Effect of Finite Grounded Dielectric Slab on the Radiation of Microstrip Antenna

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Abstract: - In this study, the method developed to find an approximate Green’s function for a finite grounded dielectric slab is summarized. This Green’s function is used in the method of moment solution derived for microstrip patch antenna in order to compute its current distribution. Next the radiation of this current is calculated. The effects of the finiteness of the dielectric substrate and the ground plane to the radiation characteristics of the antenna are considered by adding the contributions of the equivalent volume polarization currents within the slab, and the diffraction from the ground plane edges.

Key-Words: - Green’s functions, microstrip antenna, radiation pattern, diffraction, volume polarization currents

1 Introduction
In the numerical analysis of radiating printed structures, like microstrip antennas, Method of Moments (MoM) is commonly used. In most of the MoM based analysis, the dielectric substrate and/or the ground plane is assumed to be infinite. However in most of the applications, like mobile communications, the dielectric substrate of microstrip antennas are restricted to small sizes due to the space and/or cost limitations. Therefore the accurate mathematical modeling of the finite geometry becomes important in predicting the effects of finite ground plane and dielectric substrate on the radiation characteristics of printed structures.

When the ground plane and the dielectric substrate are finite, the MoM formulation results in a large number of unknowns (volume polarization currents in the dielectric and the surface currents on the ground plane), unless the Green’s function for the finite geometry is known. The Green’s function for the infinite dielectric substrate and the ground plane has been recently modified to analyze the three dimensional problems of the microstrip antennas over the finite grounded substrate by using the reflected surface waves from the air-dielectric interface at the edges found via edge admittance concept [1]. The resulting Green’s function is expressed in closed form in terms of complex images, and the incident and the reflected surface wave contributions. [1]-[2]. This Green’s function is used in the MoM solution of the mixed-potential integral equation, to predict the current distribution and the input impedance of printed antenna build on finite ground plane and dielectric substrate for different dielectric constant values and substrate and ground plane dimensions [2]-[3].

In this paper the radiation pattern of a patch antenna is calculated using the approximate Green’s function for the finite geometry.

2 Formulation

2.1 Green’s Functions for Finite Geometry
The scalar and vector potential Green’s function of a horizontal electric dipole placed on an infinite grounded dielectric slab can be obtained in closed form in terms of complex images and surface wave contributions. [1]-[2]. This Green’s function is used in the MoM solution of the mixed-potential integral equation, to predict the current distribution and the input impedance of printed antenna build on finite ground plane and dielectric substrate for different dielectric constant values and substrate and ground plane dimensions [2]-[3].

In this paper the radiation pattern of a patch antenna is calculated using the approximate Green’s function for the finite geometry.
1) Find the surface wave component of the incident electric field at the dielectric-air interface (y = a, z>0), Fig. 1,

\[ E_{sw} = f(z)k_\rho \rho H_0^2 \left( k_\rho \rho \sqrt{(x-x')^2 + (a-y')^2} \right) \hat{a}_z \]  

(1)

where \( k_\rho \) is the propagation constant of surface wave, and \( f(z) \) shows z-variation of the field.

2) In order to be able to analyze the effect of incident waves coming from various angles, use plane wave expansion of the Hankel function

\[ \pi H_0^2 \left( k_\rho \rho \sqrt{x^2 + y^2} \right) = \int_{-\infty}^{\infty} e^{-jk_\rho x} \frac{e^{-jk_\rho y}}{k_\rho} dk_\rho, \]

\[ k_\rho = \sqrt{k_x^2 + k_y^2} \]  

(2)

3) Define equivalent magnetic currents at the dielectric-air interface (y=a, z>0), \( \hat{M}_{eq} = E_{sw} \times \hat{n} \), where \( \hat{n} \) is the unit vector normal to interface using the above incident electric field component. Note that equivalent magnetic currents possess \( e^{-jk_\rho x} \) variation.

4) Place a semi-infinite perfect electric conductor at the dielectric-air interface. This fictitious PEC and finite ground plane form a 90° wedge. The Green’s function of a horizontal magnetic line current with \( e^{-jk_\rho x} \) variation, placed on a conducting 90° wedge is given as, [4]

\[ G_x^H = \begin{cases} \frac{2\omega \epsilon_0 \pi}{3} e^{-jk_\rho x} \sum_{n} \frac{1}{\sigma_n} H_2^2 (k_i z') J_{2n} (k_i z), & z < z' \\ \frac{2\omega \epsilon_0 \pi}{3} e^{-jk_\rho x} \sum_{n} \frac{1}{\sigma_n} H_2^2 (k_i z') J_{2n} (k_i z'), & z > z' \end{cases} \]

(3)

where \( \sigma_0 = 2, \sigma_n = 1, n \neq 0 \) and \( k_i = \sqrt{k_\rho^2 - k_x^2} \). Using this Green’s, the total field due to the equivalent single line current can be found.

5) Using the stationary formula [5], calculate the edge admittance as a function of \( k_i \), and find the reflection coefficient of the surface waves at the interface using the edge admittance

\[ \Gamma(k_y) = \frac{k_\rho}{k_\rho + y_{edge}(k_y)} \]  

(4)

6) Reflected surface wave at an observation point inside the dielectric can be evaluated as:

\[ E_{sw}^{ref} = f(z)k_\rho H_0^2 \left( k_\rho \rho \sqrt{(x-x')^2 + (2a-(y+y')-j\alpha_1)^2} \right) \]

(5)

7) Expand reflection coefficient in terms of complex exponentials by using GPOF (Generalized Pencil of Function) method. Finally reflected surface wave component can be expressed in the following form

\[ E_{sw}^{ref} = f(z)k_\rho \]

\[ \ast \sum_{i=1,N} \beta_i H_0^2 \left( k_\rho \rho \sqrt{(x-x')^2 + (2a-(y+y')-j\alpha_i)^2} \right) \]

(6)

\( \alpha_i \) and \( \beta_i \) values depend on the thickness and the dielectric constant of the substrate and the frequency. Since the shape of the discontinuity that causes the reflections from the edges does not change at four sides of the dielectric slab and the ground plane, the same \( \alpha_i \) and \( \beta_i \) values can be used for the reflections from other edges. Hence the total reflected electric field could be expressed as

\[ E_{sw}^{ref} = f(z)k_\rho \]

\[ \ast \sum_{i=1,N} \beta_i H_0^2 \left( k_\rho \rho \sqrt{(x-x')^2 + (2a-(y+y')-j\alpha_i)^2} \right) \]

(7)

By using the procedure outlined in this paper, the scalar and vector potential Green’s functions for a finite grounded dielectric slab can be obtained in closed-form in terms of the summation of complex.
images [6], incident and reflected surface wave contributions. These Green’s functions are used in the MoM solution of MPIE obtained for printed structures in calculation of the current distribution over the antenna.

2.2 Radiation Pattern Formulation

As the radiation pattern of the antenna is evaluated by using the obtained current distribution, not only the radiation from the surface current over antenna is taken into account but also the contributions of the dielectric substrate and the ground plane should be accounted for due to the finite structure. In the computation of the radiation field from the current over the antenna, the spectral (Fourier transform) [7] technique can be used.

The effect of the finite dielectric slab can be obtained with the application of volume equivalence theorem, i.e. the slab is replaced by equivalent volume polarization currents [8] as:

$$\overline{J}_d (r) = \frac{jk_0}{\eta_0} (\varepsilon_r - 1) \overline{E}_d (\vec{r})$$

(8)

$\overline{E}_d$ in (8) is the real electric field within the dielectric substrate, and its components can be evaluated by employing the closed-form Green’s functions reported in [6]. The radiated field due to polarization currents can be written as:

$$\overline{E}_d^f (k_x, k_y, k_z) = -j k_0 \eta_0 \frac{\exp(-jk_0 r)}{4\pi r}$$

$$ \int dz \int \int \overline{J}_d (x, y, z) \exp(k_x x + k_y y + k_z z) dxdy$$

(9)

The effect of the finiteness of the ground plane to the far field radiation can be considered by the application of the geometrical theory of diffraction [9].

The mechanism in the evaluation of the diffraction from the ground plane is shown in Fig. 2. The diffraction points (Q1, Q2) that contribute to the principal E-plane radiation are the points where the principal E-plane pattern and the edge of the ground plane intersect. The contributions of these points to the far field radiation are related to the incident field, attenuation factor, and the diffraction coefficient. The diffracted field from Q1 is

$$E_{d1} = E^i (Q_1) D_h A_1(s'_1, s_1) e^{-jk_h}$$

(10)

where $D_h$ is the diffraction coefficient, $E^i (Q_1)$ is the incident electric field to Q1, and $A_1$ is the attenuation factor. The diffraction from other point can be found similarly.

3 Results

The developed software programs are used in the calculation of the current distribution over the patch antenna fed by a microstrip line (Fig. 3), which are built on the substrate and the ground plane of different dimensions. The frequency is 7.8 GHz. The thickness of the dielectric is 0.152 cm and the relative permittivity is 3.38. The resonance frequency of the antenna is found to be 8.0 GHz.
The radiation pattern of the antenna is found by adding the contributions of the current over the patch and the equivalent volume polarization current within the substrate, and the diffraction from the edges of the ground plane. In the computations, the results are obtained for different values of the spacing between the antenna and the dielectric edges (a, b and c values in Fig. 3).

In Fig. 4 the total far-field radiation (solid line) due to the surface current over the patch and the equivalent volume polarization currents inside the dielectric slab is presented. The dash-dotted line shows the radiation due to the current over the patch and the radiation due to the equivalent current is shown by dotted line. The results are evaluated with the infinite ground plane assumption. The contribution of the polarization current should be accounted for especially at the grazing angle as seen from Fig. 4.

If the contribution of the diffraction from the edges of the ground plane is added to the result found in Fig. 4, the radiation pattern for the finite geometry will be obtained. The result for a=5.095 cm, b=5.046 cm, and c=5 cm is shown in Fig. 5.

More detailed results including the results for the H-plane radiation and the experimental results will be given during the presentation.

References: