative amplitude of the scattering pattern $g(\theta)$ for plane incident wave with different angle of incidence $\varphi_0$, perfect conducting elliptical cylinder $S2$ (with parameters: $ka2 = 1; kb2 = 3; \varepsilon 2_r = 4; xh = 2; yh = 2$ – coordinates of the cylinder’s center in coordinate system deal with dielectric cylinder) placing inside of the dialectical elliptical cylinder $S1$ (with parameters: $ka1 = 4; kb1 = 12; \varepsilon 1_r = 2$) is shown at Fig.3.

![Fig.3 Scattering pattern by metal cylinders covered by dielectric layer; 1 - $\varphi_0 = 0$, 2 - $\varphi_0 = \pi/4$, 3 - $\varphi_0 = \pi/2$.](image)

4. Conclusion
The method can easily be extended to plane-layered media, H polarized incident field, and 3D vector fields.

Acknowledgments
This work was supported by the Russian Foundation for Basic Research, project no. 00-02-17639.

References: