Optimizing the Design of IIR Filter via Genetic Algorithm

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Abstract: - The IIR filter design under the mixed-criterion of H_2 norm and ∞ norm is proposed in this paper, and genetic algorithms(GA) is introduced to realize the filter design based on such criterion. By this approach, the disadvantage of other conventional methods is avoided and the satisfied performance of the designed filter is obtained. The evaluating example shows that the filter designed by GA is superior to conventional Butterworth filter in either the optimization capability of design method or the performance of designed filter.

Key-Words: - IIR filter, Genetic algorithms, Mixed-criterion, H_2 norm, ∞ norm, GA filter

1 Introduction

Filter design is an important task of signal processing. Conventional digital filter design technique is using the parameter of analog filter and transformation method to obtain the corresponding digital filter[1]. This design approach inherits the design result of analog filter, easy and simple, but ignores the different characters between digital system and analog system[3]. So, by the conventional design approach, the performances of obtained digital filter are optimized under the particular object function and don't have universality[2]. In this paper, The IIR filter design under the mixed-criterion of H_2 norm and ∞ norm is proposed in this paper, and genetic algorithms is introduced to realize the design based on such criterion. By this techniques, the disadvantage of other conventional methods is avoided and IIR filter of satisfied performance is obtained.

2 Problem Formulation

There are many IIR filter design methods, such as Butterworth, Chebyshev, ellipse, Least Square, and so on[4]. In this section, the design problem of IIR digital filter under the mixed-criterion of H_2 norm and ∞ norm by genetic algorithms is proposed. Some definitions, problem statement, and remarks of IIR digital filter design are presented.

Definition 1 (H2 norm of frequency): Lets the particular frequency signal $S(\omega)\in R^n$, which is the function defined in $\omega\in[-\omega_0, {}_0]$, the H_2 norm of frequency is[5]

$$\|S(w)\|_2^2 = \int_{-w_0}^{w_0} |S(w)|^2 dw \tag{1}$$

similarly to (1), for the digital serial(L elements)

$$\|S(k)\|_2^2 = \sum_{k=1}^{L} |s(k)|^2 \tag{2}$$

Definition 2 (∞ norm of frequency): Lets the particular frequency signal $S(\omega)\in R^n$, which is the function defined in $\omega\in[-\omega_0, \omega_0]$, the H_2 norm of frequency is[5]

$$\|S(w)\|_\infty = \sup_{w\in[-w_0, w_0]} |S(w)| \tag{3}$$

similarly to (3), for the digital serial(L elements)

$$\|S(k)\|_\infty = \max_{k=1,2,\ldots,L} |S(k)| \tag{4}$$

IIR digital filter(DF) has the form[2]:

$$y(n) + \sum_{k=1}^{N} a_k y(n-k) = \sum_{r=1}^{M} b_r x(n-r) \tag{5}$$

Problem Statement 1(Optimizing the design of IIR filter under frequency-domain mixed-criterion): design the IIR digital filter DF, make the difference of frequency-domain between DF and the theoretical filter DF_T reach the minimum of mixed-criterion for optimize

$$\min_{\substack{a_i \in R \\ b_i \in R}} [q\|F(w)-F_T(w)\|_2 + (1-q)\|F(w)-F_T(w)\|_\infty] \quad (6)$$

In (6), θ ∈[0， 1] is the parameter need to be selected,，$F(\omega)$ and $F_T(\omega)$ is the frequency character of DF and DF_T respectively.

Remark 1: The selected parameter θ should be various in different applications. If θ=1, the problem is optimizing the design of IIR filter under frequency-domain Least-Square criterion; If θ=0, the problem is optimizing the design of IIR filter under frequency-domain ∞ criterion; if 0≤θ≤1, the problem is optimizing the design of IIR filter under frequency-domain mixed-criterion of 2 /∞ -norm.

Remark 2: The H_2 norm in criterion has a maximal value in the definition, which is hard to compute by analytical methods. And the ∞ norm in criterion has also a maximal value in the definition, which is hard to compute by analytical methods. The minimum value can be obtained by introducing genetic algorithms with less computing time[6].

Remark 3: It is reasonable because there is obvious physical mean for introducing the H_2 /∞-norm of the frequency-domain character difference between the designed IIR digital filter(DF) and theoretical IIR digital filter (DF_T) as optimizing criterion. This criterion predicates that the designed digital filter is the synthesized balance result of Least-Square resolution and minimum resolution for maximum error on the frequency-domain character.

Remark 4: It is difficult for using the analytic techniques to design the digital filter transfer function because there are integral operation caused by H_2/∞ norm in the selected filter design criterion. Theoretically, such problem can be resolved by introducing variational method, but in the cost of large computation scale. Here, it should be indicated that the Least-Square design technique is just the special example of the approach proposed in this paper in the condition of θ=1.

So, as a effective optimizing method, genetic algorithm is introduced to to resolve the optimizing problem under the frequency-domain H_2 /∞-norm mixed-criterion for IIR digital filter design.

3 Genetic Algorithm(GA)

Genetic algorithms are stochastic optimization algorithms that were original motivated by mechanisms of nature selection and evolutionary genetics. It is first introduced by Holland in 1962. Genetic algorithms have been proven to be efficient in many different areas such as function identification, image processing, system identification, and controller design.

General genetic algorithms could be expressed as program[7]

```
{
    Create the original group P(0)={x1,x2, …, xn}
    randomly ,t:=0;
    Compute the fitness value of every unit in P(0);
    while (do not satisfy the condition to end ) do
    {
        Compute the selector probability pi by
        unit's fitness value and the number in a
        group;
        Select N1(≤N) units to do evolutionary
        operation (reproduction, crossover, and
        mutation etc) and exchange the N1 worth
        units in P(t) by the resolution to create a
        new group P(t+1);
        Compute the fitness value of every units in
        P(t+1)，t=t+1;
    }
}
```

The concepts in genetic algorithms, such as reproduction, crossover, and mutation etc. [8], will not be explained in this paper. The applications of genetic algorithms in reduced order are described as follows.

The first step to solve the problem by genetic algorithms is to code the solutions. The real number code [9] method is taken after having selected a suitable value domain to each parameter, for the problem in this paper is an optimization to a nonlinear function. And the value domains should be translated into the domain [0,1] to make compute easily. The method as follows can be taken. Let $\{c_i\}$、$\{d_j\}(i=0,1,\cdots n_1, j=0,1,\cdots m_1)$ be searched in $\{(x_1,x_2,\cdots,x_{n1+m1+2}) = (c_0,c_1,\cdots c_{n1},d_0,\cdots d_{m1})$: $x_k \in [l_k,u_k]$, $k=1, 2, \cdots n_1+m_1+2$ }, where $[l_k,u_k]$ are the value domain for x_k .The x_k will be translated to t_k with:

$$t_k = \frac{x_k - l_k}{u_k - l_k} \quad (7)$$

and $t_k \in [0,1]$. On the other hand, the selection of parameter should be explained. Generally, the maximum/minimum limits in the designeded digital filter should be larger/less than the maximum /minimum parameter in the theoretical filter.

For the program of genetic algorithms is used to get the maximum units, the fitness function is defined as

$$c_x = \frac{1}{1+q\|F_T(s)-F_x(s)\|_2 + (1-q)\|F_T(t)-F_x(t)\|_\infty} \tag{8}$$

In (8): x is the unit and $F_x(s)$ is the transfer function for reduced order system to unit x.

The selection of evolution operator is presented as follows. The selector method is truncation selection, that select the D% fittest units to be parents and reproduce them randomly until the sons' number reach defined number; the crossover is global recombination [9], that the parent $x=(x_1,x_2,\cdots x_k)$ and $y=(y_1,y_2,\cdots,y_k)$ and their son is $z=(z_1,z_2,\cdots z_k)$ that

$$z_i = a x_i + (1-a) y_i \tag{9}$$

In (9): α is a random number in [0,1]; the mutation is uniform mutation[9], that gene x_i of the parent $x=(x_1,x_2,\cdots,x_i,\cdots x_k)$ is to be mutated by probability 1/k and his son is $x'=(x_1,x_2,\cdots,x_i',\cdots x_k)$ where $x_i'=(1-r)x_i$ and r is a random number in [0,1].

The original group will create the pointed accuracy resolve by several era evolutions.

4 Evaluating the performance of IIR Filter Designed by GA

In this section, the performance of IIR filter designed via GA(GA filter) is evaluated by comparing it with the conventional Butterworth filter, a typical filter designed by conventional technique, in frequency-domain and filtering performance.

Design a 7-orders Low-pass filter, the normalized cut-off frequency is 0.5.

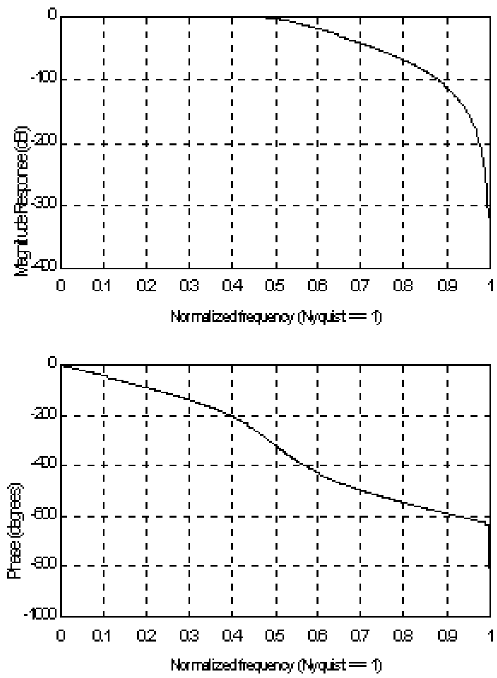
The filter coefficients can be obtained by using Butterworth filter design technique: a_1=0.0166, a_2=0.1160, a_3=0.3479, a_4=0.5798, a_5=0.5798, a_6=0.3479, a_7=0.1160, a_8=0.0166; b_1=1.0000, b_2=0.0000, b_3=0.9200, b_4=0.0000, b_5=0.1927, b_6=0.0000, b_7=0.0077, b_8= 0.0000; the frequency-domain character of conventional Butterworth filter are proposed in fig.1(a).

Design the digital filter by the technique of this paper. θ=0.5, crossover probability 0.5, mutation probability 0.01, all the parameter in domain [-1,1] 上, the designed filter coefficients : a_1=0.0182, a_2=0.1276, a_3=0.3766, a_4= 0.5218, a_5=0.5218, a_6=0.3827, a_7=0.1276, a_8=0.0149; b_1=1.0000, b_2=0.0000, b_3=1.0120, b_4=0.0000, b_5=0.1734, b_6=0.0000, b_7=0.0085, b_8=0.0003. The fig.1(b) present the frequency-domain characters of GA filter.

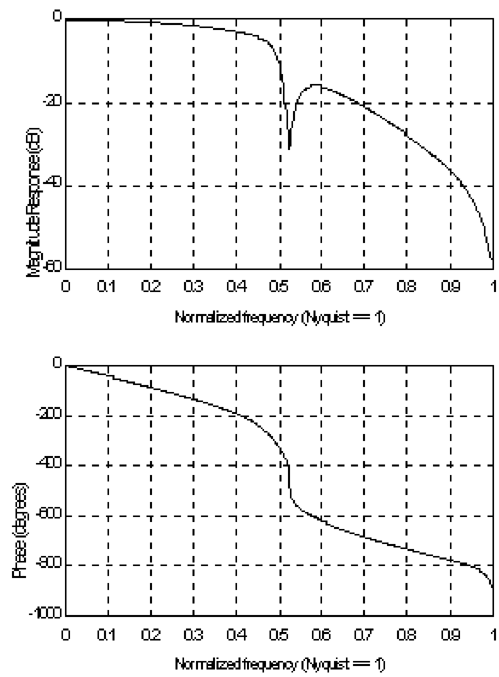
Figure 1 illustrates that the designed IIR filter via GA is superior to conventional Butterworth filter in frequency-domain performance. Compares the filter designed by GA and Butterworth filter in filtering the signal

$$s(t)=10\sin(2t)+2\sin(100t) \tag{10}$$

The SNR of output signal is 19.8871dB, 19.6485dB respectively. Obviously the filter designed by GA is superior to Butterworth filter in the filtering performance.



(a) Butterworth filter



(b) IIR filter designed by GA

Fig. 1 Frequency-domain performance of Butterworth filter and the filter designed by GA

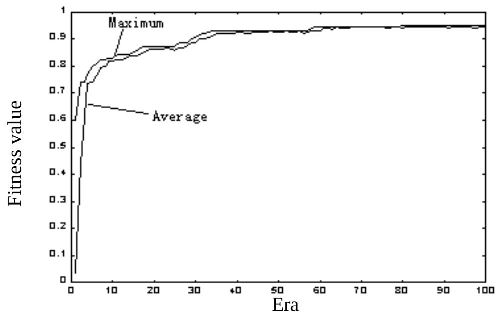


Fig. 2 The fitness value changes in computing

Fig.2 presents the evolution process of computing. The resolution is reached by 40 times' evolutions. The units in a group are 500 and they all have computed 100 times.

5 Conclusion

The IIR filter design under the mixed-criterion of H_2 norm and ∞ norm is proposed in this paper is researched for the requirement of engineering application, and the filter design is accomplished by introducing Genetic Algorithms as optimizing tool. Designing example show that the performance of the filter designed by GA is superior to conventional Butterworth filter in frequency-domain performance and filtering performance. By the approach proposed in this paper, The SNR is improved and the frequency-domain performance is approach theoretical filter. This approach is preferable for its simple in computation and realizing, is an applied IIR filter design technique in engineering.

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