Abstract
A moment method has been presented to analyze small linear arrays consisting of longitudinal slot in the broad wall of a rectangular wave guide, after taking the effect of mutual coupling, both internal and external into consideration. The wall thickness also been accounted for, arrays have been designed, fabricated and both computed and experimental results have been presented for various parameter of interest such as field distribution in the slot, slot length, radiation pattern etc. The results are excellent and radiation patterns are almost ideal.

Key Words: Slot Array, Method of Moment, Mutual Coupling, rectangular wave guide, Radiation pattern, wall thickness

1. Introduction:
The well known design procedure of slot arrays, due to Elliot et al.[1-2] dependson the active admittance concept. The principle limitation of it is the equiphase half co-sinusoidal approximation for the slot aperture field, which is not justified for slot in reduced guide wave guide with moderate to large offset [2]. Also the accuracy of procedure relies on the experimental curve obtained for isolated slots. The improvement is in [3], where no basic assumption have been made and internal coupling has been accounted for, but the wall thickness and external coupling have not been considered. In this paper, a procedure based on Moment method [4] with “generalized network formulation” for aperture problem has been presented for the design of linear slot array consisting of longitudinal slots in the broad wall of the rectangular wave guide. It takes in to account the effect of both internal and external mutual coupling due to propagating and non propagating higher order modes. Actual aperture field distribution within the slot is considered which inherently incorporates the effect of finite wall thickness of the waveguide. It takes in to account the effect of both internal and external mutual coupling due to propagating and non propagating higher order modes. Actual aperture field distribution within the slot is considered which inherently incorporates the effect of finite wall thickness of the waveguide. It takes in to account the effect of both internal and external mutual coupling due to propagating and non propagating higher order modes. Actual aperture field distribution within the slot is considered which inherently incorporates the effect of finite wall thickness of the waveguide.

2. Problem formulation:
Fig.1 shows the problem to be considered and defines the coordinates and parameters to be used. The offset and inter element spacing are arbitrary. The slots have dimensions of l_r x w_r, r=1,2,.....N. w_r is small enough to neglect cross polarization fields. The entire region is divided into (N+2) distinct regions using the equivalence principle (Fig. 2) R_j is the wave guide region having equivalent magnetic currents \( \overline{M_i} \) \( i=1,2,...,N \) placed upon conducting sheets covering the slot surfaces. Region \( R_c, c=2,...,N+1 \) are the cavity resonators bounded by lower and upper surfaces of each slot are exited by the surface current \( \overline{M_i} \), \( i'=1,2,...,N \) on the lower surfaces, and \( \overline{M_i} \), \( i'=N+1,N+2,...,2N \) on the upper surfaces. Region \( R_{N+2} \) is the half space in to which the current \( \overline{M_i},i'=N+1,N+2,...,2N \) are radiating in the presence of a complete conducting wall.

The \( i^{th} \) magnetic current is given by
\[
\overline{M_i} = \overline{E_i} \times \hat{n}
\]  

(1)

Where \( \overline{E_i} \) is the \( i^{th} \) slot surface aperture field distribution and \( \hat{n} \) is the unit normal on the slot surface.

The required boundary conditions give a system of coupled operator equations as
\[
\sum_{j=1}^{N} \overline{H_n}^{R1} (\overline{M_j}) + \overline{H_n}^{Re} (\overline{M_i}) = \overline{H_n}^{Rinc}
\]

over \( S_i \)

(2)
\[
\sum_{j'=N+1}^{2N} H_{tj}' \left( \hat{M}_{j'} \right) + \sum_{j'=N+1}^{2N} \left( M_{j'} \right) = 0
\]

over \( S_i \)

\( i=1,2, \ldots \), \( i'=N+1, 2N+2, \ldots \)

\( H_{tj}' \) is the unknown coefficient to be evaluated. Next defining identical sets of weighting functions \( \hat{W}_{pk} \), \( \hat{W}_{pk}^t \) - different practises \( \hat{M}_{ij} \) over \( S_i \), \( S_j \), \( S_{N-1} \) using Galerkin’s procedure, equation (2) can be written as:

\[
[Y] \begin{bmatrix} V \end{bmatrix} = [I]
\]

where,

\[
[Y] = \begin{bmatrix}
Y^{(1)}_{11} & Y^{(1)}_{12} & \cdots & Y^{(1)}_{1N_N} \\
Y^{(1)}_{21} & Y^{(1)}_{22} & \cdots & Y^{(1)}_{2N_N} \\
\vdots & \vdots & \ddots & \vdots \\
Y^{(1)}_{N_N,1} & Y^{(1)}_{N_N,2} & \cdots & Y^{(1)}_{N_N,N_N}
\end{bmatrix}
\]

\[Y^{(1)}_{ij} = \int\int \hat{W}_{pk} \hat{H}_{tp} \left( \hat{M}_{ij} \right) ds \]

with the magnetic field inside the wave guide being given as \[ Y^{(1)}_{ij} \] is given by;

\[ Y^{(1)}_{ij} = \int\int \hat{W}_{pk} \hat{H}_{tp} \left( \hat{M}_{ij} \right) ds \]

The entire domain sinusoidal functions have been chosen as basis and weighting functions, i.e.

\[
\hat{M}_{ij} = \hat{z} \sin \frac{j\pi(z_i + 1/2)}{l_i}, j=1,2, \ldots N_c
\]

\[
\hat{W}_{pk} = \hat{z} \sin \frac{k\pi(Z_p + 1/2)}{l_p}, k=1,2, \ldots N_c
\]

4.Wave Guide Matrix : The wave guide matrix \([Y^{(1)}]\) comprises \( N^2 \) submatrices of the order of \( N_c \times N_c \) as;

\[
[Y^{(1)}] = \begin{bmatrix}
Y^{(1)}_{11} & Y^{(1)}_{12} & \cdots & Y^{(1)}_{1N_N} \\
Y^{(1)}_{21} & Y^{(1)}_{22} & \cdots & Y^{(1)}_{2N_N} \\
\vdots & \vdots & \ddots & \vdots \\
Y^{(1)}_{N_N,1} & Y^{(1)}_{N_N,2} & \cdots & Y^{(1)}_{N_N,N_N}
\end{bmatrix}
\]

5.Coupling Matrix : Coupling is obtained via the cavity resonators formed by closing upper and lower endcap.
lower surfaces of each slot by conducting sheets, and excited by the equivalent magnetic current elements placed on respective slot surfaces \( [Y^c] \) can be thought of four black symmetric matrices:

\[
[Y^c] = \begin{bmatrix}
Y_{11}^{(c)} & Y_{12}^{(c)} \\
Y_{21}^{(c)} & Y_{22}^{(c)}
\end{bmatrix}
\] (11)

Because of Galerkin’s procedure, an element of \( [Y^c] \) can be given as:

\[
[Y^{(q)}]_{kj} = \int \int_{s} \mathbf{W}_{pk} \cdot \mathbf{H}_{tp}^{(1)} (\mathbf{M}_{ij})_{kj} \, ds
\] (12)

where \( p \) and \( i \) are the upper and lower surface of a slot forming \( q \) th cavity and \( \mathbf{H}_{tp}^{(q)} (\mathbf{M}_{ij}) = j \omega \int \mathbf{G}_{c} (\mathbf{r}' / \mathbf{r}') \cdot \mathbf{M}_{ij} \, ds \) (13)

is the field on the \( S^p \) surface. \( \mathbf{G}_{k} (\mathbf{r}' / \mathbf{r}') \) is the dyadic Green’s function for a cavity.

6. Half Space Matrix: Region \( R_{N+2} \) is a half space into which the magnetic currents, \( \mathbf{M} \) on \( S_{i} \), \( i = N+1, N+2, \ldots, 2N \), are radiating in the presence of a complete conducting plane half space matrix is further subdivided into;

\[
[Y^h] = \begin{bmatrix}
Y_{hs}^{N+1, N+1} & \cdots & Y_{hs}^{N+1, 2N} \\
Y_{hs}^{N+2, N+1} & \cdots & Y_{hs}^{N+2, 2N} \\
& \cdots & \cdots
\end{bmatrix}
\] (14)

Applying image theory, an element of the submatrix \( [Y_{pi}^{h}] \) can be written as:

\[
[Y_{pi}^{h}]_{kj} = 2 \int \int_{sp} \mathbf{W}_{pk} \cdot \mathbf{H}_{tp}^{hs} (\mathbf{M}_{ij}) \, ds
\] (15)

with \( \mathbf{H}_{tp}^{hs} (\mathbf{M}_{ij}) = -j \omega \mathbf{F}_{pi} - \nabla \phi_{pi} \) (16)

Only \( S_{11} \) and \( S_{12} \) are of interest. Assuming only dominant mode at the input and with matched termination at the load end, the reflection coefficient of the \( p \) th slot;

\[
(S_{11})_{p} = \left[ \frac{H_{p}\text{inc}}{H_{p}\text{scat}} \right]_{\mathbf{v} = \mathbf{z}} (21)
\]

\[
\mathbf{H}_{\text{scat}} = -j \frac{\mathbf{F}_{p}}{k_{o}^{2}} \left( \frac{\partial^{2}}{\partial x \partial z} F_{p} \right) \mathbf{k} \quad (22)
\]

where \( F_{p} = \frac{\varepsilon_{o}}{4\pi} \int \int_{s} \mathbf{M}_{pj} \cdot \mathbf{G}_{m} (\mathbf{r}' / \mathbf{r}') \, ds \) (23)

After a little mathematical manipulations, it comes out to be

\[
(S_{11})_{p} = \frac{-j A_{10} \pi}{2a} \sin \frac{\pi \theta_{p}}{2a} \cos \frac{\pi x_{p}}{a} \times \left[ \sum_{k=1}^{N_{c}} \mathbf{k} \mathbf{p} \mathbf{k} \right]_{(k \pi / l p)^{2} - \beta_{p}^{2}} \left\{ \begin{array}{l}
\cos \beta_{gl} / p / 2, k \text{ odd} \\
2 j \sin \beta_{gl} / p / 2, k \text{ even}
\end{array} \right. \quad (24)
\]
and 
\[
(S_{12})_p = 1 - (-1^p)(S_{11})_p \quad (25)
\]

\[
A_{10} = \left[ \frac{2}{j \omega \mu_0 \sqrt{1 - k_o^2 ab}} \right]^{1/2} \quad (26)
\]

Radiation Pattern: The directive gain, \( G_d(\theta, \phi, R_m) \) of the radiating is given by;
\[
G_d(\theta, \phi, R_m) = 4\pi R_m^2 |\bar{E}_m|^2 / \eta \text{real}(P_{rd})
\quad (27)
\]

The radiated power, \( P_{rd} = P_{inc} \left[ 1 - \left| S_{11} \right|^2 + \left| S_{12} \right|^2 \right] \quad (27) \)

\( P_{inc} \) is the incident power =
\[
\frac{1}{2} R_e \left\{ \int \int |\bar{E} \times \bar{H}| ds \right\} \quad (28)
\]

\( \bar{E}_m \) is the electric field at a point \( R_m \) in the half space, given as;
\[
\bar{E}_m = (j k_o \eta / 4\pi R_m) \exp(-j k_o R_m) [\bar{P}^m] V
\quad (29)
\]

Where, \( [\bar{P}^m] \) is the measurement vector given by;
\[
[\bar{P}^m] = \left[ \begin{array}{c} \bar{P}_1^m \\ \vdots \\ \bar{P}_N^m \end{array} \right] \quad N \times 1 
\]

\[
\bar{P}_i^m = -< \bar{M}_{ij}, \bar{H}_i^m > \quad j \times 1 \quad (31)
\]

\( k_o, \eta \) are the free space propagation constant and intrinsic impedance respectively.

9. Results: Fig. 3 shows the variation of \( S_{11}, S_{12} \) of a slot length l. The agreement with that of Lyon et al. [5] establishes the validity of computer programme. Fig. 4 shows the computed gain pattern of 3 element array. Figs. 5, 6, 7 show the measured radiation patterns of 3, 4, 5 element arrays fabricated. The slot lengths are determined after taking mutual coupling and wall thickness into account. The patterns are well formed and deviation from the theoretical values can be attributed due to various assumptions made during the analysis.

References:
Fig. 1(a) Geometry of the problem.
(b) Expanded view of the ith slot.

Fig. 2: Application of equivalence theorem to a N-element slotted array.
(a) Relevant region (R1).
(b) Expanded view of ith cavity resonator (R2 region).
(c) High-sweep region (R2a).

Fig. 3: Scattering parameters versus normalized slot length for an isolated rectangular slot: 0.025 ≤ L ≤ 0.375.

Fig. 4: Radiation pattern for a 3-slot resonant array.

Fig. 5: Experimental results for radiation patterns of a 3-element array: 0.05 ≤ L ≤ 0.0825.