Perception of Mathematical Structure and Architectural Design: Form and ‘Forming’ in Music

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Abstract: In musicology, architectural and mathematical correlations have been observed in studies of the nature of musical form, whilst Schopenhauer restated the Pythagorean view that the universality of music resembles geometrical figures and numbers. Stravinsky suggested that we could not better define the sensation produced by music than by saying that it is identical with that evoked by the contemplation of the interplay of architectural forms. This paper considers the perception of architectural form in music and reveals Debussy’s use of Golden Section as a structural determinant in his prelude La fille aux cheveux de lin for piano.

Keywords: musicology, form, Debussy, analysis, architecture, Golden Section

At the beginning of the twentieth century the increasingly extended and chromatic harmony of Richard Strauss and the early Schoenberg led to the inevitable dissolution of the tonal system. The continually deferred resolution of the structural dominant, which typified the music of Wagner and which was able, seemingly, to propel its substance indefinitely was no longer the form-building entity it had been for the last few hundred years. Composers like Schoenberg and other members of the Second Vienna School sought to abandon completely the principles of the tonal system and to seek new compositional means by which music could acquire form. At this time, too, the positivistic Zeitgeist (evident, for example, in the functional, disciplined style of Bauhaus architecture, the stark, absolute angularity of cubist art and the logical positivism expounded by Wittgenstein) led Schoenberg to develop the serial technique of composition; a compositional method which, both at the time and since, received a good deal of misguided criticism on the grounds that it is a predetermined method of musical construction and, as such, did not correspond to the nineteenth-century Romantic aesthetic, which saw the creation of any art as the spontaneous effusion of inspired genius. The idea that musical composition could resemble, or be reduced, to some kind of mathematical equation was anathema to many.

On the other hand, there were those composers who, rather than abandon tonality altogether, sought to reinvent its possibilities. Composers like Stravinsky, Vaughan-Williams and, most notably, Debussy, attempted a kind of intertextual misreading of musical styles which predated the emergence of the tonal system, evident, for example, in the echoes of organum in La cathédrale engloutie. This absorption of an obsolete musical aesthetic into the fabric of twentieth-century musical art is what the literary critic Harold Bloom would describe as the ‘antithetical completion of a precursor’. [1] In other words, these composers have found a way to retain the ‘terms’ of the tonal system, but to ‘mean’ them in another sense. This means that Debussy, et al, were able to maintain a sense of tonal allusion throughout their work but were not dependent upon the tonal rhetoric which was the essence of musical development from the seventeenth to the nineteenth centuries.

It is for this reason that the music of Debussy was more accessible to an early twentieth-century audience. In fact, Debussy is now rarely linked to the iconoclasm of the Second Vienna School; more usually, Debussy is aligned with the impressionistic aesthetic of artists and writers like Monet and Mallarmé. The terms used to describe this movement, such as impressionistic, chromatic or colouristic and non-functional all imply a kind of rhapsodic and improvisatory approach, and indeed, this is how much of Debussy’s music sounds. However, Roy Howat in his book ‘Debussy in Proportion’ [2] has argued that the rather nebulous, indistinct air of Debussy’s music conceals a more rigorous and material method of manufacture which lies below the musical surface; and I would like to examine one such example of Debussy’s compositional
process, evident in the piano prelude *La fille aux cheveux de lin*, and to consider its implications in terms of music analysis and the perception of musical structure.

Fig. 1.
In its most fundamental sense, this is a ternary piece with a simple A B A' structure. The first section is characterised by an undulating melodic line, to be played ‘très calme et doucement expressif’, within which there are several subsections (to which I will return shortly). The central section begins at b.19 (un peu animé) and is characterised by an ascending arpeggiated phrase, building in dynamic intensity until we reach the undoubted climax of the piece: the final beat of b.21 with the mf C flat major chord. The modified return to the opening gesture occurs at b.24 with the restatement of the opening four pitches and the return of the central key, G flat major. Within these main structural divisions there are small, self-contained units
which I have numbered 1-8 and which are marked in Fig. 1.

Apart from some local dominant/tonic punctuation marks, the piece has no structural dominant; the only keys to which the material moves being G flat, E flat and C flat majors. Yet the music, rather than appearing drifting and improvisatory, has an inevitability of design which is more reminiscent of mathematics and architecture than traditional musical thought. Fig. 2 highlights the harmonic divisions in the piece and also notes a recurring harmonic pattern, the focal point of which corresponds to the start of section 5 (the ‘B’ section).

***Fig. 2.***

![Harmonic divisions in the piece](image)

In visual art and architecture, structural proportion between elements is clearly of primary concern, since it is in these terms that the form of any piece is determined. Golden Section is a particular proportion between two lengths, where the relationship of the shorter length to the longer is the same as that between the longer length and the whole. (See Fig. 3.) This design has particular significance, since it also reflects structures which are evident in nature (in, for example, sea shells and fir cones).

***Fig. 3.*** Golden section

![Golden section diagram](image)

\[ \frac{C}{B} = \frac{B}{A} \]

The application of Golden Section in musical composition, however, is rather enigmatic, since the observational domains in which music and architecture exist are transparently dissimilar. Music exists in time; it is a continual, temporal process which occupies time only in the present tense. Architectural form, on the other hand, has a mode of existence which will allow us to apprehend all of its structural proportions are constantly and permanently manifest.

That said, if we examine the relative proportions in Debussy’s piece in terms of Golden Section then some interesting features of compositional style emerge.\(^1\) *La fille aux cheveux de lin* is 39 bars long and there are three beats per bar. There are, therefore, 117 beats or units in this piece. The Golden Section of the piece is calculated thus:

***Fig. 4***

\[ \begin{align*}
39 \times 3 & = 117 \\
117 \times 0.618034 & = \text{GS} = 72.309978
\end{align*} \]

To find the bar number at which the GS occurs, simply divide this number by three (since there are three beats per bar). The GS of this piece occurs, therefore, at bar 24.

\(^1\) Golden Section can be calculated using the Fibonacci series; a sequence of numbers where any number is the sum of the two previous numbers in the sequence, so that in the sequence, ... 3, 5, 8, 13, 21, 34... the GS of 21 is 13, the GS of 13 is 8 etc. Furthermore, the GS of any number can be calculated by multiplying it by 0.618034.
What is immediately apparent is that the GS coincides with one of the main structural divisions in the piece, that is, the return to the original material and key at b.24. Furthermore, if, rather than calculating the GS from b.1, we work backwards from b.39, producing what I will call the 'retrograde golden section' [RGS], we can see that this point corresponds with b.16, the mini-climax which foreshadows the real climax at b.21 (both culminating on a C flat major chord).

So what can we say about the other structural divisions in this piece? In fact, once we begin this process it becomes clear that every single structural event in the entire composition is determined in exactly the same way. So, for example, in calculating the GS of the GS (that is, the GS of bb. 1-23) we see that this falls at the start of section 4; what we might describe as our ‘second subject’ material. The GS of this passage corresponds to section 3 and of that to section 2.

The climax of the piece is calculated by finding the GS of the central section, which is only five bars in duration and begins at b.19. There are, therefore, only fifteen beats in this section, the GS of which occurs on beat nine, or, the third beat of b.21. The process of reducing the structure to its constituent parts is summarised in Fig. 5, where a recurring visual pattern begins to emerge.

Fig. 5. Debussy: La fille aux cheveux de lin
Golden Section derivation

Whilst the structural aspects of the piece exposed in Fig. 5. are undoubtedly present, questions remain regarding the significance of such structures in terms of musical perception. Like Schoenberg’s 12-note rows or the ‘magic squares’ used by Peter Maxwell-Davies, the point at which GS occurs in any music can never be directly perceived or experienced, since music is necessarily sequential and depends on the physical absence at each and every instant of all but what defines that instant. We could never know, therefore, that what we heard was, in fact, the GS until we reached the end of the piece and could, retrospectively, comprehend its significance. This mode of apprehension of some form of artistic Gestalt, in which music exists as an object of memory, reflects the way in which musicology has traditionally categorised artistic form. Indeed, I began my own account of this piece by describing it as an A,B,A structure and then went through a type of top-down processing of the
information, breaking the piece down into its constituent elements. However, rather than regarding form as a thing (noun) we might rather consider it as a process (verb) and ask how this piece ‘forms’ rather than describe what form it has. If we consider how the Fig. 6.

It then seems that this material is abandoned, if only briefly, since before it returns we encounter a short, chorale-like passage in E flat. What we can observe, then, is that the harmonic design of the first three sections of the piece, that is, up to b.13, is a direct reflection of the G flat/E flat oscillation of the opening bars. The three-note anacrusis which links sections 1 and 2 also generates much of what happens later in the piece; its minor third-major second structure recurring in various permutations throughout.

The application of Golden Section to this piece can, likewise, be observed in its micro-structural components. The first phrase of the piece, lasting just eleven beats, contains all the elements of what follows. The phrase moves between G flat and E flat, followed by the C flat/G flat plagal cadence Fig. 8. (this, therefore, contains within it the modulatory scheme of the entire piece). The G flat/E flat arpeggios (actually spanning the notes D flat and E flat) themselves imply the three-note anacrusis of b.4

The point of the cadence is also significant, since, if the opening phrase is eleven beats in duration, then its GS will correspond exactly to the first beat of b.3, the point at which the tonic is established. Rather, then, than considering this piece in the way that we might consider an architectural form, it may be more appropriate to recognise that all the structural elements of the piece are presented at the outset in, as it were, easily perceivable ‘bite-sized’ chunks, and that thereafter we experience those structures again and again in expanded forms. Seen in this way, we might conclude that the prelude has a structure not dissimilar to fractals, in that the micro- and macro-structure are one and the same, or, to put it another way, with the opening phrase resembling a leaf, which contains within it a reflection of the structure of the whole tree.

References