Abstract: In this paper, a comparative study has been provided for image enhancement applications, between three techniques, namely: the curvelet transform which is considered as relatively new method for representing edges and thus is suited for multiscale edge enhancement; the multiscale gradient using wavelet transform, and multiscale retinex. Simulation results have shown that the curvelet transform is superior for enhancement applications.

Keywords: Curvelet transform, Ridgelet transform, Radon transform, Multiscale gradients, Retinex.

1 Introduction

The curvelet transform has been introduced as an attractive method for various applications. Enhancement based on curvelet transform is a promising direction of research [1-3]. The Curvelet Transform is based on decomposing the image into different scales, then partitioning into squares whose sizes are based on the corresponding scale. Another interesting transform is the Orthonormal Ridgelet Transform (ORT) [4]. The key point in the Ridgelet Transform is to map a line singularity into a point singularity by using Radon transform, then the Wavelet Transform can be used efficiently to handle the point in the Radon domain. The process of enhancement for the curvelet coefficients uses Velde function [1] which is a nonlinear function enhancing the faint edges.

Multiscale gradient using wavelet transform is another method used for enhancement [5]. Wavelets provide an efficient way to compute multiscale gradient. [6] defines two oriented wavelets as the partial derivatives of a smoothing function. These functions can be convolved with the images to detect the edges at different scales.

The Multiscale Retinex (MSR) [7] is a generalization of the Single-Scale Retinex (SSR), which is based on the Land's center/surround retinex. The MSR approach combines dynamic range compression and color constancy.
2 Enhancement Using Curvelets

2.1 The Ridgelet Transform

The continuous ridgelet transform (CRT) in $\mathbb{R}^2$ can be defined by [1]:

$$CRT_f(a,b,\theta) = \int_{\mathbb{R}^2} \psi_{a,b}(x)f(x)dx$$ (1)

where $x = (x_1,x_2)$. The ridgelets $\psi_{a,b}(x)$ in 2-D are defined from choosing a wavelet function in 1D $\psi(x)$ as [1]:

$$\psi_{a,b}(x) = a^{-1/2}[\psi(x_1 \cos(\theta) + x_2 \sin(\theta) - b)/a]$$ (2)

this function is oriented by $\theta$ and constants along the lines $x_1 \cos(\theta) + x_2 \sin(\theta) = b$.

Ridgelet analysis may be constructed as wavelet analysis in the Radon domain. By recalling the definition of Radon transform as the collection of line integral indexed by $(\theta,t) \in [2\pi,0) \times \mathbb{R}$ :

$$Rf(\theta,t) = \int f(x)\delta(x_1 \cos(\theta) + x_2 \sin(\theta) - t)dx$$ (3)

where $\delta$ is Dirac function. The ridgelet transform is given by analysis of the Radon transform by [1]:

$$CRT(a,b,\theta) = \int R(\theta,t)a^{-1/2}[\psi((t-b)/a)]dt$$ (4)

Hence, the ridgelet transform is given by application of a 1-dimensional wavelet transform to the slices of the Radon transform where the angular variable $\theta$ is constant and t is varying.

The ortho-ridgelets are indexed using $\lambda = (j,k,l,i,\varepsilon)$, $\lambda \in \Lambda$, where $j$ indexes the ridge scale, $k$ the ridge location, $i$ the angular scale, and $l$ the angular location; $\varepsilon$ is a gender token. Here, $\psi_{\lambda}$ denotes Meyer wavelets for $R$, and periodic wavelets $(-\pi,\pi)$, indices runs as $j,k \in \mathbb{Z}, l = 0,...,2^{-j-1};i \geq ij$, & if $\varepsilon = 0,i = \max(i_0,j)$, while if $\varepsilon = 1,i > \max(i_0,j)$.

The ORs are modified to the ridgelet, they can be characterized by certain localization properties in a radial frequency $\times$ angular frequency domain. The formula (5) shows that the ORs are localized in the frequency domain into elongated wedges have radial extent $2^j$ and angular width $2^{-i}, i \geq j$.

For practical applications, the discrete ridgelet transform leads to discrete the Radon transform. By using projection slice theorem, the Radon transform can be obtained: by (a) performing a 2-d Fourier transform, (b) obtaining a radial slice of the Fourier Transform, and (c) applying a 1-d inverse Fourier Transform to the obtained slice. For a discrete data, the fast Fourier may be used on 2-d and 1-d Cartesian grids, but the problem in step (b) is that the radial slices of the Fourier domain do not intersect the Cartesian grid, so some sort of interpolation is required to take place. The pseudopolar Fourier Transform is a method that can be used as an interpolation to evaluate the 2-d Fourier transform on a non-cartesian points, which is called the pseudopolar grid. Figure(1):(a) shows the orthonormal ridgelet in frequency domain, and (b) the digital case where use pseudo polar grid that resulted for interpolation of Fourier transform.

![Figure(1): Some Ridgelets at different scales, and angles.](image1)

The ortho-ridgelet (OR) is given in frequency domain by [4]:

$$\rho_J(\xi) = |\xi|^{-1/2} [\psi_{j,k}(|\xi|\cos_j^\varepsilon(\theta)) + \psi_{j,k}(-|\xi|\cos_j^\varepsilon(\theta + \pi))/2$$ (5)

2.2 Multiscale Ridgelet Transform

Let $Q$ denote a dyadic square $Q = [k1/2^i, (k+1)/2^i] \times [k2/2^i, (k+1)/2^i]$, and The notation $Q_s$ will correspond to all dyadic squares of scale $s$. Let $w_Q$ be a window centered near $Q$, obtained after dilation and translation of a single $w$, such that the $w_Q$’s, make up a partition of unity. The multiscale ridgelets is defined by

![Figure(2): (a) Ridgelet tiling, (b) and Digital Ridgelet tiling](image2)
\[ \rho_{0,\lambda} : s > s_0, \lambda \in \Omega, \lambda \in \Lambda \]
\[ T_0 f = 2^s f (2^s x_1 - \lambda_1, 2^s x_2 - \lambda_2) \quad (6) \]

The multiscale ridgelet system renormalizes and transports the ridgelet basis, so that one has a system of elements at all lengths and all finer widths.

### 2.3 Subband Filtering

The last component required is a bank of filters \((P_0, \Delta f_1, \Delta f_2, \Delta f_3)\) with the property that the passband filter \(\Delta s\) is concentrated near frequencies \([z, 2z]\) which is a not classical standard [2]:

\[ \Delta_s = \Psi_{2s} \ast f, \Psi_{2s}(\xi) = \Psi(2^{-2s} \xi) \quad (7) \]

### 2.4 Curvelet Transform Enhancement

The curvelet decomposition can be stated in the following form. First, Subband Decomposition. \(f\) is filtered into subbands [2]:

\[ f \rightarrow (P_0, \Delta f_1, \Delta f_2, \Delta f_3) \quad (8) \]

Second, Smooth Partitioning. Each subband is smoothly windowed into "squares" of an appropriate scale \(\Delta_s f \rightarrow w_0 \Delta_s f\).

The curvelet transform have a new kind pyramid structure. First pyramid, indexed by \(Q\) whose range is recalled to be the set of all dyadic squares, which localizes the image both in space and frequency. Second pyramid, the ridgelet pyramid which analyzes each renormalized block of image data that obey spatial and frequency localization.

The curvelet transform is suited for images contains edges, so it is a good for edge enhancement. Curvelet coefficients can be modified in order to enhance edges in an image by \(y(x)\) (Veld function). This function aims to faint edges, the formula is defined by:

\[
\begin{align*}
y(x) &= \left\lfloor \frac{m}{c} \right\rfloor y \quad \text{if} |x| < c \\
y(x) &= \left\lfloor \frac{m}{|x|} \right\rfloor y \quad \text{if} c < |x| < m \\
y(x) &= 1 \text{ if } |x| \geq m 
\end{align*}
\]

Three parameters used : p, m, and c. p determines the degree of non-linearity and must be in \([0,1]\). Coefficients larger than m are not modified by algorithm. The c parameter corresponds to the noise level. Figure(3) shows the coefficients enhancement versus the original coefficients.

### 3 Wavelet Multiscale Gradients

Multiscale gradients generate by choosing a kernel function that is the first order derivative of a smoothing function , this kernel have to be satisfied the admissible wavelet condition.

The oriented wavelets as the partial derivatives of a smoothing function \(\phi(x, y)\) are defined as [5]

\[ \psi^i = \frac{\partial}{\partial x} \phi(x, y) \quad \text{and} \quad \psi^j = \frac{\partial}{\partial y} \phi(x, y) \quad (10) \]

Assume an image is a differentiable two dimensional function \(f(x, y)\), the associated 2-D dyadic wavelet transform of an image at scale 2, position \((x, y)\), and orientation \(k\) is defined as [5]

\[ W_{2^k, f}(x, y) = f \ast \psi_{2^k, f}(x, y) \quad k = 1, 2 \quad (11) \]

with \(\psi_{2^k, f}(x, y) = 2^{-2k} \psi^i(2^{-k} x, 2^{-k} y)\)

The wavelet defined by (10) produces a sequence of vector fields indexed by scale, they are the gradient of \(f(x,y)\) smoothed by \(\phi(x, y)\) at dyadic scales, or the multiscale gradients \(\nabla_{2^k, f}(x, y) = (W_{2^{k-1}, f}(x, y), W_{2^{k-1}, f})\). It is often convenient to represent multiscale gradients in magnitude, and angle pairs, as the following [6]:

\[ \rho_{2^k, f}(x, y) = \sqrt{(W_{2^{k-1}, f}(x, y))^2 + (W_{2^{k-1}, f}^2(x, y))^2} \quad (12) \]

\[ \Theta_{2^k, f}(x, y) = \arctan \left[ \frac{W_{2^{k-1}, f}^2}{W_{2^{k-1}, f}} \right] \quad (13) \]

Two fast algorithm are implemented: the wavelet transform and the inverse wavelet transform in two dimensions. The two wavelet \(\psi^i(x, y)\) and \(\psi^j(x, y)\) are characterized by the three
discrete filters H, G, K, and L. These filters are related to the spline wavelets.

The 2-D discrete wavelet transform of an image \( f(x,y) \), at each scale \( 2^j \), decomposes the image into approximation image and the horizontal and vertical wavelet coefficients as defined by formulas:

\[
W_{2^j,1d} = S_{2^j,1d} f * (G_j, D) \tag{14}
\]

\[
S_{2^j,1d} f = S_{2^j,1d} f * (H_j, H_j) \tag{15}
\]

\[
S_{2^j,2d} f = S_{2^j,2d} f * (H_j, H_j) \tag{16}
\]

\( j \) is a positive integer value where varied from 0 to \( J \). at scale \( 2^j \), the \( W_{2^j,1d}, W_{2^j,2d} \) are horizontal and vertical wavelet coefficients, and the \( S_{2^j,1d} f \) is approximation image. Where the subscript \( j \) in the digital filters means the number of zeros between the coefficients of the filters. D is Dirac filter whose impulse response is equal to 1 at 0 and 0 otherwise. The notation \( A^\oplus(H,L) \) is denoted to separable convolution of the rows and columns, respectively, of the image \( A \) with 1-D filters H.L., the original image is \( Sf \).

The reconstruction algorithm computes \( S_{2^j,2d} f \) from \( S_{2^j,1d} f \), and \( W_{2^j,1d}, W_{2^j,2d} \), defined by:

\[
S_{2^j,2d} f = W_{2^j,1d} f * (K_{j-1,1}, L_{j-1,1}) + W_{2^j,2d} f * (L_{j-1,1}, K_{j-1,1}) + S_{2^j,1d} f * (H_{j-1,1}, H_{j-1,1}) \tag{17}
\]

Based on the above discussion, contrast enhancement can be done by inserting gradients into Velde function then multiply the wavelets coefficients by Velde function.

### 4 Multiscale Retinex

The single scale retinex (SSR) methods consists of applying the following transform to each band \( i \) of the color image [7] :

\[
R_i(x,y) = \log(I_i(x,y)) - \log(G(x,y) * I_i(x,y)) \tag{18}
\]

where \( R(x,y) \) is the retinex output, \( I(x,y) \) is the image distribution in the \( i \)th spectral band \( G \) is a Gaussian function, and \( * \) is the convolution. A gain/offset is applied to the retinex output which clips the highest and lowest signal excursions.

The Multiscale retinex combines several SSR outputs to produce a single output image which has both good dynamic range compression and color constancy which may be defined as the independence of the perceived color of the light source and good tonal rendition. MSR can be defined by[1], [7]:

\[
R_{MSR} = \sum_{j=1}^{N} w_j R_{i,j} \tag{19}
\]

\( N \) is the number of scales, \( R_{i,j} \) is the \( i \)th spectral component of the MSR output, and \( w_j \) is the weighted associated with the scale \( j \). The Gaussian \( G \) is given by:

\[
G_j(x, y) = k \exp(-r^2 / c_j^2) \tag{20}
\]

where \( c_j \) defines the width of the guassian. Three scales are recommended with \( c_j \) values equal to 15, 80, 250 and \( w_j = 1/N \). The final step is gain/offset applied to make the coefficients be suitable display.

### 5 Results

In this section, the performance of the above algorithms will be explored. The color image (Kodak image) in Figure(4) are converted into YCbCr. Then the luminance component Y is decomposed for levels.

Figure(5) depicts the horizontal, vertical wavelet coefficients, the gradients, the angles, and the approximation images with no enhancement implemented.
The image gradients are enhanced by Velde function where $c = 3$, $p = 0.5$, $m = 50$. Figure(4) shows these results. Figure(7) shows the reconstructed image after enhancement. Figure(8) and Figure(9) show the analysis of curvelet transform in forward and inverse respectively for the luminance component Y. Figure(10) shows the enhancement based on curvelet transform where the parameters of Velde functions are $c = 3$, $p = 0.5$, $m = 0.1M$, $M$ is the maximum of Curvelet coefficients. Figure(11) shows SATELLITE image and the enhancement of the gray scale based on curvelet transform. Figure(12) shows FLOWER image enhanced by MSR.

In fact, enhancement of images by MSR tends to increase the grayness. In contrast, the multiscale gradient using Velde function shows better reservation of chrominance.
6 Conclusion

In this paper, a comparative study has been provided, for image enhancement applications, between three techniques, namely: the curvelet transform; the multiscale gradient using wavelet transform, and multiscale retinex. Simulation results have shown that the curvelet transform is superior for enhancement applications. The Curvelet transform has new features like the scaling law, in other words, the spatial domain related with scale by parabolic curving. Also it has new pyramid. The enhancement on curvelet transform can obtain good results, especially for images having curve edges.

Multiscale retinex is a good method, but it tends to increase the grayness of the images with respect to other methods. However, the advantage of multiscale is that it is very easy to be implemented with respect to curvelet transform and multiscale gradient.

Multiscale gradient is also good new method for detecting edges in different scales. the results based on enhancement is very close to the curvelet transform results, especially, in the images that are free of noise.

References


