GPS Based Azimuth/Gradient Determination for Land Vehicle

PEIQING XIA, KECK VOON LING, N. NAGARAJAN

School of Electrical & Electronic Engineering
Nanyang Technological University,
Nanyang Avenue,
SINGAPORE 639798

Abstract:- This paper presents the theory and simulation results of a GPS-based Azimuth/Gradient determination instrument for a land vehicle, using two GPS receivers to realize interferometry. Our purpose is to design a system, which is both robust and accurate. A simulation system is designed and developed to simulate the GPS constellation, the required carrier phase observation and ephemeris, the dynamics of the land vehicle and the signal processing for Azimuth/ Gradient determination. This system uses the interferometric processing of the GPS L1 signals to determine the Azimuth/ Gradient using Euler angle rotations. On the basis of the carrier phase observation model, the non-linear equation is solved iteratively. After coordinate system transformation, the Azimuth/ Gradient are computed in body frame. Utilizing the proper maneuver of the vehicle, the integer ambiguity is resolved by an instantaneous method, i.e. solution for the GPS integrated carrier Doppler wavelength ambiguities (integer ambiguity) doesn't depend on the observations over a long time. In this way, further improvement in the robustness of the realization is achieved because the system could be quickly reset and initialized at any time. This instantaneous property, together with the cycle slip detection routine, renders the system resistant to cycle slip. Results are presented using simulated GPS L1 signal with the effects of multi-path, ionosphere, troposphere and receiver electronic noise, proving its accuracy and stability.

1. Introduction

Azimuth determination is a problem, which is key to the control system of a land vehicle. Conventionally, the azimuth information is achieved using magnetic components in land vehicle. In this paper, a procedure for Azimuth/ Gradient determination using Global Positioning System (GPS) interferometry is presented. The use of phase difference measurements of GPS signal provides a novel approach for the Azimuth/Gradient determination [1,2,3]. This approach has been successfully applied to land, air, sea, and space based vehicle. The procedure of attitude determination generally involves a two-step process. First, in the double differenced phase observation interferometry process, the unknown number of integer wavelengths between a given pair of antennas must be found before the calculation of Azimuth/Gradient. Second, the resolution of vehicle Azimuth/Gradient is achieved using carrier phase interferometry information and the estimated integer ambiguity in the system model.

The solutions of this unknown integer ambiguity are generally available in two categories, "instantaneous" (motionless) or "dynamic" (motion-based). Instantaneous methods rely on the fact that the integer ambiguities are not completely arbitrary numbers and are constrained to be integers within a known range defined by the baseline length and GPS signal wavelength. Using an exhaustive search over all the possible ambiguity combinations, the instantaneous ambiguity resolution is achieved [4].

Dynamic techniques for resolving integer ambiguity require phase observation data collection for a certain period, which normally ranges from 10 minutes to few hours [5]. A batch solution is then carried out to get the ambiguity resolved. The duration of observations must ensure that certain amount of motion of the line of sight from the GPS receiver to the GPS satellite occurs during the observation period. This is a well-developed method, but the main disadvantage, compared to the instantaneous method is the requirement of observation for a long time. The other disadvantage is that the baseline needs to remain stationary during the observation period.

Motion-based techniques of integer ambiguity resolution rely on the fact that either the motion of GPS line of sight or the vehicle motion is significant enough. If we can utilize the proper maneuver of land vehicle, it is easy to achieve the significant motion of GPS line of sight.

2. Interferometry Process

Carrier phase observation is the only applicable observation for the purpose of Azimuth/Gradient determination using GPS [6]. Two antennas forming a baseline provide the GPS phase observations to determine the Azimuth/Gradient. The configuration of single baseline is shown in Fig.1.

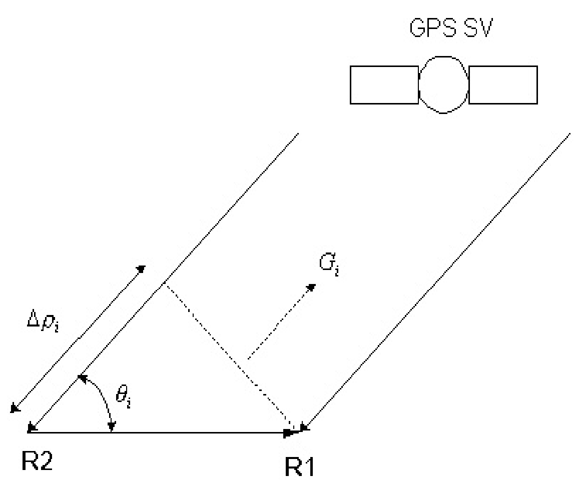

Fig 1. Single Differenced Model using GPS Carrier Phase Observation

Let R1 and R2 denote the two GPS antennas mounted at the ends of the baseline. The line of sight vector with respect to the *i*th GPS satellite is denoted as G_i. The line of sight vectors of the two antennas are same because the two antennas are near enough [7]. From the two receivers, we get two carrier phase observations.

$$\lambda\cdot\phi_1 = \rho_1^i + c\cdot(\Delta t^i - \Delta T_1) + \lambda\cdot N_1^i - d_{ion}^i + d_{trop}^i + d_{mp}^i + \varepsilon_1^i \qquad (1)$$

$$\lambda\cdot\phi_2 = \rho_2^i + c\cdot(\Delta t^i - \Delta T_2) + \lambda\cdot N_2^i - d_{ion}^i + d_{trop}^i + d_{mp}^i + \varepsilon_2^i \qquad (2)$$

The superscript *i* means this observation is from the *i*th GPS satellite. The subscript 1 or 2 means that the observations are from the GPS receivers 1 or 2. The following notations are adopted in the paper.

- λ: Wavelength of L1 carrier phase (19.03 cm).
- ϕ: carrier phase observation in units of carrier cycles
- ρ: geometric range between the GPS satellite and the GPS receiver
- c: speed of light
- Δt: the satellite clock error with respect to GPS time, including S/A effects
- ΔT: the receiver clock error with respect to GPS time
- N: Integer Ambiguity of the carrier phase measurement from the GPS receiver
- d_{ion}: the ionospheric delay error on the carrier phase measurement
- d_{drop}: the tropospheric delay error on the carrier phase measurement
- d_{mp}: the multipath effect on the carrier phase measurement
- ε: other carrier phase measurement error

Subtract Eq.(2) from Eq.(1) to eliminate Δt^i, d_{ion}^i, d_{trop}^i and define the following. $\Delta\phi_i \equiv \phi_1^i - \phi_2^i$ is the difference between the phase observations of the receivers located at the ends of the baseline. $\Delta\rho_i \equiv \rho_1^i - \rho_2^i$ is the difference between the ranges from satellite to the receivers. $\Delta T \equiv \Delta T_1 - \Delta T_2$ is the differenced clock bias between two receivers. $\Delta N_i \equiv N_1^i - N_2^i$ is the single differenced ambiguity between receiver 1 and 2 with respect to the same *i*th satellite and $\Delta\varepsilon_i \equiv \Delta\varepsilon_1^i - \Delta\varepsilon_2^i$ represents the remained error between receiver 1 and 2. From (1) and (2) we obtain,

$$\Delta\rho_i = \lambda\cdot\Delta\phi_i + C\cdot\Delta T - \lambda\cdot\Delta N_i - \Delta\varepsilon_i \qquad (3)$$

Due to the short spacing between the antennas in the attitude determination system, the orbital, tropospheric and ionospheric errors cancel out.

From the geometry shown in the Fig.1, we get,

$$B^T \cdot G_i = \Delta\rho_i \qquad (4)$$

where B is the baseline vector and G_i is the line of sight vector as is shown in Fig.1. We now obtain the second difference of the observations from different satellites because there is still one major error, receiver clock error existing in the single differenced phase observation. Thus we get,

$$\nabla\Delta\rho^{ij} = \left(\nabla\Delta\phi^{ij} - \nabla\Delta N^{ij}\right)\cdot\lambda + \nabla\Delta\varepsilon^{ij} \qquad (5)$$

where $\nabla\Delta\rho^{ij} \equiv \Delta\rho_i - \Delta\rho_j$ is the difference between two single differenced ranges, $\nabla\Delta\phi^{ij} \equiv \Delta\phi^i - \Delta\phi^j$ is the difference between two single differenced phase observation and $\nabla\Delta N^{ij} \equiv \Delta N_i - \Delta N_j$ is the double differenced integer ambiguity between receiver 1 and 2 with respect to two GPS satellite i and j. We will use the motion-based technique to resolve this unknown integer ambiguity value. $\nabla\Delta\varepsilon^{ij} \equiv \Delta\varepsilon_i - \Delta\varepsilon_j$ is the remaining error after double differenced process. Receiver clock bias between receiver 1 and 2 is cancelled out by doing double difference process. From Eq.(4) and Eq.(5), double differenced phase observation is written as follows:

$$B^T\left(G_i - G_j\right) = \nabla\Delta\rho^{ij} \qquad (6)$$

Let $B^T \cdot G = z$ (7)

where $G = G_i - G_j$, $z = \nabla\Delta\rho^{ij}$.

The complete direction cosine matrix A for the overall rotation is the matrix product of the three matrices for the individual rotations, $\delta\theta = \begin{bmatrix}\delta\theta_x & \delta\theta_y & \delta\theta_z\end{bmatrix}^T$. A is the attitude matrix, an orthogonal matrix (i.e. $A \cdot A^T = I_{3x3}$) representing the transformation between the body and reference frames [1]. The transformation matrix could be reduced into a simpler expression when the three Euler angles are small perturbations.

$$A = \delta A \cdot A_0 \approx (I + \left[\delta\theta^X\right]) \cdot A_0 \qquad (8.a)$$

In the present work the attitude states are only the azimuth ($\delta\theta_z$) and the gradient ($\delta\theta_y$) angles of the vehicle and hence $\delta\theta_x = 0$. This leads to,

$$\left[\delta\theta^X\right] = \begin{bmatrix} 0 & \delta\theta_z & -\delta\theta_y \\ -\delta\theta_z & 0 & 0 \\ \delta\theta_y & 0 & 0 \end{bmatrix} \qquad (8.b)$$

where the A_0 is the attitude matrix before Euler perturbation. The baseline vector B in Eq.(7), is in the body frame. Since the line of sight vector G is known in the reference frame, it should be rotated into body frame using the attitude matrix A. Hence by replacing the line of sight vector G in Eq. (7) by $A \cdot G$, we deduce the attitude determination model,

$$z = B_s^T \cdot A \cdot G \qquad (9)$$

With Euler perturbation $\delta\theta$, this attitude determination model is expressed as:

$$z = B_s^T \cdot \left(I + \left[\delta\theta^X\right]\right) \cdot A_0 \cdot G \qquad (10)$$

Differentiating the equations (9) and (10) [2, 5, 10], we get the linearized model for small perturbation:

$$\delta z = B_s^T \cdot \left[\delta\theta^X\right] \cdot A_0 \cdot G \qquad (11)$$

Eq.(11) can be further written as:

$$\delta z = E \cdot \delta\theta' \qquad (12)$$

where $E = \left(A_0 . G\right)^T \begin{bmatrix} B_{sz} & 0 & -B_{sx} \\ -B_{sy} & B_{sx} & 0 \end{bmatrix}^T$ and

$$\delta\theta' = \begin{bmatrix}\delta\theta_y & \delta\theta_z\end{bmatrix}^T. \qquad (13)$$

Equation (12) is written as:

$$\delta z = \left(A_0 \cdot G\right)^T \cdot \left[B_s^X\right] \cdot \delta\theta' \qquad (14)$$

The process of attitude determination consists of converting these double differenced carrier phase measurements into attitude solution. Assume that we have *m* baselines and *n* double differenced phase observations, an optimal attitude solution [8] for a given set of phase measurements is given by a function of desired attitude Euler angle as follow:

$$C(\delta\theta') = \sum_{i=1}^{m}\sum_{j=1}^{n}[\delta z_{ij} - (A_0 \cdot G_j)^T \cdot \left[B_{si}^X\right] \cdot \delta\theta']^2 \qquad (15)$$

Because only two GPS receivers are used here and they form a single baseline, m=1. Equation (15) is then reduced to:

$$C(\delta\theta') = \sum_{j=1}^{n}[\delta z_j - (A_0 \cdot G_j)^T \cdot \left[B_s^X\right] \cdot \delta\theta']^2 \qquad (16)$$

Normally, the available number of double differenced carrier phase measurements is greater than the number of unknown Euler angle parameters namely 2. In order to solve this minimization problem, we use Least Square method to estimate $\delta\theta'$.

We first rewrite Eq.(14) as:

$$\delta z = E \cdot \delta\theta' \qquad (17)$$

Where

$$\delta z = [\delta z_1 \quad \delta z_2 \quad \cdots \quad \delta z_n]^T$$

$$E_i = \left[(A_0 \cdot G_1)^T \cdot \left[B_{si}^X\right] \ \cdots \ (A_0 \cdot G_n)^T \cdot \left[B_{si}^X\right] \right]$$

Because there is only one baseline available, i=1.
A short form of Eq.(12) is as:

$$\delta z = E \cdot \delta\theta' \tag{18.a}$$

$$E = \left(A_0 \cdot G\right)^T \cdot \left[B_s^X\right] \tag{18.b}$$

The Least Square Estimation is given as:

$$\delta\theta' = \left(E^T \cdot E\right)^{-1} \cdot E^T \cdot \delta z \tag{19}$$

Minimizing Eq.(16) means finding the attitude matrix A corresponding to the Azimuth/Gradient information.

3. Integer Ambiguity Resolution

Recall that in the Double Differenced Phase Observation model in Eq. (7), the integer ambiguity is still unknown. In this paper, the ambiguity is solved instantaneously by utilizing the proper maneuver of the vehicle, known as the 'swap' method [7], i.e. the solution doesn't depend on the observation over a long time. The antenna swap method is described in Fig.2.

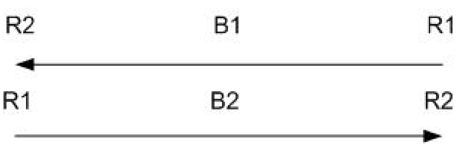

Fig. 2 Antenna swap method

On the structure of the vehicle, find 2 points forming one baseline B. Mount two GPS antennas at the ends of the baseline B, forming the baseline vector B1. The vector in the reversed direction after the 180 degree rotation of the satellite is the baseline vector B2. Now, $B2 = -B1$ (20)

From Double Differenced Phase Observation model, Vector B1 can be expressed as:

$$G \cdot B1 = \begin{bmatrix} \nabla\Delta\phi_1 - \nabla\Delta N_1 \\ \vdots \\ \nabla\Delta\phi_n - \nabla\Delta N_n \end{bmatrix} \cdot \lambda + \begin{bmatrix} \nabla\Delta\varepsilon_1 \\ \vdots \\ \nabla\Delta\varepsilon_n \end{bmatrix} \equiv \begin{bmatrix} \nabla\Delta\rho_1 \\ \vdots \\ \nabla\Delta\rho_n \end{bmatrix} \tag{21}$$

Vector B2 is then expressed as:

$$G \cdot B2 = \begin{bmatrix} \nabla\Delta\phi_1' - \nabla\Delta N_1 \\ \vdots \\ \nabla\Delta\phi_n' - \nabla\Delta N_n \end{bmatrix} \cdot \lambda + \begin{bmatrix} \nabla\Delta\varepsilon_1' \\ \vdots \\ \nabla\Delta\varepsilon_n' \end{bmatrix} \equiv \begin{bmatrix} \nabla\Delta\rho_1' \\ \vdots \\ \nabla\Delta\rho_n' \end{bmatrix} \tag{22}$$

Eq.(20) could be explicitly written as:

$$\begin{bmatrix} \nabla\Delta\rho_1 \\ \vdots \\ \nabla\Delta\rho_n \end{bmatrix} = -\begin{bmatrix} \nabla\Delta\rho_1' \\ \vdots \\ \nabla\Delta\rho_n' \end{bmatrix} \tag{23}$$

Substitute Eq.(21) and Eq.(22) into Eq.(23), we get:

$$\begin{bmatrix} \nabla\Delta\phi_1 - \nabla\Delta N_1 - \nabla\Delta\varepsilon_1 \\ \vdots \\ \nabla\Delta\phi_n - \nabla\Delta N_n - \nabla\Delta\varepsilon_n \end{bmatrix} = -\begin{bmatrix} \nabla\Delta\phi_1' - \nabla\Delta N_1 - \nabla\Delta\varepsilon_1' \\ \vdots \\ \nabla\Delta\phi_n' - \nabla\Delta N_n - \nabla\Delta\varepsilon_n' \end{bmatrix} \tag{24}$$

Treating the carrier phase measurement error as a negligible value, by taking the difference between left side and right side of Eq.(24), the ambiguities is eliminated at all. Divide it by 2, we get:

$$\begin{bmatrix} \nabla\Delta\phi_1 - \nabla\Delta\phi_1' \\ \vdots \\ \nabla\Delta\phi_n - \nabla\Delta\phi_n' \end{bmatrix} / 2 = \begin{bmatrix} \nabla\Delta\rho_1 \\ \vdots \\ \nabla\Delta\rho_n \end{bmatrix} \tag{25}$$

Omit the superscript ij for simplicity reason, Eq.(5) could be rearranged as:

$$\begin{aligned} \nabla\Delta N \cdot \lambda &= \left(\nabla\Delta\phi - \nabla\Delta\rho\right) \cdot \lambda + \nabla\Delta\varepsilon \\ &\approx \left(\nabla\Delta\phi - \nabla\Delta\rho\right) \cdot \lambda \\ &= \left(\nabla\Delta\phi - \frac{\nabla\Delta\phi - \nabla\Delta\phi'}{2}\right) \cdot \lambda \\ &= \frac{\nabla\Delta\phi + \nabla\Delta\phi'}{2} \cdot \lambda \end{aligned} \tag{26}$$

The integer ambiguity hidden in the carrier phase observation is found now, after the antenna swap initialization procedure.

$$\nabla\Delta N = \frac{\nabla\Delta\phi + \nabla\Delta\phi'}{2} \tag{27}$$

Once the integer ambiguity is resolved, provided the GPS receiver successfully keeps tracking the GPS signal, the integer ambiguity won't change subsequently. In all the future measurement, the only unknown is $\nabla\Delta\rho^{ij}$ in Eq.(5), and it could be computed directly.

Three considerations about integer ambiguity resolution appear here. One is the realization of the swap procedure. If the baseline is fixed to the vehicle, a proper maneuver is required for the Motion-based integer ambiguity resolution. Suppose that the head of the vehicle is pointing to the west when stationary, and after the initialization period, a 180-degree turn, the head of the vehicle turns to be pointing to the east. Then whole integer ambiguity resolution is completed. From that epoch onwards, if only the cycle slip doesn't happen, the azimuth/gradient information could be achieved with every epoch's phase observation.

Another consideration is about the robust property. The system could get more than 1 resolved integer ambiguity set if a rotation greater than 180-degree is performed. Theoretically, all the resolved integer ambiguity sets should have same value. With this hypothesis, more available sets of data provide the redundancy.

The last consideration is that the GPS carrier phase observation is discrete in time, and the sampling time normally won't match the exact time point that the baseline rotates half cycle. When the dynamic of the vehicle is very high, the error of mismatch can cause wrong integer ambiguity resolution. New method to achieve accurate 180 degree turn must be found.
Assume that the rotation angular velocity of the vehicle is ω, and the carrier phase sampling interval is T. In an arbitrary time epoch, the phase information in the initial direction is recorded. After n samples, we achieve the phase information in the reversed direction. n is given by this equation:

$$n * T = (k + 0.5) * 2 * \pi / \omega \text{, k=1,2,3…}$$

There is still remained error of angular rotation. But what we use here is only the integer part of the processed output. After the integer ambiguity resolution, it remains constant. The fractional part of double differenced phase observation, which is crucial to the accuracy, is obtained from the phase observation directly and is not related to the integer ambiguity resolution step.

4. Experiment Results

Currently, in order to test the workability of this algorithm, a computer simulation for azimuth/gradient determination has been carried out successfully. Based on the successful computer simulation a prototype hardware setup is being planned. This simulation uses single frequency GPS L1 signal. The accuracy is generally proportional to the length of the baseline; the longer the baseline, the better will be the accuracy we could achieve. In this experiment, we choose baseline length is assumed to be 1m.
Suppose the vehicle is running on a flat ground, the gradient is always zero, so the gradient angle is set to be zero in this simulation. Arbitrarily set the azimuth to be in a sinusoidal waveform. Use the attitude information and assumed velocity, the position is deduced iteratively, see Fig 3.

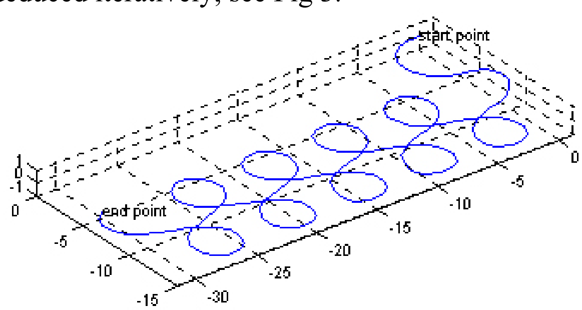

Fig.3 Simulated Driving Path

At each time epoch, the GPS satellite in view is simulated together with their carrier phase observation from the two GPS receivers. In order to test the algorithm for its convergence, the simulations were first carried out with noise free measurements.

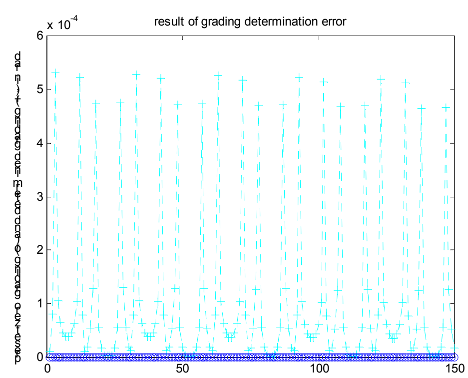

Fig.4 Gradient Determination Result (noise free)

In Fig.4, the preset true value of gradient, zero is compared with the gradient determination result; the error is in the level of 10^{-4} rad. In Fig.5, the azimuth determination result matches the preset sinusoidal waveform. The error between the determination result and preset value is shown in Fig.6.

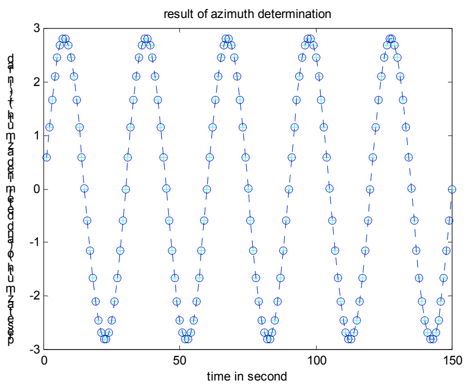

Fig.5 Azimuth Determination Result (noise free)

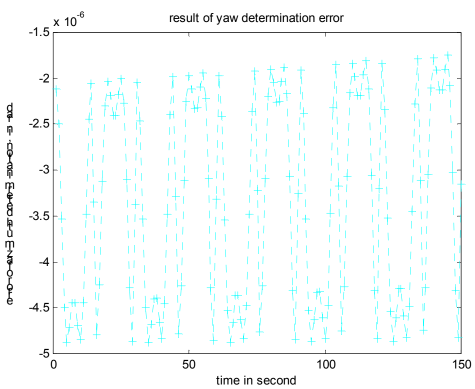

Fig.6 Azimuth Determination Error (noise free)

In the subsequent simulations, the noise of carrier phase observation is included besides the errors due to, the multipath, troposphere and ionosphere effect, receiver noise, etc. It proves the robustness of this approach in the presence of noise. The determined result still tracks the preset value, but the determination error residue is larger than the result using noise free phase observation.

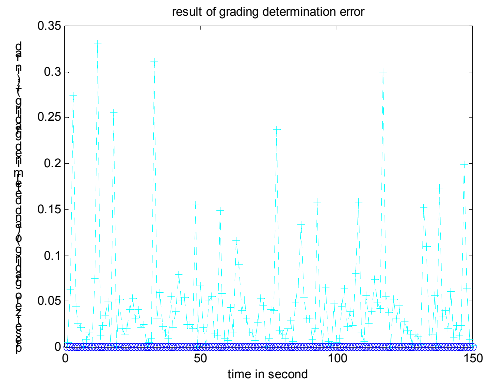

Fig.7 Gradient Determination Result (with noise)

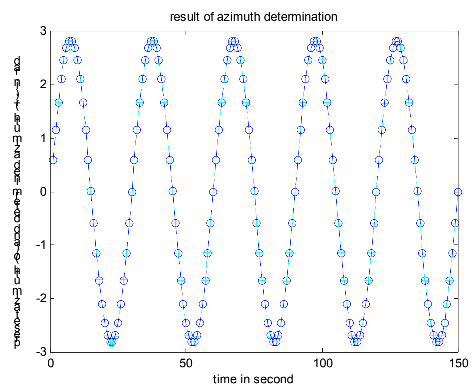

Fig.8 Azimuth Determination Result (with noise)

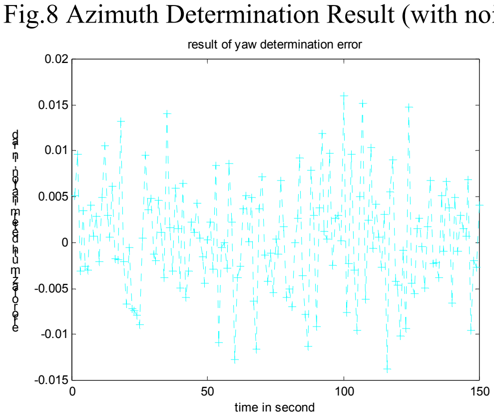

Fig.9 Azimuth Determination Error (with free)

5. Conclusion

In this paper, a comprehensive introduction and simulation about azimuth/gradient determination using GPS phase observation are presented. It utilizes proper maneuver of the land vehicle and solves the integer ambiguity quickly and accurately. This simulation uses the GPS L1 phase observation and shows the good performance of this algorithm in land vehicle application. A prototype road experiment is under development.

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