Decision-Directed Adaptive Algorithms for Mobile Radio Channel Estimation: A Comparative Study

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Abstract: In this paper two important adaptive algorithms for channel impulse response estimation are studied. The selected algorithms are the decision-directed recursive least square (RLS) and the decision-directed least mean square (LMS) algorithms, weighted and unweighted. These algorithms are tested to estimate the impulse response of the newly proposed geometrical-based hyperbolic distributed scatterer (GHDS) channel model. It is shown that the performance of both the ideal decision weighted LMS (IDWLMS) and the ideal decision weighted RLS (IDWRRLS) algorithms is better than that of the unweighted versions of these algorithms.

Keyword: Adaptive algorithm, channel estimation; decision-directed; least mean square (LMS); recursive least square (RLS).

1 Introduction

Over the past few decades, radio communication systems have undergone extensive developments. The demands that a radio system must fulfill are greater by the day. Channel estimation algorithms allow the receiver to approximate the impulse response of the channel and explain the behavior of the channel. This knowledge of the channel's behavior is well utilized in modern radio communications for improvement of the system performance. Adaptive channel equalizers utilize channel estimates to overcome the effects of inter symbol interference (ISI). Diversity techniques (e.g. in the IS-95 Rake receiver) utilize channel estimation to implement a matched filter such that the receiver is optimally matched to the received signal instead of the transmitted one [1]. Adaptive beamformers utilize some of the channel-estimated parameters to adjust their weights. Maximum likelihood detectors utilize channel estimates to minimize the error probability [2].

One of the most important benefits of channel estimation is that it allows the implementation of coherent demodulation. Coherent demodulation requires the knowledge the phase of the signal. This can be accomplished by using channel estimation techniques.

Once a model is established, its parameters should be continuously updated (estimated) in order to minimize the error as the channel conditions change. If the receiver has a prior knowledge of the information sent over the channel, it can utilize this knowledge to obtain an accurate estimate of the impulse response of the channel. This method is simply called training sequence based channel estimation. It has the advantage that can be used in any radio communications system quite easily. However, it has some drawbacks. One of the obvious drawbacks is that it is wasteful of bandwidth. Precious bits in a frame that might have been otherwise used to transport information are stuffed with training sequences for channel estimation.

Blind methods, on the other hand, require no training sequences. They utilize certain underlying mathematical information about the kind of data being transmitted. These methods might be bandwidth efficient but still have their own drawbacks. They are notoriously slow to converge (more than 1000 symbols may be required for an FIR channel with 10 coefficients).

Decision directed estimation is a subclass of blind estimation techniques; it uses the detected symbols to reconstruct the transmitted signal, and then uses this signal in place of the original signal. In [3], performance comparison was made between the decision-directed algorithm and the constant modulus algorithm (CMA), as well as the spectral self-coherence restoral (SCORE) algorithm. Results
showed that the decision-directed algorithm converges faster than other algorithms.

In this paper we compare between two adaptive algorithms, decision-directed least square (LS) and decision-directed least mean squares (LMS), weighted and unweighted, as they are applied to estimate the impulse response of the newly proposed GHDS channel. In this estimation, no training sequence is utilized and no assumption about the number of incoming signals is made.

2 System Model
In this section we discuss the signal model and the multipath channel model.

2.1 Signal Model
Although estimation algorithms are not inherently dependent upon the modulation scheme, we will consider multiple phase shift keying (MPSK) modulation for implementation and simulation purposes. In MPSK, the carrier phase takes on one of \( M \) possible symbols, namely \( \theta_i = 2(i-1)\pi/M \), where \( i=1,2,\ldots,M \). The \( i \)th complex baseband modulated waveform can be expressed as [4]

\[
s_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left(2\pi f_c t + \frac{2\pi}{M}(i-1)\right), \quad 0 \leq t \leq T_s
\]

(1)

where \( E_s = (\log_2 M)E_b \) is the energy per symbol, \( E_b \) being the energy per bit, \( T_s = (\log_2 M)T_b \) is the symbol period, \( T_b \) being the bit period, and \( f_c \) is the carrier frequency.

2.2 Propagation Channel Model
In this work we use the SIMO space-time geometrical-based hyperbolic distributed scatterers (GHDS) channel model for macrocell environment to simulate the adaptive algorithm estimators [5]. This model can estimate the power of each path, the time-of-arrival (TOA), and the direction-of-arrival (DOA) of the multipath components as well as the fading effect. The complex time-variant baseband discrete impulse response as a function of time and delay is given by

\[
h(t, \tau) = \sum_{k=1}^{L} \sqrt{P(\tau_k)} \alpha_k(t) e^{j\phi_k(t)}
\]

(2)

where \( L \) is the number of paths, \( P(\tau) \) describes the \( k \)th path attenuation, \( \alpha_k(t) \) is the fast fading of the \( k \)th path, \( \tau_k \) is the path delay, and \( \phi_k(t) = 2\pi f_d \cos(\psi_k(t) - f_d \tau_k) \), where the expression \( f_d \) is the Doppler shift, \( f_d = v/\lambda \) being the maximum Doppler shift (\( v \)=mobile velocity, \( \lambda \)=wavelength of the carrier frequency) and \( \psi_k \) is the direction of the \( k \)th scatterer with respect to the mobile velocity vector. The Doppler spread \( f_d \) is given by \( 2f_d \). These parameters may vary with time [5].

3 Decision-directed Algorithms for Channel Estimation
When the receiver does not have knowledge of the transmitted signal, it must use blind channel estimation techniques. A subclass of these techniques, called decision directed estimators, involves utilizing the detected symbols to reconstruct the transmitted signal, then using this signal in place of the original signal [6], [7], [8]. A block diagram of the system performing decision directed estimation is shown in Fig.1.

![Fig. 1 A block diagram of the decision-directed estimator.](image)

From the previous diagram, in [9] the following relations in a matrix form were defined

\[
y = Uh + w
\]

(3)

\[
e = y - \hat{y} = y - Xh
\]

(4)

Defining the error or loss function \( J(h) \) as

\[
J(h) = e^T G e = (y - Xh)^T G(y - Xh)
\]

(5)

where \( U \in \mathbb{C}^{n \times m} \), \( n \geq m \) is a complex-valued matrix representing the discrete-time transmitted signals for \( N \) sample periods, \( X \in \mathbb{C}^{m \times N} \) represents the detected versions of the transmitted signals \( U \), \( h, \hat{h} \in \mathbb{C}^m \) are the true and the estimated channel impulse response, \( w \in \mathbb{C}^n \) is a mean zero complex valued discrete time random process, \( G \) is a positive definite symmetric \( N \times N \) matrix of weighting coefficients, possibly dependent on \( X \) and \( U \), \( e \in \mathbb{C}^n \) represents the error signal, and \( y \in \mathbb{C}^n \) is the received signal. The decision weighted linear estimators are a subclass of decision directed linear estimators. The matrix \( G \) is...
a function of the decision quality, which it can be measured through mechanisms such as error-detecting codes or soft decisions [9]. The non-realizable idealized choice of $G$ such that $X^T GX = X^T GU$ is called ideal decision weighted linear estimation [9]. In this work we consider $G$ as a diagonal weighting matrix, where

$$G = \text{diag}(a_1 \beta N^{-1}, ..., a_N \beta, a_N)$$

(6)

where $a_i \in \mathbb{R}$ and $0 < \beta \leq 1$ such that $G$ is positive definite.

The decision-directed RLS and LMS algorithms, weighted and unweighted, are implemented to estimate the impulse response of the GHDS channel model [5]. These algorithms utilize detected symbols to reconstruct the transmitted sequence. A brief description of these algorithms is given below.

### 3.1 Decision-Directed Recursive Least Square Algorithm

In the decision-directed RLS algorithm, we have to differentiate eq.5 with respect to $\hat{h}$ and equate the result to zero [10], to get

$$x^T G x \hat{h} = x^T G y$$

(7)

and if $x^T G x$ is non-singular then

$$\hat{h} = (x^T G x)^{-1} x^T G y$$

(8)

The form in eq.8 above is not desirable for implementation because it requires the storage of all past data. Hence in [9], they consider its recursive form, which is suitable for digital implementation. This form can be written as follows

$$\hat{h}_n = \hat{h}_{n-1} + a_n H_n^{-1} x(n) e(n)$$

(9)

$$H_n = \beta H_{n-1} + a_n x(n) x^T(n)$$

(10)

$$e(n) = y(n) - x^T(n) \hat{h}_{n-1}$$

(11)

### 3.2 Decision-Directed Least Mean Square Algorithm

The stochastic gradient or least mean square error (LMS) algorithm, based on minimizing the error function in eq.4 at every time instant $n$. The LMS is a steepest decent algorithm; it adjusts $\hat{h}$ in the direction of maximum change of the error function $J(\hat{h})$. The algorithm has the following forms [6], [9]

$$J(h) = \rho_p e^T e$$

(12)

$$\hat{h}_n = \hat{h}_{n-1} - \frac{1}{2} \mu \frac{\partial J}{\partial \hat{h}}$$

(13)

$$\hat{h}_n = \hat{h}_{n-1} + \mu \rho_n x(n) e(n)$$

(14)

Where $\rho_n \in \mathbb{R}$ is a weighting sequence. The best value for $\mu = 1/[6(M+1)\text{var}(x)]$, where $M+1$ is the number of filter taps, and $\text{var}(x)$ is a variance of $x$. If $\mu$ is small, convergence of LMS is slow; and if is large, convergence is faster but larger fluctuation error is encountered [6], [11].

### 4 Simulation Results

A simulation study has been conducted in order to evaluate the performance of the decision directed RLS and LMS algorithms, weighted and unweighted, in estimating the impulse response of the GHDS channel model.

#### 4.1 Simulation Parameters

In this simulation we chose the system parameters as follows: number of paths $L = 10$, distance between mobile and base station $D = 1\text{km}$, and Doppler spread $f_d = 200\text{Hz}$. The GHDS has been simulated for an urban area. The transmitted constellation is QPSK. The sampling rate was set to 1 sample per second, the symbol interval to 4 samples per symbol, and the total number of symbols per individual simulation to 200 symbols. The LMS gain and the RLS forgetting factor were $\mu = 0.2$ and $\beta = 0.98$, respectively. Fig.2 shows the transmitted signal, the GHDS channel output signal, and the detected version of the transmitted signal (in-phase and quadrature components).

#### 4.2 Performance in terms of Channel Estimation

The algorithms performances were compared by evaluating the average squared absolute difference between the true (as found in the GHDS model, eq.2) impulse responses and their estimations. Fig.3 and Fig.4 show the distance squared from the true response after estimation using LMS and RLS algorithms, respectively. Ideal decision weighted RLS (IDWRLS) performs better than the blind RLS (BRLS), however, the IDWLMS and the BLMS have the same performance.
Fig. 2 The transmitted, detected, and channel output signals.

Fig. 5 and Fig. 6 show the performance of the algorithms versus SNR. We varied SNR (at the receiver input) from 0 to 40 dB in 3 dB increments. The performance-measure is the deviation of the estimated channel impulse response from the true one as a function of SNR. Both IDWLMS and IDWRLS algorithms have better performance than their unweighted versions. The weights \( \rho_n, a_n = 1 \) for the IDWLMS and IDWRLS if \( x(n) = u(n) \) and zero otherwise.

Fig. 3 The distance squared between the true and estimated responses of GHDS channel using LMS algorithms.

Fig. 4 The distance squared between the true and estimated responses of GHDS channel using RLS algorithms.

Fig. 5 Estimation error of LMS algorithms as a function of SNR at the receiver.

Fig. 6 Estimation error of RLS algorithms as a function of SNR at the receiver.
5 Conclusion

In this paper, we demonstrated how the decision directed RLS and LMS adaptive algorithms could estimate the GHDS channel impulse response. Simulation results showed that the IDWRLS and the IDWLMS have better performance at the receiver than the BRLS and the BLMS estimators. The performance was measured based on the deviation of the estimated channel impulse response from the true one as a function of SNR.

References: