A New Approach for the Decoupling with Simultaneous Disturbance Rejection (DDR) Problem for Nonlinear Systems with Application in Induction Motor Control

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ABSTRACT. A new approach to the decoupling with simultaneous disturbance rejection (DDR) problem via static state feedback is presented. In the sequel, it is applied to the nonlinear model of an induction motor. The proposed approach reduces the problem of determining the admissible state feedback control law to the solution of a system of first-order partial differential equations. Based on these equations, the following two aspects with regard to the DDR problem are derived: (a) necessary and sufficient conditions for the problem to have a solution and (b) the general analytical solution for all the admissible feedback control laws. Applying this approach to the induction motor, the general analytical expression for the control law, which leads to a decoupled closed-loop system whose outputs are not affected by the load torque (considered as a disturbance), is produced. Moreover, it is proven that appropriate selection of the arbitrary degrees of freedom of the control law leads to a closed-loop system with linear input-output (i/o) description. Furthermore, the problem of bounded angular velocity is treated and the appropriate controller is designed.

I. INTRODUCTION

The contribution of this paper is two fold: (i) A new approach to the decoupling with simultaneous disturbance rejection (DDR) problem for nonlinear systems is presented and (ii) this approach is applied to the nonlinear model of an induction motor.

With regard to the decoupling problem of nonlinear systems, many results have been reported in the literature [2]-[6]. The problem was first studied in [2], and the first systematic results are reported in [3]. In [4], a geometric approach to the
decoupling or the disturbance rejection problem is presented. In particular, necessary and sufficient criteria, of geometric nature, for these two problems to have a solution are established. Moreover, the construction of an admissible controller is proposed. A characterisation of all decoupling control laws has been reported in [5], where in the determination of the decoupling control law requires the solution of a homogeneous system of first-order partial differential equations (whose solution, in general, is not constructable). In [6], a construction algorithm of the decoupling or disturbance rejection control law has been presented. With regard to the disturbance rejection problem, the first systematic results, on a geometric framework, have been reported in [4] and [7], wherein geometric necessary and sufficient conditions for the solvability of the problem are established. The problem of the nonlinear disturbance rejection with stability is studied in [8]. Construction algorithms of an admissible controller have been reported in [6] and [9]. With regard to the DDR problem of nonlinear systems, only a few guidelines are given in [1] and [4], wherein a special admissible control law, called "standard noninteracting feedback" is derived.

The control problem of induction motors has been extensively studied in the literature, see [11]-[17]. The control methods for induction motors can be classified as follows:

1. **Field Oriented Control:** This method, which is reviewed in [12], achieves the control of the velocity and the flux amplitude. In [12], it is proven that this method is equivalent to a coordinate change in the state-space and the application of a nonlinear control law which leads to asymptotic decoupling. Both cases of static and dynamic state feedback are studied. In order to apply this method, the flux amplitude has to be constant. The disadvantages of this method are: (a) This technique cannot be used if an increase of the flux amplitude or the motor speed is required. (b) There are singularities when the motor starts to operate. (c) For the realisation of the control law, it is necessary to use flux sensors which are based on the Hall phenomenon.

2. **Decoupling Techniques on a Reduced-Order Model:** In [11] and [16], the techniques of decoupling and linearization, respectively, are applied to a reduced-order model of the motor. More specifically, the nonlinear model is of fourth-order and it does not include the motion equation of the rotor (the rotor speed is supposed to change slowly). The decoupling of the produced torque and the magnetic flux in the stator [16] or the flux amplitude [11] is achieved. The control of these variables is achieved by changing the frequency and the amplitude of the voltage supply. Geometric techniques are used to achieve decoupling. To realise the control law, the use of a flux observer is necessary.

3. **Decoupling Techniques on the Full-Order Model:** In [12], a decoupling technique is applied to a fifth-order model. The load torque is not considered as a disturbance. The controlled outputs are the angular velocity of the rotor and the flux. The control of the produced torque is achieved indirectly, via the control of the velocity. The decoupling allows the change of the flux level without any consequences on the velocity regulation. This methodology requires a constant and known load torque, whose effects are still present at the controlled outputs. The control law cannot be applied during the starting of the motor operation. The application of the control law leads to an unobservable state. As in case 2, it is necessary to use a flux observer.
4. Adaptive Control: Cases 2 and 3 have been extended in order to deal with the case of the change of some motor parameters. More specifically, in the adaptive version of case 2, two electric parameters (resistances) are considered unknown. In [12], the rotor resistance and the load torque can vary 50% around their nominal values. In [17], an adaptive algorithm for the regulation of the produced torque is developed. The algorithm is applied to a fifth-order model and requires field orientation, resulting in the disadvantages of case 1. The control law is realized via partial feedback of the state vector. Hence, a flux observer is not necessary. Moreover, the application of this method does not cause any singularities during the starting of the motor operation. It is only assumed that the load torque and the produced torque are constant. These assumptions are weaker in [16], where the produced torque can be time varying and the load torque can be time varying linearly parameterized.

5. Variable Structure Control: A review regarding the application of this method is reported in [12] and [17].

In this paper, a new approach to the DDR problem under static state feedback is presented. The proposed approach reduces the determination of the desired control law to the solution of a nonhomogeneous system of first-order partial differential equations, called DDR design equations. On the basis of the DDR design equations, necessary and sufficient conditions, of simple algebraic nature, for the problem to have a solution are established. Furthermore, based on the DDR design equations, the general analytical expression of the desired control law is derived. In particular, a constructive algorithm of all the admissible controllers is presented. This algorithm requires only simple integration.

The foregoing DDR technique is subsequently applied to the nonlinear model of an induction motor. The present control strategy has the following characteristics:
- The method is applied to the full- (fifth-) order model.
- The load torque is considered as a disturbance.
- There are no assumptions regarding the produced torque.
- The rotor resistance is assumed to be known and constant.
- The produced torque is controlled directly, and independently of the flux.
- Selecting appropriate degrees of freedom of the general DDR control law, the closed-loop system reduces to a linear system (with respect to the i/o description)
- In the case where the load torque is constant and is applied for a finite time, an additional dynamic output feedback is designed in order to achieve bounded angular velocity.

The results presented in this paper form part of the material reported in [10].

II. THE DDR TECHNIQUE

1. Preliminaries
   Consider the nonlinear analytic system
\[
\dot{x} = E_0(x) + E(x) \begin{bmatrix} u \\ \xi \end{bmatrix}, \quad x(0) = x_0
\]

where \(E(x) := \begin{bmatrix} G(x) \\ M \end{bmatrix}D(x)\), the input \(u \in \mathbb{R}^m\), the output \(y \in \mathbb{R}^n\), the disturbance \(\xi \in \mathbb{R}^{\xi}\) and the state \(x\) belongs to an open subset \(U\) of \(\mathbb{R}^n\). The vectors \(E_0(x)\) and \(h(x)\) and each column of \(E(x)\), denoted by \(E_i(x)\), are vector valued functions of \(x\). The vector \(E_0(x)\) and each column of \(E(x)\) are analytic mappings from \(U\) to \(\mathbb{R}^n\). The vector \(h(x)\) is an analytic mapping from \(U\) to \(\mathbb{R}^m\).

We list here the notations used in this paper. The gradient of a vector-valued function \(\phi(x)\) is denoted by \(d(\phi(x))\). The Lie differentiation with respect to an analytic vector field \(\tau(x) \in \mathbb{R}^n\) is defined as

\[
L_{\tau} \phi(x) = \frac{\Delta}{dx} \frac{\partial(\phi(x))}{\partial \tau(x)}
\]

Let \(\sigma(x) \in U\) be an analytic vector field. The Lie bracket operation of \(\tau\) and \(\sigma\) is defined as

\[
[\tau, \sigma](x) = \frac{\Delta}{dx} \frac{\partial \sigma(x)}{\partial \tau(x)} - \frac{\partial \tau(x)}{\partial \tau(x)} \sigma(x)
\]

In this paper \(inv(Q(x))\) denotes the involutive closure, (see [1]), of the distribution spanned, locally around \(x_0\), by the columns of the matrix \(Q(x)\).

**Definition 2.1**

For (2.1), there exist nonnegative integers \(d_i\)'s, \(\forall i \in \{1, 2, ..., m\}\), called characteristic numbers, defined as

\[
L_{\xi}^k L_{E_i}^j h_i(x) = 0, \quad \forall k \in \{1, 2, ..., m + \xi\} \text{ and } j < d_i
\]

\[
L_{E_i}^k L_{\xi}^j h_i(x) \neq 0, \text{ for some } j \in \{1, 2, ..., m + \xi\}
\]

\(\forall x\) around \(x_0\), where \(h_i(x)\) is the \(i\)-th component of \(h(x)\).

**2. The DDR Technique**

**Definition 2.2**

A system of the form (2.1) is i/o and disturbance decoupled, if the \(i\)-th element of the input \(u\) affects only the \(i\)-th element of the output \(y\) and the same time the disturbance \(\xi\) does not affect \(y\) for any \(x\) around \(x_0\).

**Statement of the DDR problem:**

Consider applying to (2.1) the control law

\[
u = a(x) + B(x)w
\]

where \(w \in \mathbb{R}^m\) and \(B(x)\) is nonsingular, i.e. \(\|B(x)\| \neq 0, \quad \forall x\) around \(x_0\), to yield a closed-loop system of the form

\[
\dot{x} = \bar{E}_0(x) + \bar{E}(x) \begin{bmatrix} w \\ \xi \end{bmatrix}, \quad x(0) = x_0, \quad y_{cls} = h(x)
\]

where

\[
\bar{E}_0(x) := E_0(x) + G(x)a(x)
\]
The DDR problem is defined as in [10]: Determine a control law of the form (2.2) such that the resulting closed-loop system (2.3) is i/o and disturbance decoupled.

The DDR problem is studied in [10]. We present here the main theorems.

**DDR Design Equations**

**Theorem 2.1.**

Assume that \( |B(x)| \neq 0, \forall x \) around \( x_0 \). Then, the feedback pair \( \{a(x), B(x)\} \) of (2.2) satisfies the following set of equations, called the DDR Design Equations

\[
E^*(x)[ \begin{bmatrix} B(x) & 0 \end{bmatrix} ] = \text{diag}[\{ \lambda_i(x) \} ] : 0 \quad (2.4)
\]

\[
\theta_i(x) \Pi_i(x) = 0
\]

\[
\xi_i(x) \Pi_i(x) = 0
\]

where

\[
\theta_i(x) = [d \phi_i(x) : k_i(x) : k_{i,0}(x) : ]
\]

\[
\xi_i(x) = [d \lambda_i(x) : p_i(x) : p_{i,0}(x) : ]
\]

where \( \phi_i(x) = L_{E_0}^{i+1} h_i, \) and \( \lambda_i(x) \neq 0, k_i(x), k_{i,0}(x), ..., p_i(x), p_{i,0}(x), ..., \Pi_i(x) \) depend on the Markov parameters of the closed-loop system, and

\[
E^*(x) = \begin{bmatrix} B^*(x) : D^*(x) \end{bmatrix} = \begin{bmatrix} d \left( L_{E_0}^{i} h_i(x) \right) \\ \vdots \\ d \left( L_{E_0}^{i} h_m(x) \right) \end{bmatrix}
\]

**Necessary and Sufficient Conditions:**

Based on the DDR design equations, the necessary and sufficient conditions for the DDR problem to have a solution are as follows:

**Theorem 2.2.**

The necessary and sufficient conditions for the solvability of the DDR problem, under the state feedback (2.2), are

\[
\det[B^*(x)] \neq 0 \text{ and } D^*(x) = 0, \forall x \text{ around } x_0.
\]

**Special Solution for the Control Law:**

If Theorem 2.2 is satisfied, then a special solution \( \{ \hat{B}(x), \hat{a}(x) \} \) for the feedback pair which satisfies the DDR problem is given by

\[
\hat{B}(x) = B^*(x)^{-1} \text{ and } \hat{a}(x) = -B^*(x)^{-1} a^*(x) \quad (2.5)
\]

where \( a^*(x) = \left[ L_{E_0}^{i+1} h_i(x) \ldots L_{E_0}^{m+1} h_m(x) \right]^T \). The special solution (2.5) is called the "standard noninteracting feedback" in [1] and [4].

Application of the feedback pair (2.5) to (2.1) results in a closed-loop system of the form
\[
\dot{x} = \hat{E}_0(x) + \hat{E}(x) \begin{bmatrix} u \\ \xi \end{bmatrix}, \quad x(0) = x_0, \text{ and } y = h(x) \quad (2.6)
\]

where \( \hat{E}_0(x) = E_0(x) + G(x) \hat{a}(x) \) and \( \hat{E}(x) = [\hat{G}(x) : D(x)] = [G(x) \hat{B}(x) : D(x)] \). It is easy to see that (2.6) may be separated into \( m \) subsystems having i/o maps of the form

\[
y_i^{(d_i+1)} = w_i, \quad i \in \{1, 2, \ldots, m\}
\]

**General Solution for the Control Law:**

Based on the DDR design equations, the general expression for the control law (2.2) is given by the following theorem.

**Theorem 2.3.**

If Theorem 2.2 is satisfied, then the general solution for the feedback pair \( \{a(x), B(x)\} \) will be given by

\[
a(x) = \begin{bmatrix} B^*(x) \end{bmatrix}^{-1} \{ a^*(x) - \phi(x) \} \quad (2.7a)
\]

\[
B(x) = \begin{bmatrix} B^*(x) \end{bmatrix}^{-1} \text{diag} \{ \lambda_i(x) \}_{i=1}^m \quad (2.7b)
\]

where \( \phi(x) = [\phi_1(x), \ldots, \phi_m(x)]^T \) and

\[
\phi_i = \phi_i \begin{bmatrix} \hat{t}_{i,1}(x), \ldots, \hat{t}_{i,\sigma_i}(x), \hat{s}_{i,1}(x), \ldots, \hat{s}_{i,n-n^*}(x) \end{bmatrix} \quad (2.8a)
\]

\[
\lambda_i = \lambda_i \begin{bmatrix} \hat{t}_{i,1}(x), \ldots, \hat{t}_{i,\sigma_i}(x), \hat{s}_{i,1}(x), \ldots, \hat{s}_{i,n-n^*}(x) \end{bmatrix} \quad (2.8b)
\]

where \( \phi_i, \lambda_i \) are arbitrary analytic functions of their arguments, with \( \lambda_i(x) \neq 0, \forall x \in U \).

To determine \( \hat{t}_{i,\rho}(x), \rho \in \{1, \ldots, \sigma_i\} \) and \( \hat{s}_{i,\rho}(x), \rho \in \{1, \ldots, n-n^*\} \), for \( i \in \{1, 2, \ldots, m\} \), as well as the integers \( n^* \) and \( \sigma_i \), the following algorithm has been proposed.

**Remark 2.1.** In [10], it is shown that if \( \sum_{i=1}^m (d_i + 1) = n^* = n \), then the solution for the functions \( \phi_i \) and \( \lambda_i \), \( i \in \{1, \ldots, m\} \), are given by

\[
\phi_i = \phi_i \begin{bmatrix} h_{i,1}, \ldots, L_{E_0}^{d_i} h_{i,1} \end{bmatrix} \quad (2.9a)
\]

\[
\lambda_i = \lambda_i \begin{bmatrix} h_{i,1}, \ldots, L_{E_0}^{d_i} h_{i,1} \end{bmatrix} \quad (2.9b)
\]

**Remark 2.2.** In [10], it is proven that there exist \( \sigma_i \) linearly independent solutions for \( \phi_i \) or \( \lambda_i \) of the DDR Design Equations. Furthermore, it is proven that the functions \( h_{i,1}(x), \ldots, L_{E_0}^{d_i} h_{i,1}(x) \) are linearly independent and they are also solutions for \( \phi_i \) or \( \lambda_i \).

Hence, \( \sigma_i \geq d_i + 1 \) holds.
Remark 2.3. If $\sigma_i = d_i + 1$ and $n = n^\ast$, then the general solution for $\phi_i$ and $\lambda_i$ may be immediately determined by (2.9).

III. THE INDUCTION MOTOR MODEL AND THE CONTROL PROBLEM

The Induction Motor Model. The full-order model of an induction motor may be written as [12]

$$\dot{x} = f(x) + g_a(x)u_a + g_b(x)u_b + d(x)\xi,$$  \hspace{1cm} (3.1)

where

$$f(x) = \begin{bmatrix} \mu (x_2x_3 - x_3x_4) \\ -ax_2 - n_p x_1 x_3 + aMx_4 \\ n_p x_1 x_2 - ax_3 + aMx_5 \\ a\beta x_2 + n_p \beta x_1 x_3 - \gamma x_4 \\ -n_p \beta x_1 x_2 + a\beta x_4 - \gamma x_5 \end{bmatrix},$$

$$g_a(x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{\sigma L_s} \\ 0 \end{bmatrix}, \quad g_b(x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{\sigma L_s} \\ 0 \end{bmatrix}, \quad d(x) = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix},$$

and $\xi = T_L / J$. The state vector $x = [x_1, x_2, x_3, x_4, x_5]^T = [\omega, \psi, \psi, i, u]$ denote rotor speed, flux linkage, current and stator voltage input to the machine, respectively. The subscripts a and b denote the components of a vector with respect to a fixed stator reference frame. The parameters $\alpha, \beta, \gamma$ and $\mu$ are defined as follows: $\alpha = R_\ell / L_\ell$, $\beta = M / (\sigma L_s L_r)$, $\gamma = M^2 R_\ell / (\sigma L_s L_r) + R_\ell / (\sigma L_r)$, and $\mu = n_p M / (J L_\ell)$, where $R_\ell, R_\ell, L_\ell, L_r, M, n_p$, and $J$ denote resistances, self-inductances, mutual inductance, the number of pole pairs of the induction motor and the moment of inertia of the rotor and of any tool attached to it, respectively. The term $T_L$ is the load torque, which is considered as a disturbance.

Next, define the change of coordinates $z = f(x)$, where $z = [z_1, z_2, z_3, z_4, z_5]^T$ and

$$F(x) = \begin{bmatrix} \mu_1(x) \\ L_f \mu_1(x) \\ \mu_2(x) \\ L_f \mu_2(x) \\ \mu_3(x) \end{bmatrix} = \begin{bmatrix} x_1 \\ \mu (x_2x_3 - x_3x_4) \\ x_2^2 + x_3^2 \\ -2ax_2^2 + 2aM(x_2x_4 + x_3x_5) \\ \arctan(x_5/x_2) \end{bmatrix}$$

The foregoing transformation is one-to-one in $\Omega = \{x \in \mathbb{R}^5 : x_2^2 + x_3^2 \neq 0\}$ but it is onto only for $z_3 > 0$ and $-90^\circ \leq z_2 \leq 90^\circ$. The state-space in z-coordinates is
System (3.1) may be written in $z$ coordinates as

$$z = E_o(z) + E(z) \begin{bmatrix} u \\ v \end{bmatrix}, \quad z(0) = z_0, \quad y = h(z)$$  \hspace{1cm} (3.2)$$

where

$$E(z) = \begin{bmatrix} 0 & 0 & : & -1 \\ L_{s_a} L_f \mu_1(x) & L_{s_a} L_f \mu_1(x) & : & 0 \\ 0 & 0 & : & 0 \\ L_{s_a} L_f \mu_2(x) & L_{s_a} L_f \mu_2(x) & : & 0 \\ 0 & 0 & : & 0 \end{bmatrix}_{x = F^{-1}(z)}$$

$$E_o(z) = \begin{bmatrix} z_2 \\ L_f \mu_1(x) \\ z_4 \\ L_f \mu_2(x) \\ L_f \mu_3(x) \end{bmatrix}, \quad h(z) = \begin{bmatrix} z_2 \\ z_3 \end{bmatrix}_{x = F^{-1}(z)}$$

**Control Objective.** The control objective in the case of induction motors is the control of one or more of the following quantities ([11]-[13] and [15]-[17]): angular velocity $\omega$ of the rotor, produced torque $\mu(\psi_{a'}\psi_{b'} - \psi_{b'}\psi_{a'})$ and flux $\psi_a^2 + \psi_b^2$. In this paper the objective is to control the torque and the flux, which are considered as the system outputs.

Furthermore, it is desired that the operation of the motor is independent of the load. To this end, the quantity $T_L/J$, where $T_L$ is the load torque and $J$ the rotor inertia, is considered as a disturbance, the effect of which on the system outputs should be rejected.

**IV. THE DDR TECHNIQUE FOR THE CONTROL OF THE INDUCTION MOTOR**

Application of the theoretical results presented in Section 2 to the mathematical model (3.2) of the induction motor yields the following results:

1. **Necessary and Sufficient Conditions**

The characteristic numbers of system (3.2) are $d_1 = 0$ and $d_2 = 1$, $\forall z \in V$.

The matrix $E'$ is given by

$$E'(z) = \begin{bmatrix} -\frac{\mu x_3}{\sigma L_s} & \frac{\mu x_2}{\sigma L_s} & : & 0 \\ \frac{2aM x_2}{\sigma L_s} & \frac{2aM x_3}{\sigma L_s} & : & 0 \end{bmatrix}_{x = F^{-1}(z)}$$  \hspace{1cm} (4.1)
The determinant det[B(z)] = \( -2 \frac{\alpha M \mu}{(\sigma L_s)^2} z_3 \neq 0, \forall z \in V. \) Obviously, \( D(z) = 0, \forall z \in V. \)

Therefore, the DDR problem is solvable for (3.2), \( \forall z \in V. \)

2. Special Solution for the Control Law

The special feedback pair \{ \hat{B}(z), \hat{a}(z) \} of the control law (2.2) is given by

\[
\hat{B}(z) = \sigma L_s \begin{bmatrix}
-1 & 1 \\
\mu & 2\alpha M & z_3 \\
\mu & 2\alpha M & z_3
\end{bmatrix} \left[ x = F^{-1}(z) \right] \\
\hat{a}(z) = -\hat{B}(z) a^*(z)
\]  

(4.2)

where

\[
a^*(z) = \begin{bmatrix}
L_j \mu_1(x) \\
L_j \mu_2(x)
\end{bmatrix} \left[ x = F^{-1}(z) \right]
\]  

(4.3)

Applying the special control law (4.2) and (4.3) to (3.2) results in the closed-loop system

\[
\dot{z} = \hat{E}_0(z) + \hat{E}(z) \begin{bmatrix}
w \\
\xi
\end{bmatrix}, \quad z_0 = F(x_0), \quad y_{cl} = h(z)
\]  

(4.5)

where

\[
\hat{E}_0(z) = \begin{bmatrix}
z_2 \\
0 \\
z_4 \\
0 \\
n_p z_1 + \frac{RJ}{n_p} z_2 \\
\end{bmatrix}, \quad \hat{E}(z) = \begin{bmatrix}
0 & 0 & -1 \\
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]  

(4.6)

The i/o description of (4.5) is

\[
y_1^{(i)} = u_a \quad \text{and} \quad y_2^{(i)} = u_b
\]  

(4.7)

System (4.5) is clearly neither BIBO stable nor asymptotically internally stable (zero input asymptotically stable).

3. General Solution for the Control Law

The general solution for the feedback pair \{ a(z), B(z) \} of the control law (2.2) is

\[
a(z) = -\hat{B}(z) \begin{bmatrix}
\phi_1(z_2) \\
\phi_2(z_3, z_4)
\end{bmatrix}
\]  

(4.8a)

\[
B(z) = \hat{B}(z) \begin{bmatrix}
\lambda_1(z_2) & 0 \\
0 & \lambda_2(z_3, z_4)
\end{bmatrix}
\]  

(4.8b)

Application of the feedback pair (4.8) to system (3.2) results in
\[
\dot{z} = \bar{E}_0(z) + \bar{E}(z) \left[ \begin{array}{c} \frac{w}{x} \\
\end{array} \right], \quad z_0 = F(x_0), \quad y_{cls} = h(z) \quad (4.9)
\]

where

\[
\bar{E}_0(z) = \begin{bmatrix} z_2, \phi_1(z_2), z_4, \phi_2(z_3, z_4), n_p z_1 + \frac{R}{n_p z_3} \end{bmatrix}^T
\]

\[
\bar{E}(z) = \begin{bmatrix}
0 & 0 & -1 \\
\lambda_1(z_2) & 0 & 0 \\
0 & 0 & 0 \\
0 & \lambda_2(z_3, z_4) & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

4. Linearization - Stabilisation of the Closed-loop System

The closed-loop system (4.9) may be linearized and stabilised by choosing the \( \phi \)'s to be linear functions of their arguments and \( \lambda \)'s to be real numbers. In this case, (4.9) can be decomposed as follows:

1st subsystem

\[
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
0 - p_2
\end{bmatrix}
\begin{bmatrix}
z_1 \\
z_2
\end{bmatrix} +
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\begin{bmatrix}
w_1 \\
k_1
\end{bmatrix} +
\begin{bmatrix}
-1
\end{bmatrix}
\begin{bmatrix}
T_L/J
\end{bmatrix}
\]

\[
y_1 = z_2 \quad (4.10a)
\]

2nd subsystem

\[
\begin{bmatrix}
\dot{z}_3 \\
\dot{z}_4
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
- p_3 - p_4
\end{bmatrix}
\begin{bmatrix}
z_3 \\
z_4
\end{bmatrix} +
\begin{bmatrix}
0 \\
k_2
\end{bmatrix}
\begin{bmatrix}
w_2 \\
\end{bmatrix}
\]

\[
y_2 = z_3 \quad (4.11b)
\]

3rd subsystem

\[
\dot{z}_5 = n_p z_1 + \frac{J R}{n_p} \frac{z_2}{z_3} \quad (4.12)
\]

where \( p_2, p_3, p_4 \in \mathbb{R} \) and \( k_1, k_2 \in \mathbb{R}^+ \). The subsystems (4.10) and (4.11) are decoupled and the subsystem (4.12) describes the unobservable state \( z_5 \) (for the evolution of \( z_5 \) see [12]). Clearly, the overall system (4.10)-(4.12) has a linear i/o description. Subsystem (4.10) is not internally stable. Subsystem (4.11) can become internally stable by choosing appropriately the constants \( p_3 \) and \( p_4 \). In conclusion, we note that, although disturbance rejection has been achieved, subsystem (4.10) is not internally stable. But since this subsystem is controllable, it may be arbitrarily stabilised.

To stabilise subsystem (4.10), apply to (4.10) the following control law

\[
w_1 = \begin{bmatrix}
\hat{f}_1 \\
\hat{f}_2
\end{bmatrix}
\begin{bmatrix}
z_1 \\
z_2
\end{bmatrix} + v_1 \quad (4.13)
\]
where $k_1 \hat{f}_1 < 0, k_1 \hat{f}_2 < p_2$. In this case, the effect of the disturbance is reintroduced.

To circumvent this difficulty, we will examine the case of the disturbance attenuation in the steady state. In the sequel, $\xi(t)$ is considered as a step function which is applied for a finite-time interval. In this case, to achieve disturbance attenuation in steady state we apply the following dynamic output feedback law:

$$ v_1(s) = \tilde{f}(s) + \dot{w}_1(s), \quad \text{where} \quad \tilde{f}(s) = \frac{1}{s^2} \quad (4.14) $$

It may be proven that the choice $k_1 < 0, k_1 \hat{f}_2 < p_2, f_1 \left( k_1 \hat{f}_2 - p_2 \right) < -1$ leads to an internally stable closed-loop subsystem, whose output is not affected by the disturbance in steady state.

5 Implementation of the Control Law

In order to implement the control laws (2.2), (4.13) and (4.14), where the matrices $a(z)$ and $B(z)$ are given by the relationship (4.8), the measurement of the state vector

$$ z = (z_1, z_2, z_3, z_4, z_5) = \begin{bmatrix} \psi_a \psi_b \psi_a \psi_b \psi_a i_b \psi_a i_a \psi_b i_a \psi_b i_b \psi_a \psi_b \psi_a i_b \psi_a i_a \psi_b i_a \psi_b i_b \psi_a \psi_b \psi_a i_b \psi_a i_a \psi_b i_a \psi_b i_b \psi_a \psi_b \psi_a i_b \psi_a i_a \psi_b i_a \psi_b i_b \psi_a \psi_b \psi_a i_b \psi_a i_a \psi_b i_a \psi_b i_b \end{bmatrix} $$

is required. The state vector $z$ is not totally measurable and therefore the use of state observers for the estimation of the fluxes $\psi_a$ and $\psi_b$ is necessary [18]. It is noted that in order that the condition $z_3 \neq 0$, which is necessary for the implementation of the nonlinear control law (2.2), to be satisfied, the control law must be implemented on the system after the beginning of its operation.

6. Asymptotic Output Tracking

To track prespecified trajectories $y_{id}, \dot{y}_{2d}$ from the overall closed-loop system, it suffices to apply the control law

$$ w_1 = \frac{1}{k_1} \left[ y^{(1)}_{id} \left( k_1 \hat{f}_2 - p_2 \right) y_{id} - k_1 \hat{f}_1 \int_0^\sigma y_{id} d\tau - k_1 \int_0^\sigma \int_0^\sigma y_{id} d\sigma d\tau \right] $$

$$ w_2 = \frac{1}{k_2} \left[ y^{(2)}_{2d} + p_4 y^{(1)}_{2d} + p_3 y_{2d} \right] $$

which leads to the error dynamics

$$ e^{(1)}_1 = \left( k_1 \hat{f}_2 - p_2 \right) e^{(1)}_1 - k_1 \hat{f}_1 e^{(1)}_1 - k_1 e_1 = 0 $$

$$ e^{(2)}_2 + p_4 e^{(1)}_2 + p_3 e_2 = 0 $$

where $e_1 = y_i - y_{id}$ and $e_2 = \dot{y}_2 - \dot{y}_{2d}$. For the derivation of the foregoing error dynamics the assumption that $\xi$ is a step function was used. Clearly, if the parameters $k_1, \hat{f}_1, \hat{f}_2, p_2, p_3$ and $p_4$ are chosen properly, the above error dynamics are exponentially stable.
V. CONCLUSIONS

In this paper, a new approach to the DDR problem under static state feedback is presented first. The main characteristic of the proposed approach is that it reduces the determination of the desired control law to the solution of a nonhomogeneous system of first-order partial differential equations, called DDR design equations. Based on the DDR design equations, necessary and sufficient conditions for the problem to have a solution are established. Furthermore, based on the DDR design equations, the general analytic expression of the desired control law is derived. Moreover, the "standard noninteracting feedback" is rederived here as a special control law satisfying the DDR problem. It is noted that application of the general solution of the control law may lead to an internally stable closed-loop system, in case where special solutions of the control law result to unstable internal dynamics.

The foregoing DDR technique is subsequently applied to control the nonlinear model of an induction motor. The present control strategy for the induction motor has the following characteristics and/or advantages over known control strategies: (a) The method is applied to the full (5th) order model. (b) The load torque is considered as a disturbance. (c) There are no assumptions regarding the produced torque. (d) The produced torque is controlled directly, and independently of the flux. (e) Selecting appropriately the degrees of freedom of the general DDR control law, the closed-loop system reduces to a linear system (with respect to the i/o description) and (f) In the case where the load torque is constant and it is applied for a finite time, an additional dynamic output feedback is designed in order to achieve bounded angular velocity.

References
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