Abstract: This paper presents the application of hybrid direct adaptive fuzzy control techniques to a missile autopilot design. The proposed control structure is to augment existing conventional controller with a fuzzy model reference learning controller (FMRLC). The conventional control law is chosen to stabilize the missile system and to provide approximate control. A feature of this scheme is that the FMRLC credit output will be added simply to the existing control signal. Moreover, the missile systems with high nonlinearities and aerodynamic uncertainties can be controlled over a wide operational envelope. A six-degrees-of-freedom flight simulation model for guided missile system is developed for verification. The results are promising and clearly demonstrate the potential of hybrid FMRLC scheme.

Keywords: Fuzzy logic, adaptive fuzzy, neural networks, missile control

NOMENCLATURE
Abbreviations
FMRLC = fuzzy model reference learning controller
6DOF = Six-Degrees-of-Freedom
STT = Skid To Turn
CoG = Center of Gravity

Symbols
$I_{xx}$, $I_{yy}$, $I_{zz}$ = Moments of inertia about airframe axes.
$U$, $v$, $w$, $V_m$ = Velocity components along the missile’s axes (X, Y, Z), and total missile velocity, respectively.
$p$, $q$, $r$ = Missile angular rates (roll, pitch, yaw).
$T_x$ = Motor thrust force.
$F_x$, $F_y$, $F_z$ = Aerodynamic forces.
$M_x$, $M_y$, $M_z$ = Aerodynamic moments.
$c_{m\alpha}$, $c_{m\delta}$, $c_{mq}$ = Pitching moment derivatives.
$c_{n\beta}$, $c_{n\delta}$, $c_{mr}$ = Yawing moment derivatives.
$c_y$, $c_y^\delta$ = Side force derivatives.
$c_z$, $c_z^\delta$ = Z-axis force derivatives.
$\delta^d$, $\delta^a$ = demanded and achieved fin deflections.
$\tau_a$ = first order actuator time constant.
$\alpha$, $\beta$ = Angle-of-Attack and sideslip angle.
$\phi$, $\theta$, $\psi$ = Euler’s angles (roll, pitch, heading angles).
$c_x$, $c_y$, $c_z$ = Aerodynamic force coefficients that describe the missile airframe.
$c_{m\alpha}$, $c_{m\delta}$, $c_{m\psi}$ = Aerodynamic moment coefficients that describe the missile airframe.
d, $\ell$, $m$, $s$ = Missile diameter, length, mass, and reference cross-sectional area ($\pi d^2 / 4$) respectively.
$\rho$, $Q$ = Air density and atmospheric dynamic pressure ($1/2 \rho V_m^2$).

1. Introduction

In the past, the field of missile guidance and control system design has been dominated by classical control techniques [1-4]. Several classical and modern control theories have been proposed for designing longitudinal and lateral control laws. Typically these techniques are either time domain or frequency domain based, are applicable to linearized and time-invariant plants. Nonlinearities and time-varying effects must be coped with by a robustness margin of the control loop. The performance of the loop is hence not constant but will change with the operating point. When the time variation and nonlinearities are severe, it may not be an easy task to find a controller that can cope with it all.

While classical control methods have produced many highly reliable and effective guidance and control systems, recent years have seen a growing interest in applications of robust, nonlinear, adaptive and intelligent control theories to missile flight guidance and control systems.

Autopilots are typically designed using linear quadratic regulator (LQR) and/or classical control technique [3,4]. Traditional missile autopilot designs involve linearizing the vehicle dynamics about several operating conditions throughout the flight envelope, designing linear controllers for each condition, and blending these point designs with an interpolation scheme. One of the most popular control design approaches that has been widely and successfully applied in the aerospace control field is gain scheduling [5-7]. While gain scheduling is conceptually simple and has been proven successful, it has no guarantee of
stability in the transitional periods between operating points and relies on the fact that scheduling variables should only change slowly. Furthermore, there is a heavy design overhead due to the large number of linear controllers, which must be derived. Thus, alternatives to linear control are being considered.

Alternative nonlinear techniques [8,9] include feedback linearization, sliding mode control, recursive back-stepping, and dynamic inversion have been developed.

While robustness of the controller and the performance of the system are sometimes two conflicting objectives, optimization between both of these goals is possible. Adaptive control [10] systems played a main role to improve the performance of nonlinear dynamics of flight control system. Adaptive control structure has been integrated with artificial intelligent expert systems as the current control technology to overcome some of the unsolved control problems.

The main advantage of intelligent over classical control is that the former can provide robust systems when there are model and environmental uncertainties. Neural networks and fuzzy logic [11-13], by giving control laws based on input-output relationships, avoid the need for accurate knowledge of system dynamics, and are thus insensitive to their changes. Examples of application of intelligent control to missile autopilot design are in [14-18].

Fuzzy sets and fuzzy logic, introduced in 1965 by Zadeh [19], are applied in a wide variety of disciplines. Many examples of fuzzy control applications exist in the literature. Hence, the growing interest in fuzzy control is understandable. There have been well-documented applications of adaptive fuzzy [11,20,21] such as self-tuning and self-organizing controllers to various practical systems. The increase in the complexity does have practical drawbacks that need to be considered and justified.

In simple systems the classical controller may be preferred while systems with more complex requirements and capabilities, the increased abilities of the fuzzy controller may be useful. In such a system, it is frequently advantageous to use hybrid intelligent systems. The resulting control system can incorporate many desirable qualities, such as robustness, ease of adaptability to new tasks, and is faster to produce than traditional methods that are heavily model dependent.

Another feature of intelligent systems is that they could combine knowledge, techniques, and methodologies from various sources. These intelligent systems supposed possess humanlike expertise within a specific domain, adapt themselves and learn to enhance the performance in changing environments.

A missile control system has requirements that coincide with the strengths of a fuzzy system. The application of fuzzy logic to the missile guidance and control system problem required the investigation of several areas of fuzzy theory and missile systems that have been under development.

In this paper, a hybrid fuzzy model reference learning control scheme is proposed where a fuzzy logic controller is placed in parallel with a conventional fixed gain controller as shown in Fig. 1. The conventional control law is chosen to stabilize the missile system and to provide approximate control. The FMRLC is used to enhance the performance of conventional controller thus the missile systems with high nonlinearity and aerodynamic uncertainties can be controlled over a wide operational envelope. The fuzzy learning process can converge rapidly due to the approximate control criteria.

The paper is organized as follows. In section 2 the design of missile lateral autopilot based on the hybrid FMRLC are presented. In section 3, a 6-DOF-missile guidance and control model is presented. The nonlinear differential equations that describe the missile dynamics in the space are given to show the nonlinearities in the system kinematics and dynamics. Evaluations of the proposed scheme are given in section 4. Finally, this paper ends with the conclusions.

2. Hybrid FMRLC

The FMRLC shown in Fig. 1, has two fuzzy logic controllers; one named fuzzy controller to be tuned and the other named fuzzy inverse model along with the knowledge base modifier. A reasonable reference model is chosen to quantify the desired lateral autopilot (pitch/yaw) performance \((a_x, ref) / (a_y, ref)\) all over the flight envelope. The output of the reference model is compared with the actual output of the lateral missile dynamics \((a_x / a_y)\) to produce an error measurement vector \([e_{mp} e_{my}\] which is then used in conjunction with a knowledge base modifier as a reference for fuzzy controller rule modification. As a result the tracking error will be minimized and the lateral autopilot performance enhancement will be accomplished. The following subsections present the components of the hybrid FMRLC.

2.1 The conventional fixed gain controller

The conventional fixed gain controller in Fig. 1 was designed on the following widely acceptable assumptions for STT missiles systems; a) constant missile mass and moments of inertia and longitudinal velocity (operating point) b) The missile is roll stabilized (p=0). c) The missile airframe has axis symmetry around Y and Z-axes. While these assumptions will result in approximately decoupled pitch and yaw missile dynamics, the bank angle \(\phi\) will still cause coupling.

The conventional fixed gain controlled quantity is the measured missile normal acceleration \((F_z/m, F_y/m)\). Basically, the difference between the desired and actual missile normal accelerations is sensed and used to
drive the control surface actuator to null the error. LQ optimal controllers are designed at several operating points (air speed) by linearizing the single-plane dynamics around certain set point and minimize a quadratic cost function. This approximate control criterion will assist the learning mechanism to converge more quickly.

2.2 The fuzzy controller

Basically, the fuzzy controller contains four main components: fuzzification, rule-base, inference mechanism, and defuzzification. The fuzzification interface simply modifies the inputs so that they can be interpreted and compared to the rules in the rule base. The rule base holds the knowledge in the form of a set of rules of how best to control the system. The inference mechanism or the decision making logic evaluates which control rules are relevant at the current time and then decides what is the input to the plant should be. The defuzzification interface converts the conclusions reached by the inference mechanism into the inputs to the plant to be controlled.

The inputs of the fuzzy controller are the system output errors, \([E_p, E_y]\), and their derivatives. This scheme uses 11 uniformly distributed triangular membership functions for each of its input universe of discourse and the minimum to represent the premise and implication. Whenever the input is high, the saturation of the left most and the right most membership functions are considered. For illustration, sample rules of the fuzzy scheme take the following form for pitch and yaw autopilot:

If \(E_p\) is \(I_1\) and \(\dot{E_p}\) is \(J_1\) then \(A_{pc}\) is \(R_p(1)\)

If \(E_y\) is \(M_1\) and \(\dot{E_y}\) is \(N_1\) then \(A_{yc}\) is \(R_y(1)\)  (1)

Variables, \(I_i, J_i, M_i, N_i, R_p(i),\) and \(R_y(i),\) take the linguistic values expressed by linguistic sets such as LN and LP that are interpreted as large negative and large positive respectively.

Tuning via scaling universes of discourse is applied. A great effort has been made to choose the proper input and output scaling gains. More emphasis should be put on finding out the optimum values.

In this paper, the triangular membership function, the minimum-maximum reasoning method, and the center-of-gravity (CoG) defuzzification method are used. The crisp output is obtained by calculating the CoG of the output fuzzy set. For continuous membership function, the CoG defuzzification method is defined as

\[
\text{CoG} = \frac{\int_{A}^{B} \mu_i(Z)Z\,dZ}{\int_{A}^{B} \mu_i(Z)\,dZ}
\]

(2)

where \(\mu_i(Z)\) is the output membership function and \(A\) and \(B\) are the intervals of the output \(Z\) in which the fuzzy set has a non-zero membership value. This method of defuzzification produces smooth output. This is the most widely adopted defuzzification strategy especially for continuous systems where the fuzzy sets heavily overlap.

2.3 Fuzzy learning mechanism

The fuzzy inverse model associated with the knowledge base modifier is responsible for the learning process. The learning mechanism modifies the rule base of the fuzzy controller (subsection 2.1) based on the reference model output error and its derivative. The inverse model is a fuzzy controller that performs a nonlinear mapping for the error from the desired performance, and the fuzzy modification sample rules take the following form for pitch and yaw plane:

If \(e_{mp}\) is \(A_1\) and \(\dot{e}_{mp}\) is \(B_1\) then \(\delta A_{pc}\) is \(pp(i)\)

If \(e_{my}\) is \(C_1\) and \(\dot{e}_{my}\) is \(D_1\) then \(\delta A_{yc}\) is \(py(i)\)  (3)

Variables, \(A_i, B_i, C_i, D_i, pp(i),\) and \(py(i),\) take the linguistic values expressed by linguistic sets. The defuzzification of the inverse model output is also carried out by the method of center of gravity as (2). Accordingly, the centers of the output membership functions of the fuzzy controller are modified. The center modification procedure is given as the following:

\[
\delta R_p(i) = \frac{\mu_i pp(i)}{A_p} \\
R_p(i) = R_p(i) + \delta R_p(i)
\]

(4a)

\[
\delta R_y(i) = \frac{\mu_i pp(i)}{A_y} \\
R_y(i) = R_y(i) + \delta R_y(i)
\]

(4b)

Thus, the learning mechanism contribution values are that, the centers of the membership functions are updated based on \(\mu_i\) certainty of their inference values.

3. Missile Mathematical Model

The missile simulation that was used to generate all results in this paper is a 6-DOF nonlinear dynamic model of a guided missile system. The missile is aerodynamically controlled via two pairs of rear control fins. It has two identical control channels, each channel has lateral acceleration autopilot loop that control the missile lateral acceleration to be very close to the target at the end of engagement. The on-board
measurement units are pneumatic fin servo, one accelerometer, one rate gyro, and the conditioning electronic circuits. In addition, a roll position control loop is utilized to keep the missile attitude fixed throughout the flight.

The equations for the missile’s CoG kinematical and dynamical motion, kinematical and dynamical rotation of the missile body around its CoG, and the on-board measuring and control devices are examined. Environmental parameter changes such as air density, velocity of sound as a function of altitude, and wind all effect the plant model. The motion of the missile in space is described by means of 6-differential equations. Referring to Fig. 2, the missile equations of motion are expressed in a state form as the following:

\[
\begin{align*}
\dot{u} &= (F_x / m) + rv - qw \\
\dot{v} &= (F_y / m) + pw - ru \\
\dot{w} &= (F_z / m) + qu - pv \\
\dot{\phi} &= p + (qS\phi + rC\phi)\tan\theta \\
\dot{\theta} &= qC\phi - rS\phi \\
\dot{\psi} &= (qS\phi + rC\phi)/C\theta
\end{align*}
\]

where \(B(3,3)\) is an Euler angles transformation matrix defined as in Eq. (6)

\[
B(3,3) =
\begin{pmatrix}
C\phi C\theta & C\phi S\theta - C\psi S\phi & C\phi S\psi + S\phi S\theta \\
S\phi S\psi - C\theta C\psi C\phi & C\phi C\psi + S\theta S\phi & S\phi C\psi - C\theta C\phi \\
S\theta & -S\theta C\phi & C\theta
\end{pmatrix}
\]

The aerodynamic coefficients are computed at several operating points and a linear interpolation procedure computes their values at any intermediate point. The aerodynamic force and moment coefficients that are presented previously are usually defined as a function of \(\alpha, \beta, \) and other parameters. Therefore, it is desirable to show the relationship between the velocity components and these angles. These relations are defined as:

\[
\alpha = \tan^{-1}(w/u), \quad \beta = \tan(v/u), \quad \theta_T = \sqrt{\frac{v^2 + w^2}{u^2}}
\]

where \(\theta_T\) is the angle between the velocity vector and the missile longitudinal axis and is referred to as the resultant angle of incidence.

The first order approximation to the actuator dynamics is given by

\[
\dot{\delta} = \frac{1}{\tau_a} (\delta_c - \delta)
\]

4. Simulation Results

A complete 6DOF-flight model for STT missile system is developed. A computer code that solves the model is implemented using BORLANDC. The modular concept is utilized in the code development. In practice there are many nonlinearities, aerodynamic uncertainties, and unmodeled dynamics. However, this work will present two sets of simulation results; tracking performance for sequence of step demanded accelerations and robustness study for aerodynamic uncertainties. An initial investigation on autopilot performance will be carried out for pitch plane only. A reference model, \(Y_{ref}(s)\), was chosen to provide the desired performance as the following:

\[
Y_{ref}(s) = \frac{\omega_{ref}^2}{s^2 + 2\zeta_{ref}\omega_{ref} + \omega_{ref}^2}
\]

\[
\omega_{ref} = \sqrt{200} \text{ (rad/sec)}, \quad \zeta_{ref} = 0.8
\]

4.1 Acceleration step command

To evaluate the performance of the hybrid FMRCL, various flight conditions have been simulated. A sequence of lateral acceleration step commands is imposed on the missile system and the reference model.

Figure 3 shows the missile acceleration response normalized with respect to the missile motor time burn out. The missile dynamic response is fast and provides adequate tracking of the output reference model. However a slight tendency to overshoot is monitored.
The initial acceleration goes in the opposite direction of each step command due to the nonminimum phase behavior of a tail controlled missile. The magnitude of the initial acceleration is due to the fast response of the autopilot. Thus, the proposed hybrid fuzzy structure has successfully forced the missile systems to follow the desired performance fast enough.

The control fin deflection normalized with respect to the maximum allowable fin deflection is presented in Fig. 4. While the control fin deflections tend to change rapidly to initiate the fast turn, lateral acceleration is maintained by almost a constant fin deflection. The history of the adaptation mechanism is presented in Fig. 5, where the rate of convergence is satisfactory.

4.2 Robustness test

To evaluate the robustness of the hybrid FMRLC with respect to uncertainties, various aerodynamic uncertainties have been considered. Each aerodynamic coefficient has been multiplied by series of scales [0.8 0.9 1 1.1 1.2]. This means we assume that -20%, -10%, +10%, and +20% error of the missile aerodynamics are uncertain.

Simulations have been performed for these suggested uncertain cases along with the nominal one. The results are presented in Fig. 6. The proposed scheme showed satisfactory robustness to the variations of the aerodynamic coefficient.

5. Conclusions

In this paper, we have proposed a hybrid fuzzy model reference learning control scheme to compensate for aerodynamic nonlinearities and uncertainties in a conventional fixed gain controller of a STT missile system. A direct benefit of this work is that the proposed scheme can be augmented with an existing conventional controller. The proposed control structure has been evaluated using a square wave of acceleration demand. Robustness for the missile aerodynamic uncertainties is among the advantages of the proposed approach.

This study is a part of an ongoing research to develop a complete full envelope robust stable missile guidance and control system. A possible extension for this work is underdevelopment to include a similar hybrid FMRLC for the yaw plane. A further study on the cross coupling nonlinearities and environmental noise effects will be considered.

References

Fig. 1 Hybrid FMRLC

Fig. 2 Missile airframe reference axes

Fig. 3 Normalized missile pitch acceleration response.

Rear View

Fig. 4 Normalized control fin deflection.

Fig. 5 Normalized FMRLC parameters.

Fig. 6 Normalized missile pitch acceleration response with aerodynamic uncertainties.