Elliptical Fourier Descriptors for shape retrieval in biological images

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Abstract: - The retrieval of information in a standard database can be obtained in different ways and in some cases a number of steps are necessary to extract the sought for information. More difficulties exist when we are looking for a pictorial information in a database. This paper presents a method for finding an image in a biological database based on elliptical Fourier descriptors . Fourier descriptors are considered here as textual information and a distance function is proposed to evaluate the best result. Fourier descriptors and distance function are used to retrieve image in pictorial database.

Key-Words: elliptical Fourier analysis, pictorial database, features extraction, edge, classification, retrieval.

1 Introduction

The relation between the object's representation and their features is, in many cases, very laborious to find. The search of the best features characterizing an object can be a good solution to the problem of object's research in pictorial data base or to recognition's approach. Frequently, edge have been used to describe objects and their properties have been considered to study such objects. Elliptic Fourier Analysis (EFA), Freeman chain, linear approximation and Hartley descriptors are some of the main methods used to describe the properties of an edge [8]. Foresti and Murino in [6] proposed a method, based on a voting approach, in which objects are recognized in non standard environment, where those images are considered with not completely deformed shapes. A lot of papers have been presented over the last years and some research's models have been used in different perspective. Yip et al. in [12] analyze a method, based on the elliptic Fourier Descriptors(FD), to detect symmetry in a binary image. There are several advantages to represent a closed edge with a set of Fourier Descriptors (FD): invariance to translation, rotation and scaling. Leon et Sucar in [4] proposed an approach to recognize human silhouettes based on FD, they obtained a 97% correct recognition but no study was done on harmonics's selection, so they proposed the first 40 descriptors as best choice. Yu and Lo in [13] combined FD and Convex Hull to improve the recognition process of 3-D planar objects, such a method allows to recognize objects under occlusion when only a limited number of segments are affected by occluded objects. The recognition problem is strictly related to the classification one, underwater objects and handwritten objects are some of most complex problems in the field of classification, Boulinguez and Quinquis put their attention on the classification of underwater objects. In [2] they adopted FD for analyzing an acoustic signal, like sonar, to extract descriptors from each projection of the underwater objects. Taxt and Bejerde in [11] studied the applicability of FD to classify handwritten objects, they divide each single object in subclasses of fourier descriptors.
The characteristic of invariant, in some cases, can be hazardous for the recognition or classification problems (ie: a tower can be identified as a glass and viceversa), otherwise they can help to identify objects. FD do not work at the best in the case of distortion; normalized descriptors and parametric methods were pro-

posed in the past to resolve those outlets: Zahn and Roskies in [14], Kuhl and Giardina in [7], Lin and Hwang in [9], Arbet and Snyder in [1], Diaz et al. in [5] are only some examples of them. Our contribution intend to study the use of EFA to extract a set of descriptors and then to characterize releted objects by them. A good identification of a shape gives a good starting point to extract its Elliptic Fourier Descriptors (EFD); standard edge detection gives a good solution to this problem. Moreover, Freeman encoding allows to represent edges in a numerical data structure which can be manipulated algebraically for extracting the EFD.

In this paper, EFD has been used to classify objects in biomedical images, and a distance function is introduced in section 2.1.3 for the classification of input images. It returns the best fit among fourier descriptors of input image and of stored images. A general description of the method is given in section 2, and a short presentation of the Fourier analysis can be found in the section 2.1.2, more details can be found in [7]. Experimental results and finale remarks are given in section 3 and section 4 respectively.

2 Description of the method

The method is based on Fourier coefficients to describe a closed contour by a function. In the case of objects classification, text information has been used to retrieve such objects from a pictorial database. This method intends to extract fourier coefficients from an image to classify it in a database of bi-dimensional images. Two different phases have been adopted to reach this goal, each of one is stepped as follow:

1. Pictorial Indexes Computation
   (a) Edge detection
   (b) Edge encoding
   (c) Fourier coefficient extraction
   (d) Storing in the database
2. Searching Phase
   (a) Edge detection
   (b) Edge encoding
   (c) Fourier coefficient extraction
   (d) Search in the data base

2.1 Pictorial Indexes Computation

The main aim of this phase is to define the database, which will contain not only the picture objects but also the elliptical fourier coefficients describe the shapes in the image need to retrieve images when a query will be launched (searching phase).

To extract elliptical fourier descriptors for a closed contour, it is necessary to identify at the best the closed contour of the object. In a grey level image, the edges of a shape are not well evident and for its correct extraction several standard edge detection algorithms can be used. In our implementation of the methodology we used one of the most common edge detection algorithms proposed in literature [3].

2.1.1 Edge encoding

The edge encoding starting from the result of edge detection; it is based on the Freeman encoding of a closed contour; following the description of Freeman, a closed contour can be codified by a set of eight standard lines drawing on the 3×3 matrix in which the central element is the foot of the line and the head of it is given by one of elements of the matrix (see fig.1).

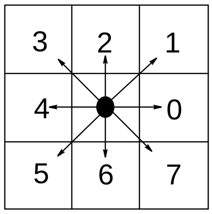


Figure.1 : Vectorial representation of Freeman encode.

With freeman encode a closed contour can be describe by a chain

$$C = u_1u_2u_3u_4....u_n,$$

where $u_i \in \{0, 1, ...7\}$ is an oriented vector on the $(\pi/4)u_i$ direction. The module of each u_i will be equal to 1 or $\sqrt{2}$ when each u_i will be even or odd respectively (see equ.1). For example the Freeman encoding of the edge shown in fig.2 will be given by the following chain:

$$000776676654433444432111$$

. Note that the starting point of the contour is given by the pixel on the up left corner.

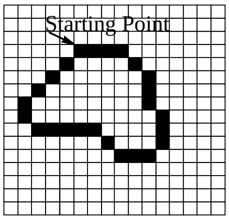


Figure.2 : Example of binary image contour.

Let u_i be an element of the Freeman chain; its length (dt_i) will be given by the following expression:

$$|u_i| = dt_i = 1 + \left(\frac{\sqrt{2}-1}{2}\right)(1-(-1)^{u_i}), \quad (1)$$

therefore, if n is the number of elements in the chain of the whole contour, the length of the chain will be given by:

$$t_n = \sum_{i=1}^{n} dt_i.$$

In the following T will identify such length and t_p will give the length of p elements of the chain.

Let dx_i and dy_i the projections of u_i on the X and Y axis respectively; their values will be given by the following ruled:

$$dx_i = sign(6-u_i) * sign(2-u_i)$$

$$dy_i = sign(4-u_i) * sign(u_i)$$

where:

$$sign(\Omega) = \begin{cases} 1 & \text{if } \Omega > 0, \\ 0 & \text{if } \Omega = 0, \\ -1 & \text{if } \Omega < 0. \end{cases}$$

Given a generic element p of the Freeman chain the projection of all elements $1..p$ on the X and Y axis will be respectively:

$$x_p = \sum_{i=1}^{p} dx_i,$$

$$y_p = \sum_{i=1}^{p} dy_i.$$

On this new image will be evaluated the elliptical coefficients of the fourier analysis.

2.1.2 Elliptical Fourier Analysis

The main aim of the elliptical fourier analysis step consists to approximate a closed edge as a sum of elliptic harmonics.

Kuhl and Giardina in [7] use four fourier coefficients a_i, b_i, c_i e d_i for each harmonic, and to identify the closed contour of k elements they consider N harmonics.

The a_i, b_i, c_i e d_i coefficients are given by:

$$a_n = \frac{T}{2n^2\pi^2}\sum_{i=1}^{k}\frac{dx_i}{dt_i}\left[\cos\frac{2n\pi t_i}{T} - \cos\frac{2n\pi t_{i-1}}{T}\right],$$

$$b_n = \frac{T}{2n^2\pi^2}\sum_{i=1}^{k}\frac{dx_i}{dt_i}\left[\sin\frac{2n\pi t_i}{T} - \sin\frac{2n\pi t_{i-1}}{T}\right],$$

$$c_n = \frac{T}{2n^2\pi^2}\sum_{i=1}^{k}\frac{dy_i}{dt_i}\left[\cos\frac{2n\pi t_i}{T} - \cos\frac{2n\pi t_{i-1}}{T}\right],$$

$$d_n = \frac{T}{2n^2\pi^2}\sum_{i=1}^{k}\frac{dy_i}{dt_i}\left[\sin\frac{2n\pi t_i}{T} - \sin\frac{2n\pi t_{i-1}}{T}\right].$$

From the geometrical point of view a_i e b_i represent the projection on the X axis of the semimajor and therefore the semiminor axis of an i harmonic and c_i e d_i on the Y axis.

The inverse process is easy to develop and it allows to identify the closed contour from the N harmonics, in particular the coordinates of an i point of contour are calculated by:

$$X_i = X_c + \sum_{n=1}^{N} a_n\cos\frac{2n\pi t_i}{T} + b_n\sin\frac{2n\pi t_i}{T},$$

$$Y_i = Y_c + \sum_{n=1}^{N} c_n\cos\frac{2n\pi t_i}{T} + d_n\sin\frac{2n\pi t_i}{T}.$$

where X_c e Y_c represent the coordinates of the centroid with the following meaning:

$$X_c = \frac{1}{T}\sum_{i=1}^{k}\frac{dx_i}{2dt_i}(t_i^2 - t_{i-1}^2) + \gamma_i(t_i - t_{i-1}),$$

$$Y_c = \frac{1}{T}\sum_{i=1}^{k}\frac{dy_i}{2dt_i}(t_i^2 - t_{i-1}^2) + \delta_i(t_i - t_{i-1}).$$

In our case, because the centroid is allocated at the center of the axis, its coordinates have to be considered equal to 0 .
Because the starting point can be chosen randomly and such choice can change the values of the coefficients. It is necessary to have a representation which is independent from the starting point. This is archived computing the phase's shift from the first major axis:

$$\theta_1 = \frac{1}{2}\text{arctg}\frac{2(a_1 b_1 + c_1 d_1)}{a_1^2 + c_1^2 - b_1^2 - d_1^2}.$$

Then, the elliptic coefficients can be rotated until a shift phase is equal zero, that is:

$$\begin{bmatrix} a_n^* & b_n^* \\ c_n^* & d_n^* \end{bmatrix} = \begin{bmatrix} a_n & b_n \\ c_n & d_n \end{bmatrix} \begin{bmatrix} \cos n\theta_1 & -\sin n\theta_1 \\ \sin n\theta_1 & \cos n\theta_1 \end{bmatrix}.$$

The translation, rotation and size features have to be considered when a classification problem has to be treated, the invariance of these features allows to have a efficient classification and also a more accurate retrieval data method. Because our challenge is to have an efficient tools to retrieve image those features have been considered.

2.1.3 Classification and Retrieval

Our classification is based on the coefficient of fourier, and each quadruple (a_i,b_i,c_i,d_i) defines an elliptical harmonic of a closed contour. Moreover, each quadruple will be modified to be invariant from starting point, translation, rotation and scaling(a_i^{***},b_i^{***},c_i^{***},d_i^{***}: for more details see section 3.1). In figure 3 is shown an example of some representative measures of one image of database.

The images of database are classified by a set of harmonics for each closed contour of them, in an other words by 50×4 invariant coefficients ($\text{N} = 50 \equiv$ harmonics).

Given the set of harmonics e_n^{***}, f_n^{***}, g_n^{***} e h_n^{***} ($\forall n \in [1, .., N]$) for retrieving linked image, they can be compared with all coefficients in the database. The function to compute the minimum distance between input image and pictorial database images(M) is given by:

$$Dist(i) = \sum_{n=1}^{N} |\Delta_n - \Delta_{i,n}|. \qquad (2)$$

where:

$$\Delta_n = |e_n^{***}| + |f_n^{***}| + |g_n^{***}| + |h_n^{***}|$$

$$\Delta_{i,n} = |a_{i,n}^{***}| + |b_{i,n}^{***}| + |c_{i,n}^{***}| + |d_{i,n}^{***}|$$

and i=Card(M).

To obtained the best choice a minimum distance was adopted:

$$\eta = \min_i(Dist(i)).$$

It represent the image in the pictorial data base nearest to input image, moreover a percentage of similarity was computed by:

$$\%Sim = \left(1 - \frac{Dist(i) - \eta}{\lambda - \eta}\right) \times 100 \qquad (3)$$

where

$$\lambda = \max_i(Dist(i))$$

n	A $Axis$	B α_{Axis}	C	D
1	-59.254 30.284	73.619 322	82.065	25.779
2	-1.268 1.030	-2.351 76	0.624	-4.193
3	0.807 1.758	4.792 314	3.452	-3.499
4	1.047 0.761	1.382 69	2.544	0.574
5	1.758 0.543	0.233 41	0.487	-1.727
6	0.504 0.529	1.210 70	-0.725	1.644
7	0.938 0.361	0.610 33	0.511	0.743
8	0.583 0.229	-0.693 328	-0.433	-0.035
9	-0.366 0.385	-1.363 359	-0.085	0.089
10	-0.440 0.139	0.221 300	0.267	0.365

Figure.3 : Table of Fourier Descriptors.

3 Experimental results

Experimental results concern an application of such method in the biological field, in particular a retrieval task was implemented on a special pictorial database on which standard and nonstandard data were involved. Edges and Fourier descriptors for each image were calculated, each database image and input image were evaluated with the distance function given in (2) for retrieving the input image in data base. In other words the degree of similarity with each image of database was evaluated by (3).

In particular, for each image, our database consists of: original image (an example is given in figure 4), the extracted edges (see fig.5) and a table of descriptors (fig.3). The descriptors' table, given in figure 3, is organized by rows, where each row means four Fourier descriptors(A,B,C,D) of the harmonic (n), length of semimajor axis($Axis$) and its angle with X axis (α_{Axis}).

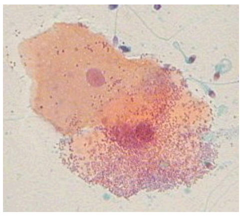


Figure.4 : Example of biological image.

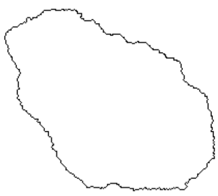


Figure.5 :Edge Extracted from the image in Figure 4.

3.1 Implementation note

The main aim of the application regards the retrieval of input image in a pictorial data base. The strategy is based on the invariance for translation, rotation and scaling of the methods. We tested the method on a biological pictorial database and its implementation was realized with Matlab [10] on a Pentium IV machine.

A rotational invariant can be introduced easily: the major axis of the first harmonic of each image has been aligned to the X axis, to do that a rotation matrix has been calculated by:

$$\begin{bmatrix} \cos\phi_1 & \sin\phi_1 \\ -\sin\phi_1 & \cos\phi_1, \end{bmatrix}$$

where the angle ϕ_1 is given by: $\phi_1 = \mathrm{arctg}\frac{c_1^*}{a_1^*}$ with a_1^* e c_1^* Fourier descriptors of the first harmonic, invariants by starting point.

These coefficients have been normalized in rotation by the following equation:

$$\begin{bmatrix} a_n^{**} & b_n^{**} \\ c_n^{**} & d_n^{**} \end{bmatrix} = \begin{bmatrix} a_n^{*} & b_n^{*} \\ c_n^{*} & d_n^{*} \end{bmatrix} \begin{bmatrix} \cos\phi_1 & \sin\phi_1 \\ -\sin\phi_1 & \cos\phi_1 \end{bmatrix}$$

In the same way a size invariant has been obtained dividing by the length of the semimajor axis L of the first harmonic:

$$L = \sqrt{a_1^{*2} + c_1^{*2}}.$$

Therefore the final coefficients, invariant to rotation, translation and scaling, have been obtained by:

$$\begin{bmatrix} a_n^{***} & b_n^{***} \\ c_n^{***} & d_n^{***} \end{bmatrix} = \begin{bmatrix} a_n^{**} & b_n^{**} \\ c_n^{**} & d_n^{**} \end{bmatrix} \frac{1}{L}$$

A set of images have been stored in a data base and a lot of tests have been performed to retrieve images from it. To test the goodness and reliability of the method we considered either distorted images from stored images and their result can be observed in figure 6 and 7, either new other biological images and their result is shown in figure 8. Figure 6 shows the result of an experiment with systematic distortion, each image of database is distorted in X with a range of distortion d included among -50 and 50 ($-50 \leq d \leq 50$), in figure 7 we give the results of an experiment with a random distortion where was randomly changed the curvature of image; the percentage of distortion is given by the percentage of changed pixels on the edge ($-50 \leq d \leq 50$). Particular examples of distorted image are shown in figure 9, on the table, *Original* image is the stored image in the database, dil_{xx} are the distorted images, $Edge$ are extracted contours, and $\%Sim$ are the percentage of similarity to the original image. In figure 8 it is shown a plot referencing other new biological images not present in our database, a systematic view of image retrieval can be seen in figure 10.

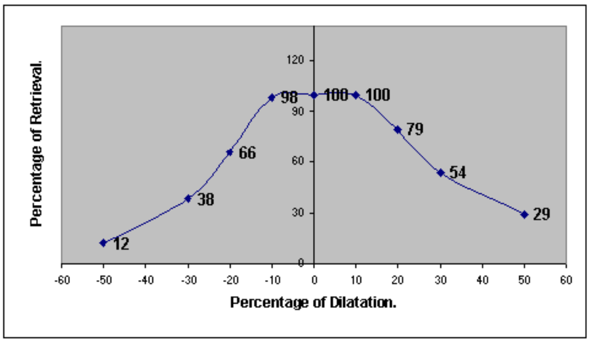


Figure.6 : Percentage of retrieval images on database when a systematic deformation was imposed.

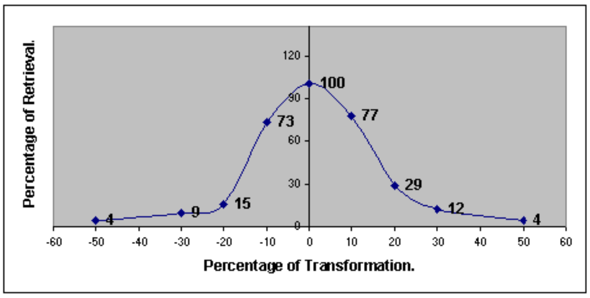


Figure.7 : Percentage of retrieval images on database when a random deformation was imposed.

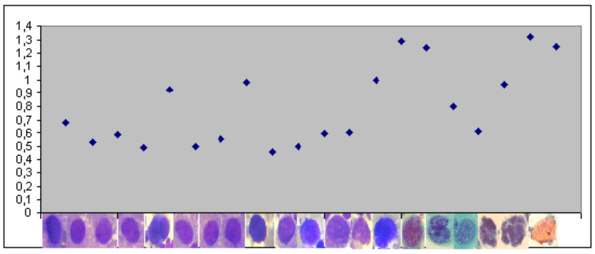


Figure.8 : Minimum distance from database images when new biological image was searched.

4 Final remarks

In this paper a retrieval method for pictorial database has been presented, and it has been tested on biological images. The database has been organized with images and their EFD.

The method gave excellent results either with rotated, translated and scaled images, because such method is invariant to these transformations. It gave good result when a lot of distorted images were searched in database. Two different distortions have been considered: systematic distortion gave a percentage of retrieval larger than 98% for a distortion smaller than 10%, otherwise when a random distortion is forced the percentage of retrieval is greater than 80% for a distortion smaller than 10%. More interesting result has been obtained when similar biological images have been searched in the database (skewed images, images drawn by the user, sub-images, etc...).

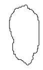
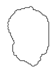
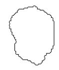
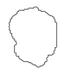
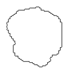
	Image	*Edge*	*%Sim*
dil_{-50}			88
dil_{-30}			97
dil_{-20}			100
dil_{-10}			100
Original			
dil_{10}			100
dil_{20}			100
dil_{30}			93
dil_{50}			78

Figure.9 : Example of retrieval with systematic distortion.

The table in figure 10 reports results of the search of a new sub-images in database, it is composed by 2 columns and each row identifies a *inputimage* to search and the best *found images* together its distance η. The image found in figure 10 seem to be, in many cases, far from the nearest image, but because such method is based only on shape retrieval, and moreover, the intrinsic characteristics of the method (invariance to rotation, translation, and scaling) can be considered as the best image inside the database. Unlike it is not clear just how it is possible to reduce the number of useful harmonics to identify an image. The obtained results encourage us to modify the method and find a minimum set of harmonics (how and what harmonics) to reduce the computational time of the research without reduce the goodness of it.

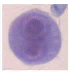
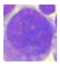
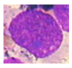
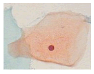
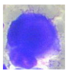
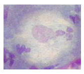
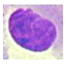
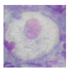
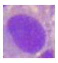
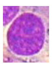
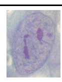
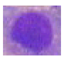
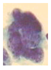
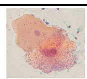
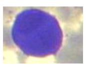
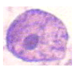
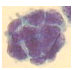
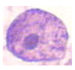
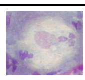
Input Image	*Found Image/η*	*Input Image*	*Found Image/η*
	 0,682		 0,593
	 0,531		 0,600
	 0,587		 0,995
	 0,494		 1,286
	 0,923		 1,239
	 0,496		 0,802
	 0,555		 0,608
	 0,983		 0,968
	 0,456		 1,322
	 0,496		 1,246

Figure.10 : Example of retrieval with new biological images.

5 Acknowledgement

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References:

[1] K. Arbet and W. Snyder. Application of affine-invariant fourier descriptors to recognition of 3-d objetcs. *IEEE Transaction on Pattern Analysis and Machine Intelligence*, 8(7):640–647, 1990.

[2] D. Boulinguez and A. Quinquis. Classification of underwater objects using fourier descriptors. In *Proc. of Seventh International Conference on Image processing and its applications*, volume 1, pages 240–244, 1999.

[3] J. Canny. A computational approach to edge detection. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 8(6):679–697, 1986.

[4] R. DeLeon and L. Sucar. Human silhouette recognition with fourier descriptors. In *Proceedings. 15th International Conference on Pattern Recognition*, volume 3, pages 709–712, 2000.

[5] G. Diaz, A. Zuccarelli, I. Pelligra, and A. Ghiani. Ellictic fourier analysis of cell and nuclear shapes. *Computer and biomedical research*, 22:405–414, 1989.

[6] G. Foresti and V. Murrino. A voting-based approach for fast object recognition in underwater acoustic imagess. *IEEE Journal of Oceanic Engineering*, 22(1):57–63, 1997.

[7] F. Kuhl and C. Giardina. Elliptical fourier features of a closed contour. *computer graphics image processing*, 18:236–258, 1982.

[8] L. Legrand, K. Khalil, and A. Dipanda. Representing plane closed curves with hartley descriptors. In *Proceedings., International Conference on Image Processing*, volume 3, pages 344–347, 1995.

[9] C. Lin and C. Hwang. New form of shape invariants from elliptical fourier descriptors. *Pattern Recognition*, 20:535–545, 1987.

[10] W. PalmIII. *Introduction to MATLAB for Engineers*. McGraw-Hill, 1999.

[11] T. Taxt and K. Bjerde. Classification of handwritten vector symbols using elliptic fourier descriptors. In *Proceedings of the 12th IAPR International Conference on Pattern Recognition*, volume 2, pages 123–128, 1994.

[12] R. Yip, P. Tam, and D. Leung. Application of elliptic fourier descriptors to symmetry detection under parallel projection. *IEEE Transaction on Pattern Analysis and Machine Intelligence*, 16(3):227–286, 1994.

[13] M. Yu and K. Lo. Object recognition by combining viewpoint invariant fourier descriptor and convex hull. In *Proceedings of International Symposium on Intelligent Multimedia, Video and Speech Processing*, pages 401–404, 2001.

[14] C. Zahn and R. Roskies. Fourier desciptiors for plane closed curves. *IEEE Transaction on Computing*, C-21(3):269–281, 1972.