**JPEG image compression process based on the use of optics as calculation mainspring**

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**Abstract:** In order to reduce the information exchanges volumes, it is necessary, indeed even of paramount importance, to compress them. Moreover, this will subsequently facilitate their storage and transmissions. This increasing need for fast and reliable image compression led us to think about a new all optical image compression based on the JPEG standard. To meet these requirements, we propose, in this paper, an all-optical image compression architecture based on the JPEG compression standard. This method benefits, first, from the optical properties such as the possibility offered by coherent optics to carry out a two-dimensional Fourier transformation quasi instantaneously by using a convergent lens, and also from those of the new technologies based on electro-optical interfaces (spatial light modulator: SLM). Indeed, in this method we propose to carry out the two principal stages of JPEG compression optically, namely the DCT and the quantification, with a simple convergent lens. This realization is made possible thanks to the introduction, in the Fourier plane, of a phase filter which allowed us to obtain the DCT component starting from the FFT and by the introduction of a hologram allowing the optical quantification. We will see that these two operations can be amalgamated in only one. The simplicity and the good performances of this method shown by numerical simulations enabled us to propose an all optical experimental set-up of this method.

**Key-Words:** Image compression, JPEG, DCT, FFT, optical DCT, optical FFT, SLM modulator.

1 **Introduction**

In the last decade we noted a very fast increase in the use of images in various fields such as: the multimedia, the games and the medical imagery.... This use, which requires a very good images quality, poses many problems as for their transmissions or their storage. To become powerful, it became necessary to compress these images very quickly while keeping their good qualities.

However this image, which is originally optical, is generally treated with a computer, **thus numerically**. This image optical property led optoelectronics to be interested in the most powerful digital compression techniques in order to study the possibilities of implement optically them.

In a general way, compression is carried out by reducing all redundancy forms present in an image [1][2][3]:

- The spacial redundancy: This form of compression is based on the fact that all pixels are identical within a uniform Zone of a given image. Therefore, it is enough to code one to characterize all the considered Zone.
- The subjective redundancy: it comes from the imperfections of the human eyes. Pixels presenting enough characteristics can be perceived in an identical waynd so can be treated like identical pixels.

In this study we look an active interest in this second method of compression known as: method of compression at with loss).

Among several compression methods suggested in the literatures, the method of JPEG compression based on the use of the Discrete Cosine Transform (DCT), appeared to us more adapted to our application and more simple for an optical implementation. Indeed the Discrete Cosine Transform (DCT) used in JPEG algorithm is a particular case of the Fourier Transformation which is very simple to implement optically [4][5][6].

2 **Principle of the optical architecture JPEG**

In order to define our optical compression method, we will start this study by pointing out the principle of numerical JPEG compression, while trying to find the stages of compression likely to be implemented optically.

2.1 **Numerical standard JPEG Recall**

at the end of the Eighties, the Joint Photographic Experts Group have created a standard of compression “JPEG”, in order to file and facilitate the exchange of images[7][8].
The principle diagram of the JPEG compression [8] is presented Fig.1.

**Fig. 1 : JPEG principle Diagram**

We can differentiate, on this Fig. 1, the four great stages necessary to carry out the JPEG compression to namely :

- **Spatial sampling of the image** : The sampling stage consists in cutting out the image (N×N size) in blocks of 8x8 size.

- **Application of the DCT on the sampled image** : The DCT (Discrete Cosine Transform) is a Fourier Transform where we keep the cosine. That is to say n(i,j) a coefficient of a block (8x8) of the matrix image sampled (i,j): co-ordinates. The coefficients of the DCT matrix F(u,v) (u,v): the frequencies are given by the Equation. (1) [1][8].

\[
F(u,v) = \frac{C(u)\times C(v)}{4} \times \sum_{i=0}^{7} \sum_{j=0}^{7} n(i,j) \times \cos\left(\frac{2i+1}{16}u\pi\right) \times \cos\left(\frac{2j+1}{16}v\pi\right)
\]

with \( C(x) = \frac{1}{\sqrt{2}} \) if \( x=0 \),
if not \( C(x) = 1 \).

The equation (1) shows that the calculation of a DCT coefficient requires (8x8=64) additions. It is for this reason that we use matrices 8×8. As for the Fourier Transformation, to rebuild the image we must apply a reverse transformation of the DCT: IDCT Equation (2).

\[
n(i,j) = \frac{1}{4} \times \sum_{u=0}^{7} \sum_{v=0}^{7} C(u)\times C(v)\times F(u,v) \times \cos\left(\frac{2i+1}{16}u\pi\right) \times \cos\left(\frac{2j+1}{16}v\pi\right)
\]

- **Quantification of the coefficients obtained by the DCT** : This stage is the principal stage of compression. Indeed the principle of the quantification is to divide each term of the DCT matrix obtained \( F(u,v) \) by a quantum \( q(u,v) \). The quantified term \( F_q(u,v) \) is given by Equation (3).

\[
F_q(u,v) = F(u,v) / q(u,v) \]

Several types of quantification could be considered according to the desired treatment. As for example the uniform quantification which consists in dividing all DCT terms by the same quantum. Or the nonuniform quantification where each DCT term will be divided by a specific quantum.

- **Coding of the quantified coefficients** : Several types of coding could be considered according to the desired treatment. For example, we chose the RLE coding (Run Length Coding). Its principle is based on the replacement of a succession of identical character by the number of these identical character follow-up of the character. For example, “AAAAAAA” will be to replace by 7A.

### 2.2 Optical implementation of the JPEG

As mentioned above, the method of JPEG compression is divided into two great parts :

- selection of information : In this part, we carry out the elimination of redundant information contains in an image by quantifying its DCT.

- the coding of this information.

Among these two stages, the first one is more complicated and it requires more time devoted to JPEG compression. Therfore, in this study we propose to implemente optically (Fig 2) this first stage in order to reduce its realization time.

**Fig. 2: Diagram principle of the JPEG optics**

The image, to be compressed, is placed at the input plane. After the realization of DCT optically, we proceed to its optical quantification, according to a well-defined criterion. The choice of this criterion is very important because it will determine the importance of information and thus the compression ratio. To recover the image, after having transmitted it numerically or optically, we proceed to the dequantification then we carry out the inverse optical DCT, in order to recover the image in the output plane.

#### 2.2.1 Optical DCT: passage formulate between DCT and FFT

Thus to implement our method optically, we must start by optically carrying out the DCT of the input image. Knowing firstly the simplicity which opto-electronics carries out a Transformation of Fourier "TF" by using a simple convergent lens [4], and secondly the similarities between the TF and the DCT, we will carry out the DCT by using a convergent lens. For that
it will be necessary to find a passage relation between the TF and the DCT. In this objective, we start by recalling the formula of the two Dimensions TF (Equation 4) for an \((MxN)\) pixels image.

\[
F(u,v) = \frac{1}{MN} \times \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m,n) \times e^{-j \pi \left( \frac{2m+1}{2M} + \frac{2n+1}{2N} \right)} \tag{4}
\]

with \(u \in [0, M-1] \), \(v \in [0, N-1]\) 

By applying the formula of the FFT to an image \((2Mx2N)\) pixels, the equation (4) becomes:

\[
F(u,v) = \frac{1}{4MN} \times \sum_{m=0}^{2M-1} \sum_{n=0}^{2N-1} x(m,n) \times e^{-j \pi \left( \frac{2m+1}{2M} + \frac{2n+1}{2N} \right)} \tag{5}
\]

with \(u \in [0, 2M-1] \), \(v \in [0, 2N-1]\) 

Considering the symmetrical properties of the cosine and sinus functions, we can write \(i \in [0, N-1]\) :

\[
\cos \left( \frac{\pi (2i + 1)}{2N} \right) = \cos \left( \frac{\pi (2N - i - 1) + 1}{2N} \right) \tag{6}
\]

\[
\sin \left( \frac{\pi (2i + 1)}{2N} \right) = -\sin \left( \frac{\pi (2N - i - 1) + 1}{2N} \right) \tag{7}
\]

Moreover, if we duplicate the image \(I(M,N)\) in the way indicated on Fig. 4, we can write:

\[
x(m,n) = x(m,2N-n-1) \quad ; \quad n \in [0, N-1] \tag{8}
\]

\[
x(m,n) = x(2N-m-1,n) \quad ; \quad m \in [0, N-1] \tag{9}
\]

![Fig. 4 : duplication of the number 2](image)

Then while multiplying the TF of the new image \(l'(2Nx2M)\) (Fig. 4), point by point, by \(e^{-j \pi \frac{m+h}{2N}}\) we can write:

\[
C(k,l) = e^{-j \pi \frac{k}{2N}} \times e^{-j \pi \frac{l}{2N}} \times \frac{1}{4N^2} \times \sum_{m=0}^{2N-1} \sum_{n=0}^{2M-1} x(m,n) \times e^{-j \pi \left( \frac{2m+1}{2M} + \frac{2n+1}{2N} \right)} \times e^{-j \pi \frac{m+h}{2N}} \times e^{-j \pi \frac{l+h}{2N}} \tag{10}
\]

By developing the Equation 10, we can then write :

\[
C(k,l) = \left[ \frac{1}{4N^2} \times \sum_{m=0}^{2N-1} \sum_{n=0}^{2M-1} x(m,n) \times \cos \left( \frac{\pi (2m + 1)}{2N} \right) \times \cos \left( \frac{\pi (2n + 1)}{2N} \right) \right] 
\]

\[
- \sin \left( \frac{\pi (2m + 1)}{2N} \right) \times \sin \left( \frac{\pi (2n + 1)}{2N} \right) \times j \times \left[ \cos \left( \frac{\pi (2m + 1)}{2N} \right) \times \sin \left( \frac{\pi (2n + 1)}{2N} \right) \right] 
\]

\[
+ \sin \left( \frac{\pi (2m + 1)}{2N} \right) \times \cos \left( \frac{\pi (2n + 1)}{2N} \right) \times \cos \left( \frac{\pi (2m + 1)}{2N} \right) \right]
\]

Let us pose:

\[
U_1 = \frac{1}{4N^2} \times \sum_{m=0}^{2N-1} \sum_{n=0}^{2M-1} x(m,n) \times \cos \left( \frac{\pi (2m + 1)}{2N} \right) \times \cos \left( \frac{\pi (2n + 1)}{2N} \right) \tag{11}
\]

\[
U_2 = \frac{1}{4N^2} \times \sum_{m=0}^{2N-1} \sum_{n=0}^{2M-1} x(m,n) \times \sin \left( \frac{\pi (2m + 1)}{2N} \right) \times \sin \left( \frac{\pi (2n + 1)}{2N} \right) 
\]

\[
U_3 = \frac{1}{4N^2} \times \sum_{m=0}^{2N-1} \sum_{n=0}^{2M-1} x(m,n) \times \cos \left( \frac{\pi (2m + 1)}{2N} \right) \times \sin \left( \frac{\pi (2n + 1)}{2N} \right) 
\]

\[
U_4 = \frac{1}{4N^2} \times \sum_{m=0}^{2N-1} \sum_{n=0}^{2M-1} x(m,n) \times \sin \left( \frac{\pi (2m + 1)}{2N} \right) \times \cos \left( \frac{\pi (2n + 1)}{2N} \right) 
\]

Thereafter we will simplify the equation (11) by developing separately each one of these four terms. 

Let us start with \(U_1\). By using the equation (6,7,8) we can write:

\[
\begin{align*}
U_1 &= \frac{1}{4N^2} \times \sum_{m=0}^{2N-1} \sum_{n=0}^{2M-1} x(m,n) \times \cos \left( \frac{\pi (2m + 1)}{2N} \right) \times \cos \left( \frac{\pi (2n + 1)}{2N} \right) \\
&= \frac{1}{4N^2} \times \sum_{m=0}^{2N-1} \cos \left( \frac{\pi (2m + 1)}{2N} \right) \times \sum_{n=0}^{2M-1} x(m,n) \times \cos \left( \frac{\pi (2n + 1)}{2N} \right) \\
&= \frac{1}{4N^2} \times \sum_{m=0}^{2N-1} \cos \left( \frac{\pi (2m + 1)}{2N} \right) \times \sum_{n=0}^{2M-1} x(m,n) \times \cos \left( \frac{\pi (2n + 1)}{2N} \right) \\
&= \frac{1}{4N^2} \times \sum_{m=0}^{2N-1} \cos \left( \frac{\pi (2m + 1)}{2N} \right) \times \sum_{n=0}^{2M-1} x(m,n) \times \cos \left( \frac{\pi (2n + 1)}{2N} \right) \\
\end{align*}
\]

While varying \(m\) of 0 to 2N-1, we obtain a series of functions :

\[
\begin{align*}
\frac{1}{4N^2} \times \sum_{m=0}^{2N-1} \cos \left( \frac{\pi (2m + 1)}{2N} \right) \times \sum_{n=0}^{2M-1} x(m,n) \times \cos \left( \frac{\pi (2n + 1)}{2N} \right) \\
&= 2 \times \cos \left( \frac{\pi (2m + 1)}{2N} \right) \times \sum_{n=0}^{2M-1} x(m,n) \times \cos \left( \frac{\pi (2n + 1)}{2N} \right) + \ldots \\
&= \ldots \ldots \ldots \\
&= + 2 \times \cos \left( \frac{\pi (2m + 1)}{2N} \right) \times \sum_{n=0}^{2M-1} x(m,n) \times \cos \left( \frac{\pi (2n + 1)}{2N} \right) + \ldots \\
&= \ldots \ldots \ldots \\
&= + 2 \times \cos \left( \frac{\pi (2m + 1)}{2N} \right) \times \sum_{n=0}^{2M-1} x(m,n) \times \cos \left( \frac{\pi (2n + 1)}{2N} \right) + \ldots \\
&= \ldots \ldots \ldots
\end{align*}
\]

with \(i \in [0, N-1]\)

\[
U_1 = \frac{1}{4N^2} \times \sum_{m=0}^{2N-1} \sum_{n=0}^{2M-1} x(m,n) \times \cos \left( \frac{\pi (2m + 1)}{2N} \right) \times \cos \left( \frac{\pi (2n + 1)}{2N} \right)
\]

By holding account again equations (6, 7, 8), we obtain finally:

\[
U_1 = \frac{2}{4N^2} \times \sum_{m=0}^{N-1} \sum_{n=0}^{M-1} x(m,n) \times \cos \left( \frac{\pi (2m + 1)}{2N} \right) \times \cos \left( \frac{\pi (2n + 1)}{2N} \right)
\]

\[
= \frac{1}{N^2} \times \sum_{m=0}^{N-1} \sum_{n=0}^{M-1} x(m,n) \times \cos \left( \frac{\pi (2m + 1)}{2N} \right) \times \cos \left( \frac{\pi (2n + 1)}{2N} \right)
\]
On the other hand we can write $U_2 = 0$. Indeed:

$$\sum_{m=0}^{2N-2} \sum_{n=0}^{N-1} x(m, n) \times \sin(\frac{\pi k (2m+1)}{2N}) \times \sin(\frac{\pi h (2n+1)}{2N})$$

$$= \sum_{m=0}^{2N-2} \frac{\pi k (2m+1)}{2N} \times \sum_{n=0}^{N-1} x(m, n) \times \sin(\frac{\pi h (2n+1)}{2N}) = 0$$

$\Rightarrow U_2 = 0$ (according to the properties of the sine equation 7).

After simplification, of $U_1$ et $U_2$ we will try to find the terms $U_3$ and $U_4$.

$$U_3 = \frac{1}{4N^2} \sum_{m=0}^{2N-2} \sum_{n=0}^{N-1} x(m, n) \times \cos(\frac{\pi k (2m+1)}{2N}) \times \sin(\frac{\pi h (2n+1)}{2N})$$

$$= \frac{1}{4N^2} \sum_{m=0}^{2N-2} \frac{\pi k (2m+1)}{2N} \times \sum_{n=0}^{N-1} x(m, n) \times \sin(\frac{\pi h (2n+1)}{2N}) = 0$$

(according to the properties of the sine equation 7).

However $U_4$ can be written:

$$U_4 = \frac{1}{4N^2} \sum_{m=0}^{2N-2} \sum_{n=0}^{N-1} x(m, n) \times \sin(\frac{\pi k (2m+1)}{2N}) \times \cos(\frac{\pi h (2n+1)}{2N})$$

$$= \frac{1}{4N^2} \sum_{m=0}^{2N-2} \frac{\pi k (2m+1)}{2N} \times \sum_{n=0}^{N-1} x(m, n) \times \cos(\frac{\pi h (2n+1)}{2N}) $$

$$= 2 \times \frac{1}{4N^2} \sum_{m=0}^{2N-2} \sin(\frac{\pi k (2m+1)}{2N}) \times \sum_{n=0}^{N-1} x(m, n) \times \cos(\frac{\pi h (2n+1)}{2N})$$

While varying $m$ of 0 with $2N-1$, $U_4$ becomes:

$$U_4 = \frac{1}{4N^2} \times \left( 2 \times \frac{\pi k (2m+1)}{2N} \times \sum_{n=0}^{N-1} x(0, n) \times \cos(\frac{\pi h (2n+1)}{2N}) + \ldots \right)$$

$$+ \ldots \ldots \ldots$$

$$+ 2 \times \frac{\pi k (2m+1)}{2N} \times \sum_{n=0}^{N-1} x(1, n) \times \cos(\frac{\pi h (2n+1)}{2N}) + \ldots$$

$$+ \ldots \ldots \ldots$$

$$+ 2 \times \frac{\pi k (2m+1)}{2N} \times \sum_{n=0}^{N-1} x(N-1, n) \times \cos(\frac{\pi h (2n+1)}{2N}) + \ldots$$

$$+ \ldots \ldots \ldots$$

with $i \in [0, N-1]$.

By taking the equations (6,7,8) into account, we obtain $U_4 = 0$.

Finally, we can write:

$$c(k, l) = \frac{1}{N^2} \times \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} x(m, n) \times \cos(\frac{\pi k (2m+1)}{2N}) \times \cos(\frac{\pi h (2n+1)}{2N})$$

(12)

This formula is nothing than the formula of the DCT except for constants. This calculation can be obtained by duplicating an image in a particular form (Fig. 4) and by carrying out the FFT of this new image, we obtain its DCT in the spectral domain.

### 2.2.2 The standardization of the formulas of passage

To have a complete standardization of the DCT, let us try to find the constants missing. For that, let us recall the definition of the DCT for an image of size $(MxN)$:

$$R_{M} = \alpha_p \times \alpha_q \times \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} A_{m,n} \times \cos(\frac{\pi (2m+1)p}{2M}) \times \cos(\frac{\pi (2n+1)q}{2N})$$

(13)

with:

$$\alpha_p = \frac{1}{\sqrt{M}} \quad \alpha_q = \frac{1}{\sqrt{N}}$$

For $p = 0$ or $q = 0$, $\alpha_p \times \alpha_q = \frac{1}{\sqrt{2MN}}$.

However match the equation (12) to (13), it is necessary to multiply (12) by:

- $\frac{1}{N}$ for $p=q=0$
- $\sqrt{2 \times N}$ for $p=0$ ou $q=0$
- $2 \times N$ for $p \neq 0$ et $q \neq 0$

Thus with these parameters, we succeeded in carrying out the discrete cosine transform while using a traditional transform of Fourier. So, we managed to carry out an optical DCT while using a simple convergent lens Fig. (5).

![Synoptic diagram of the optical DCT](image)

Fig. 5: Synoptic diagram of the optical DCT

This assembly is divided into two parts:
3 All optical assembly of JPEG compression

We have seen until now that it is possible to carry out optically the discrete cosine transform DCT, the quantification and the filtering. In this paragraph, we will present an all optical assembly which associates the two stages in order to carry out the optical JPEG compression.

3.1 Optical Compression

The all optical assembly of JPEG compression is presented Fig. (7):

The input plane, lit by He-Ne laser, does consist of a duplication of the image to compress (as shows the Fig. 4). The Transformation of Fourier of this image is carried out by the lens L2 [4] [6] [9]. The spectrum of this image is in the P2 plane. In this spectral plane, we multiply this spectrum by the hologram (H0) defined in the figure (6). Then we multiply the DCT by a first hologram "H1" (hologram of quantification) and by "H2" (filter). In this plane (P2) we finally obtain a spectrum containing the quantified DCT and of size \( (C \times C) \) with \( C < N \).

To make these various multiplications optically, firstly we form a hologram gathering H0, H1 et H2 as shown in Equation (15).

\[
H = H0 \times H1 \times H2
\]

Then we introduce this result hologram into our optical diagram by displaying it on a modulating EASLM placed in the Fourier plan of the L2 lens.

3.2 Decompression JPEG

After coding, transmission and decoding of the DCT spectrum (CxC) we will proceed to decompression in order to rebuild the image. For that it is necessary to start with:

\[
Tc = N \times N \times C
\]
1. the dequantification: This stage consists in multiplying the received spectrum by a hologram which cancels the effect of the quantification "H 3" figure(8). H3 contains the inverse of the quanta used at quantification.
2. then we multiply the result by "H4" in order to restore the spectrum of the image.
3. Then we carry out an inverse transformation of Fourier with the L3 lens in order to find the decompressed image.

4 Results by simulation

To test the feasibility and the performances of our method, we took the example of compression of the Lena.bmp image table (1-a). After having applied the various stages of our method described above the results obtained by numerically simulations are giving Table (1).

<table>
<thead>
<tr>
<th>Original Image</th>
<th>Lena avec Tc=3</th>
<th>Lena avec Tc=6</th>
<th>Lena avec Tc=10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>Rate of transmission 33.33%</td>
<td>Rate of transmission 16.66%</td>
<td>Rate of transmission 10%</td>
</tr>
<tr>
<td>(b)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lena avec Tc=20</td>
<td>Rate of transmission 5 %</td>
<td>Rate of transmission 2.5 %</td>
<td></td>
</tr>
<tr>
<td>(d)</td>
<td>(e)</td>
<td>(f)</td>
<td></td>
</tr>
</tbody>
</table>

Table (1): simulations results

To measure the performances of our method we chose the transmission rate criterion. This criterion is defined as being the relationship between the size of the spectrum of the original image and the size of the spectrum transmitted with our method.

These different results attests the feasibility of our method and its good behaviour. Indeed we notice that it is completely possible to decipher the image even with a rate of transmission around the 2.5%.

5.Combustion

In this study we proposed and validated by numerical simulations an all optical method of realization of the quantification DCT. By this realisation we succeeded in reducing the size of the image at the source before any numerical processing. Thus after having made possible to carry out an optical DCT with a simple convergent lens, it was possible for us to propose an all optical assembly of realization of compression JPEG. Then the different results obtained by numerical simulations shows the well effectiveness of this method and gives results comparable with numerical compression. With noted finally that we are implementing optically this method in our laboratory GOSI-ISEN.

References: