Investigation of Transmission Line Parameter Sensitivities in Congested Power Systems

CANAN ZOBİ KARATEKİN
Department of Electrical Engineering
Istanbul Technical University
Istanbul, TURKEY

CANBOLAT UÇAK
Department of Electrical & Electronics Engineering
Yeditepe University
Istanbul, TURKEY

Abstract: - Transmission congestion management may be categorized into short–term and long–term management. The short–term transmission congestion management is based on rules and pricing and the long-term congestion management is based on planning to strengthen transmission grid in deregulated power systems. Frequent congestion between two zones or nodes may require straightening of the appropriate lines for the long-term cost minimization. In this paper, congestion relief and cost minimization are investigated as a function of line susceptances and line limits. First, mathematical derivations to calculate sensitivities of the generation cost to the line parameters such as line susceptances and line limits are given. Second, numerical studies are done for an example system by using Sequential Quadratic Programming to determine congestion boundaries, congested lines and, cost of generation when line capacities and line susceptances are varied. The results are given in the conclusion section.

Key-Words: - Congestion management, sensitivity analysis, transmission planning

1 Introduction
The restructuring of power systems has led to an unusual usage of transmission grids not foreseen at the design stage [1]. In the restructuring environment, generators and loads can access transmission system in a non-discriminatory and equitable manner. Open access to transmission systems are very important for healthy competition in power industry. In a deregulated environment, producers and consumers sell and buy electrical energy through transactions in a market. These transactions when implemented may sometimes cause the electric power network to become congested. Therefore, congestion management has become more important and difficult in the emerging deregulated electricity markets as the number and magnitude of power transactions increase [2]. In a power market, transactions between producers and consumers are limited by line parameters, when the limitations exist, cost of electrical energy production will increase. Also, because of the congestion, in some regions market power may prevail. Transmission congestion management may be categorized into short–term and long–term. The short–term transmission management is based on rules and pricing [3-7]. The FERC expresses its preferences for market-based plans, and notes that the pricing and expansion program should be compatible with the pricing signals for shorter-term solutions to congestion management so that market participants can choose the least-cost response [8]. Congestion always increases the cost of electrical energy and congestions persisting in similar zones of the transmission line must be removed in long–term so that competitive energy market can be accomplished again. The statistical methods which are proposed in literature may be used in congestion studies for long-term planning [9]. Grid planning is a long-term approach which requires decision tools to determine which part of the network should be developed in the future. The objective of [9] is to forecast which lines might be simultaneously congested. Competitive markets expose transmission planners to new uncertainties. These are handled using a decision–analysis approach whose key contribution is quantifying and minimizing risk [10]. In vertically integrated systems, many methods have been used for transmission expansion planning [11-13]. Transmission planning in these studies are accomplished to respond to load and generation increases. However, in this paper, congestion relief is considered for horizontally integrated power systems.

Long-term congestion relief between zones can be achieved by the strengthening the transmission lines. The strengthening of a transmission line can be accomplished either by building another transmission line between the congested zones or by increasing the capability of the original line. This will cause susceptibility change and line limit increase between the two points. In this paper, congestion relief as a function of line susceptance and line limit is investigated. The goal is to eliminate line congestion with transmission line planning in the long-term. Therefore, the effect of changing line susceptance and line limit to the generation cost is studied. To achieve this, mathematical
derivations to calculate sensitivities of the generation cost to the line susceptances are given. And then, numerical studies are done by using sequential quadratic optimization programming (SQP) to determine the congestion region and the cost of generation when line capacities and line susceptances are changed.

This study shows that it’s possible to relieve congestion on the line while the line susceptance is kept in a certain interval. It’s interesting to note that sometimes it’s sufficient to decrease the line susceptance to relieve congestion. The results are given in the conclusion.

2 Sensitivity of Line Susceptance

The effect of altering the susceptance of various transmission lines in the system is calculated by the procedure described in the references [14] and [15]. Many numerical techniques in power flow studies, as in the case of congestion management and transmission pricing in a market driven power system environment, concentrate on the use of DC power flow analysis, as they can give a clear picture of the consequences on system for changes in parameters, and because a large size power system generally requires a reduction in computational time, especially for on-line analysis [16].

In this study, the so-called ‘DC’ approximation is used for the reasons mentioned above, that is, the real power losses are ignored and bus voltage magnitudes are approximated to 1.0 pu. Under these assumptions, reactive power flow is zero on each line. Each transmission line connecting the busses is specified by its susceptance $b_j$ and its maximum power flow limit $P_{ij}^{\text{max}}$. The real power flow on line $ij$ is

$$P_{ij} = b_j (\theta_i - \theta_j)$$

where $\theta_i$ is the voltage angle at bus $i$. The total injection at bus $i$ is

$$P_i = \sum_j P_{ij} + P_{\text{load}_i}$$

The total generated and the total consumed power balance relation is given by

$$\sum_i P_i = \sum_{\text{load}_i}$$

The objective is to minimize generation costs given by

$$L = \sum_{i=1}^{n_{\text{bus}}} F_i(P_i) + \sum_{i\neq j}^{n_{\text{bus}}} \lambda_{ij} h(P_i, \theta_j, b_j, P_{\text{load}_j})$$

$$+ \sum_{i\neq j}^{n_{\text{bus}}} \mu_{ij} (P_{ij} - P_{ij}^{\text{max}})$$

Variables and multipliers are given by

$$z = [P_{\text{bus}}, \theta_{\text{bus}}, \theta_{\text{bus}}, \lambda_{\text{bus}}, \mu_{\text{bus}}, \mu_{\text{branch}}]^T$$

Explicitly including $b_j$ in the problem statement, and denoting the optimal solution for a given $b_j$ by $z'(b_j)$, we still must have the first order condition

$$g(z'(b_j)) = \frac{\partial L}{\partial z} = 0$$

For a small change in $b_j$, it can be assumed that the set of binding inequality constraints does not change. We can differentiate the first-order condition with respect to $b_j$ using the chain rule of calculus

$$\left(\frac{\partial g(z'(b_j))}{\partial b_j}\right) \left(\frac{dz'(b_j)}{db_j}\right) + \left(\frac{\partial g(z'(b_j))}{\partial b_j}\right) = 0$$

It is defined $\frac{\partial g}{\partial b} = W(b_j)$ at every $z$, and this equation is integrated

$$g(z(b_j)) = W(b_j)z + g_0(b_j)$$

The sensitivity $\frac{dz(b_j)}{db_j}$ can be obtained by solving

$$W(b_j) \frac{dz^*}{db_j} = -\frac{\partial W(b_j)}{\partial b_j} z^*(b_j) - \frac{\partial g_0(b_j)}{\partial b_j}$$

This value of $g_0$ can be substituted into the previous equation,

$$\frac{dz^*}{db_j} = \frac{\partial W(b_j)}{\partial b_j} z^*(b_j)$$

So, the sensitivity $\frac{dz^*(b_j)}{db_j}$ is obtained as

$$\frac{dz^*(b_j)}{db_j} = W^{-1}(b_j) \left(-\frac{\partial W(b_j)}{\partial b_j}\right) z^*(b_j)$$

The value of $\frac{dz^*(b_j)}{db_j}$ can be used to calculate the marginal effect on total system cost due to a change in $b_j$. We can just multiply the portion of $\frac{dz^*(b_j)}{db_j}$ corresponding to power injections, $\frac{dp^*(b_j)}{db_j}$, by the corresponding marginal costs, or, equivalently, the multipliers $\lambda^*$, to obtain

$$\frac{dTC}{db_j} = (\lambda^*)^T \frac{dp^*}{db_j}$$
Now let us combine the effect of sensitivities of line susceptance and line capacity to see what the effect of marginally ‘strengthening’ a line (increasing both the capacity and the magnitude of the susceptance) will be. For that, we choose to increase the magnitude of the susceptance by the same ratio as we increase the line capacity. Let us consider the line $i$-$j$, and increase the capacity by $\varepsilon$ MW. The corresponding change in the susceptance would be $\varepsilon \frac{b_{ij}}{P^\text{max} _{ij}}$. The resulting change in the total cost for congested line, divided by $\varepsilon$, is then

$$\frac{\partial TC}{\partial P^\text{max} _{ij}} = -\mu _{ij} - \left( \frac{-b_{ij}}{P^\text{max} _{ij}} \right) \frac{dTC}{db_{ij}}$$

(16)

Similar calculations can be done for other lines as

$$\frac{\partial TC}{\partial P^\text{max} _{ij}} = -\mu _{ij} - \left( \frac{-b_{ij}}{P^\text{max} _{ij}} \right) \frac{dTC}{db_{ij}}$$

(17)

### 3 Example System

Sensitivity analysis described above has been carried out on an example system shown in Fig. 6, [17]. The data for the system is given in Table 1 (cost functions, generation capacities). Each bus has 100 MW load. Maximum power flow limits of all transmission lines are 100 MW and line susceptances are 5 pu for all lines and line resistances are omitted. By selecting the angle $\theta_1 = 0$ as the reference angle, equality constraints for the system can be written as

$$-P_1 - b_{13}\theta_2 - b_{14}\theta_3 + 100 = 0$$
$$-P_2 + b_{13}\theta_2 + b_{24}(\theta_2 - \theta_4) + 100 = 0$$
$$-P_3 + b_{13}(\theta_3 - \theta_5) + b_{36}(\theta_3 - \theta_6) + 100 = 0$$
$$-P_4 - b_{24}(\theta_2 - \theta_4) + b_{45}(\theta_4 - \theta_5) + b_{46}(\theta_4 - \theta_6) + 100 = 0$$
$$-b_{12}\theta_2 - b_{45}(\theta_4 - \theta_5) - b_{55}(\theta_1 - \theta_2) + 100 = 0$$
$$-b_{46}(\theta_4 - \theta_6) - b_{36}(\theta_3 - \theta_6) + 100 = 0$$

(18)

The supply – demand balance is

$$P_1 + P_2 + P_3 + P_4 = 600$$

(19)

Vector of variables and Lagrange multipliers are

$$z = [P_1, P_2, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \mu_{46}]^T$$

(20)

In this case, we know in advance that the flow constraint $P_{46} \leq P_{46}^\text{max}$ is binding, so we treat this constraint as an equality constraint, with Lagrange multiplier $\mu_{46}$. The other inequality constraint are nonbinding, so we ignore them. We can differentiate the Lagrange function with respect to all variables and Lagrange multipliers and, thus it is given by $Wz = g_0$. $W$ matrix depends on susceptances. In this example system, line 4-6 is congested. Therefore, $W(b_{46})$ matrix will be

$$\frac{\partial W}{\partial b_{46}}$$

is calculated by taking the differential with respect to $b_{46}$.

The sensitivity $\frac{dz^*}{db_{46}}$ is obtained by

$$\frac{dz^*}{db_{46}} = W^{-1}(b_{46}) \left( -\frac{\partial W}{\partial b_{46}}(b_{46}) \right) z'(b_{46})$$

(21)

To calculate the marginal effect on total system cost due to a small change in $b_{46}$, we can multiply the portion of $\frac{dz^*}{db_{46}}$ corresponding to power injections, $\frac{dP^*}{db_{46}}$ by the corresponding marginal costs or the multipliers $\lambda^*$ to obtain

$$\frac{dTC}{db_{46}} = \lambda^* \frac{dP^*}{db_{46}} [P_1, \lambda_2, \lambda_3, \lambda_4] = 14.75$$

($/h)/pu$

(22)
Similar calculations for other lines yield
\[
\begin{bmatrix}
\frac{dTC}{db_{12}} & \frac{dTC}{db_{15}} & \frac{dTC}{db_{24}} & \frac{dTC}{db_{35}} & \frac{dTC}{db_{45}} \\
-2.57 & -1.95 & 1.81 & -3.9 & -8.13
\end{bmatrix}
\] (23)

The sensitivity of the line 4-6 is found to be higher than the others. It’s the congested line and its susceptance will have the most effect on the total cost of the system. To see the effect on the total cost, a numerical study using SQP has been carried out for the susceptance range between 0 and 5 pu. The results are given in Fig. 1. The figure shows that there is a optimum susceptance interval where there’s no congestion in the system and the total cost is minimum. The susceptance values above or below this interval may cause congestion on the investigated line or on the other lines in the system. This shows that strengthening a line beyond a certain value may sometimes cause an adverse effect and may increase the total cost for a specific case. In Fig. 1, congestion appears at the line 4-5 when the line 4-6 susceptance is between 0 and 2.5 pu and it appears at the line 4-6 when the susceptance of 4-6 is between 3.5 and 5 pu. No congestion exists for the susceptance interval between 2.5 and 3.5 pu and, thus the total cost is minimum.

Fig. 1. The effect on total cost of altering the susceptance of line 4-6.

Fig. 1 shows that when the susceptance of the line 4-6 changes, there is congestion on the line 4-5. Therefore, the susceptance of both lines are varied between 0 and 5 pu to see the total cost change in the example system. When there’s no congestion and all the busses have the same prices, the total cost is minimum. When the susceptance value of the line 4-5 and the line 4-6 is 0 pu., the supply-demand balance can not be satisfied and there’s no feasible solution. The total cost is minimum at susceptance values between 1.5 and 3.5 pu for the line 4-6 and between 2.5 and 5 pu. for the line 4-5. If the line susceptances are chosen between these values, there is no congestion in the entire system. The results are also given graphically in Fig. 2.

Fig. 2. The effect on the total cost of altering the susceptances of line 4-6 and line 4-5.

The congestion regions and the congested lines are shown in Fig. 3 when both susceptances of the line 4-6 and the line 4-5 are changed between 0 and 15 pu. The values of b_{45} and b_{46} have to be in the region shown as "no congestion" for a minimum cost. Outside of this region, total cost will always be higher.

When the line capacities are kept constant and only line susceptances are changed, congestions may not be eliminated. The reason for that is, as the susceptance value increases, power flow also increases. But, when the power flow on the line reaches the line capacity, the increase on line susceptance will not alter the flow on that line anymore because the line is congested. Instead, since voltage angles at busses change as the susceptance variation, the power flow on the other lines will be modified and the congestion may appear on some other transmission lines.
Susceptance and line limit are not independent from each other in general. Increasing the susceptance will also lead to an increase in the capacity of the line. Therefore, in this part, the effect on total cost of increasing both the capacity and the magnitude of the susceptance of the line 4-6 and the line 4-5 in the same ratio is investigated and the result is shown in Fig. 4. As it can be seen from Fig. 4, the congested region became smaller because of the increase of both line capacity and line susceptance.

Fig. 4. The effect on total cost of increasing both the capacity and the magnitude of the susceptance of the line 4-6 and the line 4-5.

In Fig. 5, the congested lines and regions are shown in detail when both the line capacity and the magnitude of the susceptance of the line 4-6 and the line 4-5 are altered. In this case, "no congestion" region is can be seen much better and it is quite wide.

Fig. 5. The congestion boundaries and the congested lines when the ratios of the line capacity and the magnitude of the susceptance of the line 4-6 and the line 4-5 are changed.

4 Conclusion

In this paper, a sensitivity analysis and a numerical study have been carried out to see the effect of variations in transmission line parameters such as line susceptances and line limits, which will be important to be considered by the planning engineers.

The result of the study shows that the sensitivity of the total cost to the congested line is found to be higher than the lines with no congestions. There may be an optimum susceptance range which above and below values will increase the total cost when only the susceptance is varied and the line limit is kept constant. Optimum susceptance may be important when the line parameters are changed dynamically by using FACTS devices widely in the future. The operation of the network for relieving congestions may require tracking these optimal parameter ranges when system state changes in time.

Because the susceptance and the line limit are dependent to each other in general, increasing both the line susceptance and the line limit will eliminate congestion better. Therefore, congestion management for long-term requires a careful study of cost sensitivity to the line parameters. These studies will guide planners to decide which lines must be strengthened primarily to eliminate long-term congestion efficiently.

References:
Appendix

Fig. 6. Example system with six buses.

Table 1. Cost and related parameters of each generator

<table>
<thead>
<tr>
<th>Generator bus</th>
<th>Cost Function</th>
<th>Min Gen.</th>
<th>Max Gen.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_i(P_i) = a_i P_i^2 + b_i P_i + c_i$</td>
<td>50</td>
<td>170</td>
</tr>
<tr>
<td>1</td>
<td>0.0120, 12.0, 105</td>
<td>50</td>
<td>170</td>
</tr>
<tr>
<td>2</td>
<td>0.0096, 9.6, 96</td>
<td>50</td>
<td>250</td>
</tr>
<tr>
<td>3</td>
<td>0.0130, 13.0, 105</td>
<td>50</td>
<td>170</td>
</tr>
<tr>
<td>4</td>
<td>0.0094, 9.4, 94</td>
<td>50</td>
<td>250</td>
</tr>
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