An Autonomous Chaotic CNN Hysteresis Circuit

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Abstract: - A new implementation of an autonomous hysteresis chaotic circuit using only two primitive cellular neural networks (CNN) cells is presented. Both computer simulations and laboratory measurements have confirmed the chaotic behavior and show the existence of the double scroll attractor.

Key-Words: - CNN, chaotic circuits, chaos, attractor, bifurcation, oscillator.

1. Introduction

In the last decade, there is a surge in designing simple electronic circuits whose behavior is chaotic. For example, the Chua circuit [1], the Colpitts oscillator [2] and the hysteresis chaos generator [3] are among the circuits in which a variety of dynamical behaviors including chaos have been reported. It is the understanding of the operation of these simple circuits that gave birth to the development of sophisticated applications that use this behavior. Such applications are found today in the areas of communications, control, pattern recognition, and measurement device [4].

Recently, there is an interest to design analog signal processing system with complex dynamical behaviors using an array of simple, locally interconnected cells called cellular neural network (CNN) [5-7]. For instance, the Chua circuit was implemented using three generalized connected CNN cells [8], and the colpitts-like oscillator was realized using also a suitable connection of three state controlled SC-CNN cells [9]. In both circuits, a strange chaotic behavior was observed.

In this paper, it is shown how the hysteresis chaos generator [3] can be designed and implemented using a connection of only two CNN cells. In the following sections, some background concepts are summarized; the circuit design and implementation is reported. Finally, simulation results using both Matlab Simulink [10] and Electronics Workbench [11] are shown. Experimental results confirm the existence of the double scroll attractor.

2. Background

2.1. The Hysteresis Chaos Generator

The state equations of the dimensionless simplified hysteresis chaos generator are [3]:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
\sigma & \omega \\
-\omega & \sigma
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
0 \\
h(x_1)
\end{bmatrix}
\] (1)

Where

\[
h(x_1) = \begin{cases}
1 & \text{for } x_1 \geq -1 \\
-1 & \text{for } x_1 \leq 1
\end{cases}
\] (2)

and \(\sigma\) and \(\omega\) are both positive with \(\omega = \sqrt{1 - \sigma^2}\).

In this case the eigenvalues \(\lambda = \sigma \pm j\omega\) of system (1) are complex with positive real part (i.e. the system is unstable). There are two unstable equilibrium points located at \(Q^+ = (\omega, -\sigma)\) and \(Q^- = (-\omega, \sigma)\). The hysteresis function \(h(x_1)\) is shown in

\[
h(x_1) = \begin{cases}
1 & \text{for } x_1 \geq -1 \\
-1 & \text{for } x_1 \leq 1
\end{cases}
\]
Fig. 1. When \( x_1 \) hits the right threshold value, \( h(x_1) \) switches to 1 and when \( x_1 \) hits the left threshold value, \( h(x_1) \) switches to -1.

![Hysteresis function](image1)

Fig. 1. Hysteresis function

Fig. 2. Shows the block diagram of the hysteresis chaos generator described by (1).

![Block diagram](image2)

Fig. 2. Block diagram of the hysteresis chaotic system (1)

The simulation of the dynamical system (1) displays a chaotic attractor as shown by the phase portrait of Fig. 3 for \( \sigma = 0.05 \), and the initial conditions \( x_1(0) = 0.5 \) and \( x_2(0) = 0 \).  

![Chaotic attractor](image3)

Fig. 3. Chaotic attractor (\( \sigma = 0.05 \)), (horizontal axis: \( x_1 \); vertical axis: \( x_2 \))

The reader is referred to [3, 9] for more details about the sufficient condition for chaos generation. In the next section, we describe the CNN model that will be used to implement system (1).

### 2. 2. CNN Model

The generalized CNN model that will be used in the design of the hysteresis chaos generator is described by the following dimensionless nonlinear state equation:

\[
x_i = -x_i + \sum_{j=1}^{m} a_{ij} y_j + \sum_{k=1}^{n} s_{ik} x_k + i_i \quad (3)
\]

Where \( j \) is the cell index, \( x_i \) the state variable, \( y_i \) the cell output is defined as

\[
y_i = \begin{cases} 
1 & \text{for } x_i \geq -1 \\
-1 & \text{for } x_i \leq 1 
\end{cases} \quad (4)
\]

\( a_{ij} \) and \( s_{ik} \) are constant, and \( i_i \) is a threshold value.  

In the case of two fully connected CNN cells, the dynamic model is described by the equations

\[
\begin{align*}
\dot{x}_1 &= -x_1 + a_{11} y_1 + a_{12} y_2 + s_{11} x_1 + s_{12} x_2 + i_1 \\
\dot{x}_2 &= -x_2 + a_{21} y_1 + a_{22} y_2 + s_{21} x_1 + s_{22} x_2 + i_2 
\end{align*} \quad (5)
\]

Where \( x_1 \) and \( x_2 \) are the state variables, and \( y_1 \) and \( y_2 \) are the corresponding outputs. In order to implement the hysteresis chaos generator, the following parameters \( a_{ij} \) and \( s_{ij} \) must be selected such that system (5) will have the same behavior as the one shown by equation (1). This will lead to the following parameters setting:
\[ i_1 = i_2 = 0 \]
\[ a_{12} = a_{22} = 0; a_{11} = 0; a_{21} = 1 \]
\[ s_{11} = s_{22} = \sigma + 1; s_{12} = -s_{21} = \omega \]  \hspace{1cm} (6)
\[ y_1 = h(x_1) \]

In the next section, it is shown how to implement the hysteresis chaos generator using two CNN cells.

3. CNN Design And Implementation

The proposed circuit to implement the hysteresis chaos generator is shown in Fig. 4(a). It is inspired by [8] and is constituted with only two cells.

Each cell consists of three blocks: Block B1 is the main block, which can be described by the following equation:
\[ C\dot{x}_i = -\frac{x_i}{R_5} - \frac{x_i}{R_6} + \frac{R_2}{R_5} V_1 + \frac{R_4}{R_2 R_5} V_2 + \frac{R_4}{R_3 R_5} V_3 \]  \hspace{1cm} (7)

To simplify our calculation we select \( R_6 \) the input impedance of block B2 to be much higher than \( R_5 \) (i.e. in this case block B2 will not load the capacitor C). Equation (7) becomes:
\[ C\dot{x}_i = -\frac{x_i}{R_5} + \frac{R_4}{R_1 R_5} V_1 + \frac{R_4}{R_2 R_5} V_2 + \frac{R_4}{R_3 R_5} V_3 \]  \hspace{1cm} (8)

Block B2 is an inverting amplifier with unity gain. This implies that
\[ R_5 = R_6 \]  \hspace{1cm} (9)

Block B3 implements the nonlinear hysteresis function as shown in Fig. 4. \( V_o \) is described as:
\[ V_o = \begin{cases} +V_{sat} & \text{if } V_{in} \geq V_{LT} \\ -V_{sat} & \text{if } V_{in} \leq V_{UT} \end{cases} \]  \hspace{1cm} (10)

Where the lower threshold voltage \( V_{LT} \) and the upper threshold voltage \( V_{UT} \) are related to the operational amplifier saturation voltage \( V_{sat} \) by the following equations
\[ V_{LT} = \frac{R_9}{R_8 + R_9} (V_{sat}); V_{UT} = \frac{R_9}{R_8 + R_9} (-V_{sat}) \]  \hspace{1cm} (11)

\( R_9 \) and \( R_{10} \) are used to scale down the output voltage in the range [-1, 1]. This will lead to the following design equations:
\[ \frac{R_9 + R_9}{R_9} = \frac{V_{sat}}{V_{int}} \]  \hspace{1cm} (12a)
\[ \frac{R_{10} + R_{11}}{R_{11}} = \frac{R_8 + R_9}{R_9} \]  \hspace{1cm} (12b)
Where \( V_{\text{inT}} \) is the input threshold voltage, which is set to 1 in our case. \( V_{\text{sat}} \) is determined by the power supplies and internal structure of the op amps. Block B4 is a voltage follower. In the next section it is shown how to select the resistor and capacitor values.

### 4. Design Procedure

In order to implement equations (1) and (2) using the proposed circuit, each cell component parameters must be set appropriately using only the given value of \( \sigma \). In the simulation of system (1), no units were given for the state variables \( x_1, x_2 \). However, since the voltages in the proposed CNN cell described by (8) are measured in Volts and currents in milliAmperes, it is important to rescale all currents by a factor \( k_1 \). The effect is to reduce the capacitance by a factor \( k_2 \) and to increase resistances by the same factor. Since capacitors in the order of nF are easier to use and more available then a time rescaling by a factor \( k_2 \) is needed. The effect is to reduce the capacitance by a factor \( k_2 \); resistances are not affected by a time scaling. The step by step procedure to calculate the CNN cells parameters is as follows:

**Step 1:** Given \( \sigma \), calculate 

\[
\omega = \sqrt{1 - \sigma^2}, \quad a_{21} = 1, s_{11} = s_{22} = \sigma + 1, s_{12} = -s_{21} = \omega
\]

**Step 2:** Set cell 1 inputs: \( V_1 = x_1, V_2 = x_2, V_3 = 0 \).

Choose \( k_1 = 10^3; k_2 = 2 \times 10^6 \)

Select \( C = 50nF, R_3 = R_4 = R_5 = 1k\Omega \) (taking into account the rescaling process)

**Step 3:** Calculate 

\[
R_1 = \frac{R_1}{s_{11}}, \quad R_2 = \frac{R_1}{s_{12}}
\]

**Step 4:** Set \( R_6 = R_7 = 100k\Omega, R_9 = R_{11} = 1k\Omega \) 

Calculate 

\[
R_8 = R_9 V_{\text{sat}} - R_9
\]

\[
R_{10} = R_{11} V_{\text{sat}} - R_{11}
\]

**Step 5:** Set cell 2 inputs:

\( V_1 = -x_1, V_2 = x_2, V_3 = y_1 \)

Choose \( k_1 = 10^3; k_2 = 2 \times 10^6 \)

Select \( C = 50nF, R_3 = R_4 = R_5 = 1k\Omega \)

**Step 6:** Calculate 

\[
R_1 = \frac{R_4}{s_{21}}, \quad R_2 = \frac{R_4}{s_{22}}, \quad R_3 = \frac{R_4}{a_{22}}
\]

**Step 7:** Set \( R_6 = R_7 = 100k\Omega, R_9 = R_{11} = 1k\Omega \) 

Calculate 

\[
R_8 = R_9 V_{\text{sat}} - R_9
\]

\[
R_{10} = R_{11} V_{\text{sat}} - R_{11}
\]

### 5. Simulation and Experimental Results

From the above design procedure, the parameters values for the two CNN cells for \( \sigma = 0.05 \) are:

**Cell 1:** \( R_1 = 952\Omega, R_2 = R_3 = R_4 = R_5 = 1k\Omega, R_6 = R_7 = 100k\Omega, R_8 = R_{11} = 14k\Omega, R_9 = R_{10} = 1k\Omega, C = 50nF \).

**Cell 2:** \( R_1 = R_2 = R_3 = R_4 = R_5 = 1k\Omega, R_6 = R_7 = 100k\Omega, R_8 = R_{11} = 14k\Omega, R_9 = R_{10} = 1k\Omega, C = 50nF \).

The results of the Electronic Workbench [10] simulation, the closest to implementing real circuits, are shown in Fig.5.

![Fig. 5. The simulated attractor of the CNN circuit](image)

The CNN circuit of Fig 4 was implemented in the laboratory. Fig. 6 shows the bifurcation phenomena. A qualitatively distinct trajectory projected on the \((Vx_1, Vx_2)\)-plane is shown. The remarkable agreement between experiment and simulation shows the ideal system of Fig. 2 is an excellent model of the physical circuit of Fig.4.
Fig. 6. The phase portrait \( V_{x_1}-V_{x_2} \) for \( R_1=835 \Omega \). All other components values are kept the same as in the simulation.

It should be noted that we have kept the architecture of the two CNN cells to be similar although many components may not be used. This is a small disadvantage in comparison to (1) simplicity of the design and (2) as programmable CNN have become more available [12,13]; this circuit can then be easily programmed.

6. Conclusions

An autonomous hysteresis chaotic circuit was realized using only two CNN cells. The systematic proposed design uses only operational amplifiers, resistors and capacitor hence it can be built in a single chip. With the emerging programmable generalized CNNs, this circuit can then be easily programmed. Both computer simulations and laboratory measurements have confirmed the chaotic behavior and show the existence of the double scroll attractor. Next we will try to (1) apply this circuit to secure communication, (2) study higher dimensional chaos with several interconnected CNNs.

References