Complex Wavelet Packets-based DWMT for Digital Subscriber Line

Ahmed muthana M. Nadhim*, Abdul-Karim a-r. Kadhim **, and Zongkai Yang*

* Electronics and Information Engineering Department, Huazhong University of Science and Technology, Wuhan, 430074, China
ahmed_m_nadhim@hotmail.com, zkyang@public.whhb.cn http://itec.hust.edu.cn
** Electronic and Communications Engineering Department, Baghdad University, Baghdad, Iraq

Abstract: - It is well known today that Discrete Multi-Tone (DMT) modulation is an attractive technique for copper wire access technology (e.g. Asymmetric and Very-high-rate Digital Subscriber Line). Discrete Wavelet Multi-Tone (DWMT) is a variant high-performance transmission technique, which is based on the application of wavelet filter bank. However, it was shown in the work that complex wavelet system is very attractive for DWMT. It is usually implemented using parallel filter bank, which have strict design constraints. Instead, two design methods, based on wavelet packet modulation, were proposed depending on the orthogonality conditions between the subcarriers. The performances of DWMT system and the standard DMT system were examined under different noise conditions in an ADSL and VDSL environments.

Keywords: - Multicarrier modulation, DWMT, Wavelet packets, VDSL, and ADSL

1. Introduction

Fig. 1 illustrates a general N-dimensional discrete-time multitone system model. The discrete-time basis vectors \{g_i\} have a finite length of N samples over a duration of time T, where T is the symbol (or block) period [1]. Each basis vector g_i multiplies a subsymbol element X_i before being summed to form the transmit symbol vector x.

\[
x = \sum_{i=1}^{N} X_i g_i
\]  

(1)

At the receiver the matched filters are discrete-time and also finite-length. There are N input samples that lead to N matched-filter output samples. The modulator attempts to use basis vectors \{g_i\} that will remain orthogonal after undergoing through dispersive ISI channel. Thus \{g_i\} modulation bases must satisfy the orthogonality condition. For the special case where the bases vectors are the exponential functions then \{g_i\} are simply the columns of the DFT square matrix. Therefore, the receiver and transmitter basis vectors for DMT becomes the well-known DFT process. Then, the set of modulators and demodulators (in Fig. 1) are replaced by IDFT and DFT respectively. However, during the last decade, wavelet transform has been proposed as a possible transform to generate the subchannels in a multicarrier system instead of the DFT [2], with its advantages being the flexibility of the design of the wavelet bases and suppression of sidelobes compared with the rectangular window of the Fourier transform.

2. The Wavelet System and the Wavelet Packets

The word system is used in communications to describe a set of elements (or functional blocks) that are connected together in such a manner as to achieve a desired objective. The objective of the wavelet system is to implement the wavelet transform as realizable building blocks. Wavelet packets is one of the methods to realize the wavelet system. Wavelet packets represent an extension of the classical two-band wavelet system that yields basis functions with better frequency localization at the cost of more computational complexity. A particular form of the wavelet packets is the wavelet packet division multiplexing or Wavelet Packet Modulation (WPM) [3]. A transmultiplexer implementation of the WPM is shown in Fig. 2a. The WPM is, essentially, a multiple signal transmission technique in which the input signals are waveform modulated into wavelet packet basis functions for transmission. The overlapping nature in time and frequency provide a capacity improvement over the commonly used DFT.
Thus the Perfect Reconstruction condition ensures that the output is a replica of the input with no estimating error to the corresponding realizations of the filter length. This means more is omitted. Then (3) reduces to:

\[ L_{\text{eff}} = (L-1)(K_j-1) + 1 \]  

Equation (4) shows that the effective length of the WPM filters is much larger than the designed prototype two-band filter length. Although this will increase system latency, the advantage is being better frequency characteristics of the subchannels.

3. The Complex Wavelet based-DWMT

The wavelet packet decomposition is a convenient technique by which waveforms provide self and mutual orthogonalities can be obtained. The parallel transmultiplexer implementation of the wavelet system can be considered as a lapped transform when satisfying the orthogonality conditions [5]. For a lapped transform, the synthesis (also the analysis) filters are uniform band FIR filters, and they are defined from the columns of the matrix \( G \), in the form

\[ g_{k,i} = \sum_{j=1}^{N} g_{j,i} G_{j,k}, \quad k = 1, 2, \ldots, N \]

where \( g_{k,i} \) is the element in the \( i \)th row and \( k \)th column. The parameter \( q \) called the overlap factor. The overlap factor specifies how many input blocks are used to compute an output block. When \( q = 1 \), the lapped transform is simply a block transform as the case with DFT-based DMT modulation. Lapped transform can be considered as a special case of the generalized WPM. The lapped transforms are cascaded by connecting them hierarchically. Thus, this system called Hierarchical Lapped Transform (HLT). If we consider the HLT system as a parallel filter bank, then for a multistage tree, the value of \( q \) may be non-integer value. Recently, Adhikary [6] proposed two approaches for designing complex wavelet packets. These assume direct parallel implementation of the wavelet system. Also, there is a strong constraints in the design of wavelet bases and sometimes yield long filter lengths, this means more computation. However, any set of complex wavelet bases, to be used for MCM, should exhibit the desired orthogonality. And the real and imaginary parts of the complex bases should be spectrally similar and also orthogonal to each other. The idea of complex wavelet based-MCM is exactly similar to the quadrature carriers centered at the same carrier frequency of the analog QAM signal, but in that case, they are in phase quadrature. From this view, an alternative approach of designing complex wavelet packets is proposed to be used for tree-structured transmultiplexers. The complex wavelet system require two sets of bases functions and then filters. Consider the two-band transmultiplexer of Fig. 2b. The complex bases are denoted by \( h_k(n) \) and \( h_i(n) \) corresponding to the analysis filters and by \( g_k(n) \) and \( g_i(n) \) corresponding to the synthesis filters. Two methods are proposed corresponding to the required orthogonality-constraint between the subchannels.

3.1 Orthogonal PR - Constraints

Not any set of bases (filter coefficients) produce \( X_j = \mathcal{X}_j \). Thus the Perfect Reconstruction condition ensures that the output is a replica of the input with no estimating...
errors and no distortion if ideal channel is considered. In DWMT, the spectral overlap between subchannels is maintained to achieve maximum bandwidth utilization. Therefore, the orthogonality of the wavelet and scaling basis functions and hence the orthogonality between subchannels is an essential condition. And in terms of scaling and wavelet filters, \( h(n) \) and \( h(n) \), respectively, the orthogonality condition is stated as [7]

\[
\sum_{k=0}^{L-1} h_k(n) h_k(n+2l) = \delta(l)
\]

(6)

where \( k \in \{0,1\} \) and \( l \) is an integer. Equation (6) assumes that all the filters are \( L \) taps, where \( L \) is even. A necessary condition on \( h(n) \) that ensures not any set of coefficients will support a solution is [7]

\[
\sum_{n=0}^{\infty} h^2(n) = \sqrt{2}
\]

(7)

Not only the sum of \( h(n) \) must equal \( \sqrt{2} \), but for the orthogonality of the solution, the sum of the squares must also equal one [7]. Orthogonal wavelet system gives a clean, robust, and symmetric formulation with Parseval's theorem. However, the second requirement is the orthogonality of the complex bases that occupy the same band. To ensure this, \( h'_x(n) \) and \( h'_x(n) \) are given by,

\[
h'_x(n) = (-1)^x h_x(L-1-n)
\]

(8)

\[
h'_x(n) = (-1)^x h_x(L-1-n)
\]

(9)

Equations (8) and (9) imply that \( h'_x(n) \perp h_x(n) \) and \( h'_x(n) \perp h_x(n) \) respectively, where \( \perp \) denotes the orthogonality. Therefore, the wavelet-based multicarrier system design that satisfy equations (6) - (9) ensures orthogonal subchannels and orthogonal complex bases. Then by careful filter design with good spectral characteristics of the resulted DWMT system predicted to outperform the traditional DMT system. The synthesis filters are computed using the following formulas;

\[
h_x(n) = (-1)^x h_x(L-1-n)
\]

(10)

\[
g_x(n) = h_x(L-1-n)
\]

(11)

3.2 Biorthogonal PR-Constraints

Requiring orthogonality sets up a large number of the degrees of freedom and prevents linear phase analysis and synthesis filters. The PR-linear phase analysis and synthesis filters in the two-band case can be realized by relaxing the orthogonality constraints, leading to the so-called biorthogonal solution [7]. The two-channel biorthogonal PR condition is given by

\[
\sum_{n=0}^{\infty} h(n) g(n+2l) = \delta(l)
\]

(12)

To make orthogonal complex scaling bases, such that, \( h'_x(n) \perp h_x(n) \), then \( h'_x(n) \) is found by

\[
h'_x(n) = (-1)^x h_x(L-1-n)
\]

(13)

And in the case of wavelet filter to be orthogonal, such that \( h'_x(n) \perp h_x(n) \), we have to find \( g'_x(n) \) by

\[
g'_x(n) = (-1)^x g_x(L-1-n)
\]

(14)

Both of the design methods considered earlier ensure an orthogonal basis functions. The difference is that in case two, some of the orthogonality between the subchannels is used to produce the orthogonal bases. The performance of these design methods will be compared under different noise and channel environments with the standard DMT system in the next discussion.

4. Simulation Results

A computer simulation program was designed to generate a Gray coded \( M \)-level QAM symbols for any value of \( M \). The program was investigated and compared with the theoretical formula given in [8]. Fig. 3 shows the theoretical and simulated Symbol Error Rate (SER) of 64-level QAM. It is clear that the difference in SER is notable at lower SNRs, and decreases as the value of SNR increases.

![Fig. 3 Theoretical and simulated performance of QAM with \( M=64 \)](image)

System parameters were chosen to agree with the standard DMT based ADSL modem (ANSI T1.413 or ITU G992.1) [9], with simplifications. The number of tones is 256, tone spacing: 4 kHz which is equal to the symbol rate. A 24-AWG twisted-pair channel of length 1200 ft with a relative maximum usable frequency of 1.1 MHz was assumed. Chow, Cioffi and Bingham [10] loading algorithm was applied. Linear Time domain EQualizer TEQ (using feedback transversal filter) was used for DWMT and nonlinear Frequency domain EQualizer (FEQ) for DMT system. D26 was used as the wavelet basis functions. Using Eq. 4, \( L_{eff} \) is 6376 samples. The transmitted DWMT signal spectrum is shown in Fig. 4 below, it shows the multitone waveforms for eight tones only.
In biorthogonal design method, a relaxation in the orthogonality requirement between the subchannels has been done. This significantly degrades system performance. Therefore, during simulations it was seen that the second method suffers from relatively high SNR degradation. As a result, it can be say that the orthogonality of the subcarriers in a multicarrier system seems to be the first condition for multicarrier system design. So, on the other hand, orthogonal bases showed performance improvement over the DMT system, because of the better frequency containment of the DWMT signal. The performances of the DMT and DWMT subject to FEXT (which is given by [11]) are shown in Fig. 5. FEXT is the dominant noise in an ADSL environment. The SNR saving by DWMT is 2.1 dB at 10^{-4} BER. This states that if an external crosstalk disturbs the transmitted multitone signal that occupies the same frequency band, the DWMT signal is more robust than the DMT signal.

For VDSL modem, the NEXT noise is the major cause of disturbance. The simplified loss model for NEXT is given by [12]. The systems performances are shown in Fig. 6. The performance gain is 1.7 dB in SNR for DWMT over that of DMT. This is due to the improved frequency characteristics of the DWMT signal. More specifically, Daubechies bases have better frequency localization than the Fourier bases. This effectively reduces the Inter-Channel Interference (ICI). The D26 basis has lower stop-band attenuation approximately 47 dB as shown in Fig. 4 for eight tones only. While the traditional rectangular Fourier basis is only 13 dB. This has the effect of reducing the crosstalk between the subchannels. As well as, the out-of-band attenuation is significantly increases to mitigate the Inter-Block Interference (IBI).

5. Conclusion

Two multicarrier systems were simulated DWMT system and the standard DMT system. There performances were examined in different channel and noise conditions in ADSL and VDSL environments. A general simulation program was designed to generate Gray coded QAM symbols and verified successfully. Computer simulation tests showed that 2.1 dB SNR improvement at 10^{-4} BER can be achieved with DWMT if compared to the convolutional DMT system. This is achieved at the cost of increasing system latency as much as q increases. Simulations showed that the second method suffers from relatively high SNR degradation. On the other hand, orthogonal bases showed performance improvement over the DMT system.

6. References


