Optimization of Bus Stop Location Using an Agent-Based Model

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Abstract: Bus-stop spacing has a strong influence on transit accessibility. However frequent in abstract models, uniform spaced bus stops may not be the best solution for real cities in which population density is not uniformly distributed. This paper describes an agent-based model that maximizes bus stop accessibility with respect to urban density. To formulate the optimization model, a segment of a bus route is given. The proposed algorithm is able to move two bus stops along the segment to find the optimal combination. Finally, model’s efficiency is compared with the simplest solution of the problem, which is considering all possible combinations of bus stop locations and choosing the best one.

Key-Words: Agent-based model, optimization algorithm, bus stop location, transit accessibility

1 Introduction
Bus stops accessibility is a very debated issue in transport researches. Several previous studies were focused on planning optimal bus stop spacing. These previous studies were usually based on supplier costs, acceleration/deceleration delay and dwell time, which are typical variables in transport planning issues, see for example [1]. Moreover, in these studies the number of bus stop was usually unknown. Consequently, the goal was to search its optimal value with respect of the above listed variables as in [2].

The distribution of urban population density has rarely been considered in previous studies, although urban density affects bus stop accessibility and consequently accessibility of the whole bus route. Therefore, this paper aims to find an optimal combination of bus stops location that maximizes transit accessibility with respect to urban density, under the assumption that the number of stops is given. Maximizing transit accessibility reduces the distance that each user has to walk to reach the nearest bus stop. In addition, the present study is based under on the hypothesis that walking is the only mode to access transit service.

The paper is organized as follows. First in Section 2, a geometric description of the problem is proposed. In section 3, the proposed model’s structure and its assumptions are described. In section 4, a measure of the model efficiency is presented. For this purpose, the number of iterations of the algorithm implementing the model is compared to the number of all possible combinations of bus stops. Finally, in section 5 some considerations are exposed.

2 Problem Formulation
The objective of this study is to develop an algorithm by which bus stop location can be optimized. The following assumptions are made to formulate the model:

- the model operates on a generic urban scenario;
- this scenario (Fig. 1) includes a segment of a bus route;
- four bus stops lie on the bus route segment;
- all buses follow the route only in one direction, that is the direction of access to the center of the city, from the left terminus to right central point;
- users are rational decision makers, thus they always walk to the nearest bus stop.

Fig. 1 The studied area.

The proposed model follows a discrete approach, therefore, the space is considered as a regular grid of square cells.

According to this approach, the studied area is covered by a rectangular 5x17 grid, as shown in Fig. 2. As a consequence, the bus route segment can be represented as a set of 17 cells, which occupies the central row of the grid, as exposed in Fig. 2. The two red cells at the opposite sides of the segment represent fixed bus stops. Between them, there are 15 numbered white free cells in
which the model is allowed to locate both two intermediate bus stops.

![Fig. 2 The grid covering the studied area.](image1)

The two bus stops possible positions are identified with couples of numbers like (4,8). Each couple represents the way in which bus stops are located: the first one is in the 4th cell (from left to right, as shown in Fig. 2), while the second one lies in the cell number 8. According to Combinatorial Theory, there are $\binom{15}{2} = 105$ possible couples of bus stop locations. Now, what rule has to be followed in order to try to guess the optimal combination among those 105 ones? For the purposes of this study, a cost function has been chosen as objective function. Before describing the cost function, it is necessary to introduce two matrices. These two matrices have both the same size of the considered grid in which the studied area is divided. Each element of the first matrix, say $p_{ij}$, represents the number of transit users of the $i,j$ cell, given by the product between density and cell area dimension. Each element of the second matrix, for example $d_{ij}$, stands for the distance between the same $i,j$ cell and the nearest bus stop. Distances are valued according to a so-called ‘Manhattan’ metric, as described in Fig. 3.

![Fig. 3 The used ‘Manhattan’ metric.](image2)

This metric is suitable for urban analysis, because the road network is usually made up of perpendicular crossroads. The cost function can be written as:

$$C(a, b) = \sum_{i,j} p_{ij} d_{ij}, \quad \text{with } a \neq b \quad (2)$$

where $(a, b)$ represents the couple of abscissas of the two bus stops whose locations will be optimized. Fig. 4 illustrates the distance matrix. Note that all zeros lie in cells occupied by a bus stop, that are the stops at the opposite sides of the segment, and the other two intermediate ones in position (3,6), according to the above introduced ‘couple notation’. Also note that, whether the cost function value is divided by the population of the entire studied area, the resulting ratio represents the average distance that a transit user has to walk toward the nearest bus stop.

The set of all possible couples of the two intermediate bus stops locations within the bus route segment can be represented in a plane. The results is a squared surface whose diagonal is occupied by couples like (i,i). Those diagonal couples are not considered for the purposes of this study, because they take place when the two intermediate bus stops are laid one upon the other.

![Fig. 4 The distance matrix.](image3)

Moreover, the diagonal cuts the surface in two symmetric triangles. In both portions, the cost function has a lowest value, which is the goal of the proposed model. A qualitative continue representation of this surface is shown in Fig. 5. Cost function is represented with contour lines; lighter gray shades correspond to lower values of the cost function.

![Fig. 5 A contour representation of the cost function.](image4)
3 The Model
Since the optimal couple is unknown, the proposed model has to explore the surface shown in Fig. 5. The simplest way is, of course, represented by calculating the cost function in every cells and choosing the lowest value. Unluckily, this way is the most expensive, in terms of calculation, because it requests as many evaluations of the distance matrix as combinations of possible stop locations. In addition, evaluation of the distance is the most onerous part of the entire algorithm, since this matrix has 5x17 elements.

Thus, the object of this study is to develop an algorithm optimizing the bus stop location, whose runtime needs few evaluations of the distance matrix. The proposed model tries to fit this criterion.

As defined in [3], an agent is “a software entity which functions continuously and autonomously in a particular environment, often inhabited by other agents and processes”. The proposed model has only one agent. In this section, discrete elements of the surface (Fig. 9) will be called ‘cells’.

The agent is able to move across the cells of the surface. Besides, it is also able to check the value of the cost function in its range of action, that is the neighborhood of eight cells around it. Such a neighborhood is known, specially among developers of Cellular Automata models, as Moore’s Neighborhood.

Moreover, the agent is also able to store continually two information:

- its current cell position, represented by a couple like (a,b);
- C, i.e. the value of the cost function in its current position.

The procedure of the proposed model (shown in Fig. 6) is stated as follows:

Step 1. Start with the cell (a,b) chosen by the user, which means that the first cell to locate is initially placed in the available position a of the bus route segment, and the second one in position b;
Step 2. The agent explores the neighborhood of its cell: it evaluates the cost function C in each cell of its cell neighborhood, then the agent move in the neighborhood cell with the lowest value of C.
Step 3. Previous step defines an iteration. If current C is not lower than C of previous iteration, then the algorithm stops and current couple is chosen as the optimal solution; else the agents continues moving, that is algorithm reiterates Step 2.

Consequently, the resulting movement of the agent across the surface is a continuous descent from the initial position to the cells with lower C values.

4 Efficiency of the Model
The proposed model has been tested on three fictitious urban scenarios, each simulating a typical urban density distribution. The total number of transit users in the studied area (425 units), is the same for all the three scenarios with an average of 5 users per cell, but each scenario has a different density distribution, as shown in Fig. 7.

The first one exhibits an uniform density as in Fig. 7 (a), while in the second one density is uniform only within the three zones in which studied area is divided (b). Finally, in the third urban scenario density decreases in likely way from the center of the city (segment’s left end) toward the outskirts (right end), according to a not uniform distribution, as in Fig. 7 (c).

For each scenario, the proposed model has been tested three times, each one with a different initial couple of locations for the two bus stops to locate. The three initial configurations are shown in Fig. 8,
in which the bus route is represented in different colors.

![Grid](image)

(a) (b) (c)

![Initial Configuration](image)

Fig. 7 The studied area grid.

Red cells are bus stops at the opposite sides of the segment, that model will not move; green cells are the bus stops whose position will be optimized by the model.

![Initial Configuration](image)

Fig. 8 Initial configuration of the bus stops.

![Moving Across](image)

Fig. 9 Moving across the surface of admissible bus stops configuration: agent’s path towards optimum.

Table 1 Measuring the proposed model efficiency.

<table>
<thead>
<tr>
<th>scenario</th>
<th>initial position</th>
<th>all possible combinations</th>
<th>proposed model</th>
<th>ratio</th>
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<td>58</td>
<td>1.81</td>
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<tr>
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<td>105</td>
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<td>2.33</td>
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<td>2.33</td>
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</tbody>
</table>

The results obtained from the set of nine model’s runs are shown in Table 1. The numbers in column (b) are lower than ones in column (a) for all the proposed configurations. In other word, the number of iteration required by the proposed algorithm to search the optimal bus stop configuration is always smaller than the number of possible combinations. It is also shown that better initial bus stops configurations improve model’s performances, as reported in column (c) of Table 1.

5 Conclusion

In this paper, an agent-based model that optimizes two bus stops location is presented. The implemented agent-based model operates on a discrete grid space composed of square cells. Besides, the proposed model is able to move two...
bus stops along the segment to find the optimal combination. The efficiency of the proposed model has been measured in nine cases. It is also shown that the number of iteration required by the proposed agent-based model to optimize the bus stop configuration is always lower than the number of possible combinations.

In addition, it has been demonstrated that agent-based models may be effectively used in optimization problems. Optimization of urban bus stop spacing using agent-based models, therefore, is a viable alternative that deserves further explorations.

References: