A Robust Optimal Control for Robot Manipulators

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Abstract- A mixed Optimal/robust control is proposed in this paper for the tracking of rigid robotic systems under parametric uncertainties and external perturbations. The design objective is that under a prescribed disturbance norm level, an optimal control system is to be designed as well as a robust control to overcome the effect of uncertainties. The optimal control is based on the solution of a nonlinear Ricatti equation, which by virtue of the skew symmetry property of manipulators and an adequate choice of state variables becomes an algebraic equation easy to solve. We then investigate the design of the robust control of the uncertain system by a continuous state feedback function. It will be shown that our approach globally asymptotically stabilizes the uncertain dynamical system. We illustrate our approach by applying our control approach to a 2-DOF manipulator.

Key-Words: Robot Manipulators, optimal control, robust control, uncertainties.

1 Introduction
Robust control problem for robot manipulators has received considerable interest by many researchers the last two decades. By robust control strategies, we mean strategies that are designed to yield good dynamics behavior in the presence of modeling errors and unmodeled dynamics. Indeed, model uncertainties are frequently encountered in robotics due to unknown or changing payload, friction, backlash, flexible joints or robot parts for which only simplified dynamical models are available.
Robot manipulators possess complex, nonlinear and coupled dynamics. Many different robot control strategies have been published in the literature. Classical control strategies can be found in textbooks on robotics (e.g. [1]-[2]). A survey on robust control of robot manipulators is given in [3] and [4]. Various approaches are classified into five categories (e.g. [5]-[9])
1- Linear multivariable approach;
2- Passivity-based approach;
3- Variable structure controllers;
4- Robust saturation approach;
5- Robust adaptive approach.

This paper deals with the design of robust/optimal control of robot manipulators. An optimal control problem to robust control robot manipulators has been addressed in [10]. In this paper, a robust control problem is introduced using the state space representation, where the uncertainty is a function of state and the matching condition was assumed. In [11]-[12] a control strategy for the robust control of robot manipulators was also addressed in the basis of H2/H∞ problem.

In this paper we address the problem differently from what is presented in [10]-[12], we investigate first the nonlinear optimal control to ensure both zero-error convergence of the nominal system and to minimize the torque input of to manipulator. We then investigate the robust control problem for the uncertain system. The first control is based on Johansson’s work [14], where he addressed the optimal control problem for robot manipulators which minimizes a quadratic performance index involving system error and torque input, as an explicit solution of the Hamilton-Jacobi equation. The proposed robust control consists on a state feedback of a continuous time function. The stability analysis based on Lyapunov theory is investigated to prove the global asymptotic stability of the uncertain system. The continuity of the solution will be also investigated and proved.

2 Manipulator dynamics and problem formulation
The motion equations of robot manipulator with external disturbances can be expressed as [1],

\[ M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q, \dot{q}) = \tau + \tau_d \] (1)

where \( q \in \mathbb{R}^n \) is the joint position, \( M(q) \in \mathbb{R}^{n \times n} \) denotes the generalized masses matrix of the manipulator, \( C(q, \dot{q})\dot{q} \) represents the centripetal and Coriolis terms, \( G(q, \dot{q}) \) includes the gravitational and frictional forces, \( \tau \) is the applied torque vector and \( \tau_d \) is the external disturbances vector.
Two key properties related to dynamics characterize dynamics (1), [1].

**Property 1:** For all \( q \in \mathbb{R}^n \) the matrix \( M(q) \) is symmetric and uniformly positive definite.

**Property 2:** A suitable definition of \( C(q, \dot{q}) \) makes the matrix \( M - 2C \) skew-symmetric, so that
\[
x^T (M - 2C)x = 0 \quad \forall x \in \mathbb{R}^n
\]
The desired reference trajectory to be followed is assumed to be bounded functions of time in terms of the generalized positions \( \dot{q}^d \in C^2 \) where \( C^2 \) is the class of functions at least twice continuously differentiable.

Define the state tracking error as
\[
e := \left[ \begin{array}{c} \dot{\tilde{q}} \\ \tilde{q} \end{array} \right] = \left[ \begin{array}{c} \dot{q} - \dot{q}^d \\ q - q^d \end{array} \right]
\]
(2)

Then the tracking problem of the generalized position \( q \) and velocity \( \dot{q} \) is reduced to the regulation problem of the state error \( e \). Combining (1) and (2), the dynamic equation for the state tracking error \( e \) is obtained as
\[
\dot{e} = A(q, \dot{q})e + B_0(\dot{q}^d, \dot{q}^d, \dot{q}, q) + BM^{-1}(q)(\tau + \tau_d)
\]
where
\[
A(q, \dot{q}) = \left[ \begin{array}{cc} -M^{-1}(q)C(q, \dot{q}) & 0_{ns} \\ I_{ns} & 0_{ns} \end{array} \right]; B = \left[ \begin{array}{c} I_{ns} \\ 0_{ns} \end{array} \right]
\]
(3)

Define the filtered link tracking error \( r(t) \), [14] as
\[
r(t) = \alpha \tilde{q}(t) + \Gamma \tilde{q}(t)
\]
for some constant scale \( \alpha > 0 \) and constant positive matrix \( \Gamma \in \mathbb{R}^{ns \times ns} \) which should be adequately determined.

Define
\[
T = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ 0_{ns} & I_{ns} \end{bmatrix} = \left[ \begin{array}{c} \alpha I_{ns} \\ 0_{ns} \end{array} \right] \Gamma
\]
(5)
then the error dynamics equation can be modified as the compact form
\[
\dot{e} = T^{-1} \begin{bmatrix} \dot{\tilde{q}}(t) \\ \tilde{q}(t) \end{bmatrix} = A_T(e, t)e + B_T(e, t)T_{11}(-f(e, t) + \tau) + B_T(e, t)w
\]
where
\[
A_T(e, t) = T^{-1} \left[ \begin{array}{cc} -M^{-1}(q)C(q, \dot{q}) & 0_{ns} \\ T_{t_{11}} & -T_{t_{11}}T_{12} \end{array} \right] T
\]
\[
B_T(e, t) = T^{-1} \left[ \begin{array}{c} I_{ns} \\ 0_{ns} \end{array} \right] M^{-1}(q)
\]
(6)

\[
f(e, t) = M(q)(\dot{q}^d - T_{t_{11}}^{-1}T_{12}\dot{q}) + C(q, \dot{q})(\dot{q}^d - T_{t_{11}}^{-1}T_{12}\dot{q}) + G(q, \dot{q})
\]
\[
w = \alpha \tau_d
\]
\[
\tau_d \text{ represents here the vector of parameter uncertainties, unmodeled dynamics and the energy of external disturbances } \tau_{ext} \text{ such that}
\]
\[
\tau_d = \Delta M(q)\ddot{q} + \Delta C(q, \dot{q})\dot{q} + \Delta G(q) + \tau_{ext}
\]
Assumption 1: For any \( \tilde{e} \in \mathbb{R}^{2n} \), \( A_T(\tilde{e}, t), B_T(e, t) \) and \( w \) are Lebesque continuous functions, and \( \forall t \in \mathbb{R}, A_T(t, 0) = 0 \)

Assumption 2: For any \( (\tilde{e}, t) \in \mathbb{R}^{2n} \times \mathbb{R} \), the Euclidean norm of \( w \) is bounded by some known continuous function, i.e.
\[
\|w\| \leq \rho(e, t)
\]
(8)

where \( \rho(\tilde{e}, t) \) is a Lebesque continuous function.

Assumption 3: For a given set \( E \subset \mathbb{R}^{2n} \) and for \([a, b] \subset \mathbb{R} \), there exist a Lebesque integral function \( m_i(\cdot) : [a, b] \to \mathbb{R} \), \( i = 1, 2 \), such that for any \( (e, t) \in E \times [a, b] \)
\[
\|A_T(e, t)e\| \leq m_1(t)
\]
\[
\|B_T(e, t)\| \leq m_2(t)
\]
(9)

Assumption 4: The origin is an asymptotic stable equilibrium for the nominal system \( \dot{e} = A_T(e, t)e \). In particular, there exist a positive definite function \( V(.) : \mathbb{R}^{n} \times \mathbb{R} \to \mathbb{R}^+ \) and continuous decreasing scalar functions \( \gamma_i(.) : \mathbb{R}^+ \to \mathbb{R}^+ \), \( i = 1, 2, 3 \) satisfying
\[
\gamma_i(0) = 0 \quad i = 1, 2, 3
\]
\[
\lim_{\tau \to \infty} \gamma_i(\tau) = \infty \quad i = 1, 2
\]
(10)
(11)

such that for any \( (\tilde{e}, t) \in \mathbb{R}^{2n} \times \mathbb{R} \)
\[
\gamma_1(\|e\|) \leq V(e, t) \leq \gamma_2(\|e\|)
\]
\[
\frac{\partial V(e, t)}{\partial t} + \nabla_e^T V(e, t)A_T(e, t)e \leq -\gamma_3(\|e\|)
\]
(12)
(13)

where \( V(.) \) is a Lyapunov candidate function for the nominal system
\[
\dot{e} = A_T(e, t)e + B_T(e, t)T_{11}(-f(e, t) + \tau)
\]
(14)

The objective of the tracking design for nominal system involves designing an optimal control whose effect is to minimize the control effort to the input of the system. The nonlinear optimal control which should possess some physical meanings must be concerned in the quadratic performance criterion. One of the noble contributions in the work of Johansson [14] is that a selective applied torque is applied as
\[
u = M(q)\dot{\tilde{e}} + C(q, \dot{q})\dot{\tilde{e}}
\]
(15)

which only affects the kinetic energy of the system since, during the process of motion, it may be
unnecessary to consider the consumption due to the potential energy and to optimize gravitational-dependant torques or forces. The relation between the control variable \( u \) and the applied torque \( \tau \) in (14) is given by

\[
\tau = f(e,t) + T_{11}^{-1} u \tag{16}
\]

Then the state tracking error equation, driven by the selective applied torque \( u \), for the uncertain system can be written as

\[
\dot{e} = A_T (e,t)e + B_T (e,t)u + B_T (e,t)w \tag{17}
\]

The control problem goal can be stated as: Given the uncertain dynamical equation (6) of an a robotic system satisfying assumptions A1-A4 design a robust optimal control to:

1. achieve global asymptotic stabilization of the nominal system using an optimal control approach
2. ensure global asymptotic stability for the uncertain dynamical system.

3 Optimal control for the nominal system
We consider in this section the nominal system

\[
\dot{e} = A_T (e,t)e + B_T (e,t)u \tag{18}
\]

We look here for an optimal control law \( u^* \) that minimizes the quadratic index performance

\[
J(u) = \frac{1}{2} \int_0^T \left( e^T (t)Qe(t) + u^T (t)Ru(t) \right) dt \tag{19}
\]

where \( Q \) and \( R \) are positive definite matrices to be chosen.

Johansson in [14] shows that the optimal solution of (19) is given by

\[
u^* (t) = -R^{-1}B^T_P (e,t)P(e,t)e \tag{20}\]

where \( P(\tilde{e},t) \) is the solution of the nonlinear Ricatti equation

\[
P + PA_T + A_{T}^T P - PB_T R^{-1}B_{T}^P P + Q = 0 \tag{21}
\]

considering Property 2 related to the dynamics of the manipulator, one can show that \( P(\tilde{e},t) \) can be explicitly considered as

\[
P(e,t) = T^T \begin{bmatrix} M(q) & 0_{n \times n} \\ 0_{n \times n} & K_{n \times n} \end{bmatrix} T \tag{22}
\]

where \( K \) is a positive \( n \times n \) matrix.

The nonlinear Ricatti equation can be simplified as

\[
PA_T + A_{T}^T P = \begin{bmatrix} 0_{n \times n} & K_{n \times n} \\ K_{n \times n} & 0_{n \times n} \end{bmatrix} + T^T \begin{bmatrix} -\dot{M}(q,q) & 0_{n \times n} \\ 0_{n \times n} & 0_{n \times n} \end{bmatrix} T
\]

and

\[
B_{T}^P P = B^T T \tag{24}
\]

From equations (20) and (24), \( u^* \) can be rewritten as

\[
u^* (t) = -R^{-1}B^T_T (e,t)Te \tag{25}\]

and the nonlinear Ricatti equation becomes an algebraic equation as

\[
\begin{bmatrix} 0 & K \\ K & 0 \end{bmatrix} + QTBR^{-1}B^T T = 0 \tag{26}
\]

Theorem 1
Consider the nominal error manipulator dynamics (14), the optimal control law

\[
u^* (t) = -R^{-1}B^T_T (e,t)Te
\]

stabilizes asymptotically the system (18), where \( K > 0 \) and a nonsingular \( T \) solving the algebraic matrix equation (26).

Proof: Consider the Lyapunov candidate function

\[
V(e,t) = \frac{1}{2} e^T Pe \tag{26}
\]

The quadratic function \( V(e,t) \) is a suitable Lyapunov function candidate because it is positive radially growing with \( \| e \| \). It is continuous and has a unique minimum at the origin of the error space. It remains to show that \( \dot{V} < 0 \) for all \( \| e \| \neq 0 \). It can be shown that for \( u^* \) as in (25), the function \( V \) satisfies the partial differential equation

\[
- \frac{dV(e,t)}{dt} = \left( \frac{\partial V(e,t)}{\partial e} \right)^T \dot{e} + \frac{\partial V(e,t)}{\partial t}
\]

But

\[
\frac{\partial V(e,t)}{\partial t} = -L(e,u^*) - \left( \frac{\partial V(e,t)}{\partial e} \right)^T \dot{e}
\]

where \( L(\ldots) \) is the Lagrangian as

\[
L(e,u^*) = \frac{1}{2} e^T Qe + \frac{1}{2} u^* Ru \tag{29}
\]

then

\[
\frac{dV(e,t)}{dt} = -\frac{1}{2} e^T Qe - \frac{1}{2} u^* Ru
\]

and finally

\[
\frac{dV(e,t)}{dt} = -\frac{1}{2} e^T (T^T BR^{-1}B^T T + Q)e < 0 \tag{31}
\]

\( \forall t > 0, \ e \neq 0 \)

4 Robust optimal control stabilization of the uncertain system
In this section we design a robust control law for the asymptotic stabilization of the uncertain dynamical system.

The proposed control law is as

\[
u = u_1 + u_2 \tag{32}\]

where \( u_1 \) is the optimal control given by

\[
u_1 = -R^{-1}B^T_T Te \tag{33}\]

and \( u_2 \) is the robust control to be designed to overcome the effect of the perturbation vector \( w \) satisfying (8).
Substituting (33) in (17), one get
\[ \dot{e} = f(e, t) + B_T(e, t)(w + u_1 + u_2) \] (34)
where
\[ f(e, t) = (A_T(e, t) - B_T(e, t)K)e \] (35)
and
\[ K = -R^{-1}B^T e \] (36)
Consider the Lyapunov function candidate
\[ V(e, t) = \frac{1}{2} e^T T [M(q) 0 0 0] Te + \frac{1}{2} e^T T [\nabla e M(e, t) 0 0 0] Te \] (37)
The derivative of \( V \) with respect to \( e \) is given by
\[ \nabla e V = T [M(e, t) 0 0 K] Te + \frac{1}{2} e^T T [\nabla e M(e, t) 0 0 0] Te \] (38)
Define
\[ \mu(e, t) := B^T(e, t)\nabla e\rho(e, t) \] (39)
Reporting (37) in (38), we get
\[ \mu(e, t) = \left[ \begin{array}{c} t e \\ 0 \\ 0 \end{array} \right] + [M(q) 0 0 0] e^T \left[ \nabla e M(e, t) 0 0 0 \right] Te \rho(e, t) \] (40)
It is easy to verify that
\[ \gamma_1(\|\|) \leq V(e, t) \leq \gamma_2(\|\|) \] (41)
and that
\[ \nabla e V + \nabla e V A_T(e, t)e \leq -\gamma_3(\|\|) \] (42)
\[ \text{where} \]
\[ \lambda_1 := \lambda_{\min} \left\{ T^T \left[ \begin{array}{c} M(q) 0 \\ 0 & K \end{array} \right] T \right\} \]
\[ \lambda_2 := \lambda_{\max} \left\{ T^T \left[ \begin{array}{c} M(q) 0 \\ 0 & K \end{array} \right] T \right\} \]
\[ \lambda_3 := \min \left\{ \lambda_{\min}(K), \lambda_{\min}(T^T M(q) T), \lambda_{\min}(T^T M(q) M(q) T) \right\} \]
\[ \lambda_{\min}(\cdot) \text{ and } \lambda_{\max}(\cdot) \text{ denote respectively the minimum and maximum eigenvalues of } (\cdot). \]
\[ \text{Considering (8), Corless and Lietmann proved that the following discontinuous state feedback function} \]
\[ u_2(e, t) = \begin{cases} \mu(e, t) & \text{if } \|\mu(e, t)\| > \varepsilon \\ \frac{\mu(e, t)}{\varepsilon} \rho(e, t) & \text{if } \|\mu(e, t)\| \leq \varepsilon \end{cases} \] (43)
can be used ultimately stabilize the nonlinear uncertain dynamical system
\[ \dot{e} = f(e, t) + B_T(e, t)(w + u_1 + u_2) \] (44)
\[ \text{where } \varepsilon > 0. \text{ One can notice here the discontinuity of the robust control law when } \varepsilon \rightarrow 0. \]
To overcome the discontinuity problem related to the control (41), define the new class of control as
\[ u_2(e, t) = \begin{cases} \mu(e, t) & \text{if } \|\mu(e, t)\| > \varepsilon \phi(t) \\ \frac{\mu(e, t)}{\varepsilon \phi(t)} \rho(e, t) & \text{if } \|\mu(e, t)\| \leq \varepsilon \phi(t) \end{cases} \] (45)
where \( \phi(t) \) is class of uniformly continuous functions such that \( 0 < \phi(t) \leq 1 \) and \( \omega(t) := \int \phi(t)dt \) satisfying \( \omega(t) \leq 0 \).
\[ \text{Lemma 1} \]
The control law (42) is continuous and stabilizes asymptotically the uncertain dynamical system if there exist a Lyapunov function candidate verifying
\[ V(e, t) : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^+ \] such that
\[ i. \gamma_1(\|\|) \leq V(e, t) \leq \gamma_2(\|\|) \] \( \forall e(t) \in \mathbb{R}^n \times \mathbb{R} \)
\[ ii. V(e, t) \leq -\gamma_3(\|\|) + \gamma(\eta)\phi(t) \] \( \forall e(t) \in \mathbb{R}^n \times \mathbb{R} \)
\[ \text{where } \eta \text{ is a positive constant, } \lim_{t \rightarrow \infty} \gamma_i(t) = \infty, \]
\[ i.i. \text{ Consider the Lyapunov candidate function} \]
\[ u_1(t) = u_1(t_0) + \int_{t_0}^{t} \phi(t)dt \] (46)
\[ \text{Theorem 2} \]
Consider the uncertain dynamical system (34), the control law
\[ u = u_1 + u_2 \] (47)
and
\[ u_2 = \begin{cases} \frac{\mu(e, t)}{\varepsilon \phi(t)} \rho(e, t) & \text{if } \|\mu(e, t)\| \leq \varepsilon \phi(t) \\ \frac{\mu(e, t)}{\varepsilon \phi(t)} \rho(e, t) & \text{if } \|\mu(e, t)\| > \varepsilon \phi(t) \end{cases} \] (48)
stabilizes asymptotically and optimally system trajectories of (34).
\[ \text{Proof: To prove asymptotic stability of the uncertain system, we should prove that} \]
\[ i. \text{ every solution } e(t; e_0, t_0) \text{ is stable} \]
\[ ii. \text{ } \lim_{t \rightarrow \infty} e(t; e_0, t_0) = 0. \]
\[ \text{i. Consider the Lyapunov candidate function} \]
\[ V(e, t) = \frac{1}{2} e^T T \left[ \begin{array}{c} M(q) 0 \\ 0 & K \end{array} \right] Te \] (49)
\[ \text{the total time derivative of } V \text{ is given by} \]
\[ \frac{dV(e, t)}{dt} = \frac{\partial V(e, t)}{\partial t} + \left( \frac{\partial V(e, t)}{\partial t} \right)^T \dot{e} \] (50)
there exists a continuous positive definite function $\gamma$ such that
\[
\frac{dV(e,t)}{dt} \leq -\gamma(e) + \gamma(\eta)\varphi(t)
\]
where $\eta$ is a positive scalar constant and
\[
\gamma(e) = \lambda_{\text{max}} (Q + T^T BR^{-1} B^T T) e + \int_{t_0}^t \dot{V}(e,\tau)d\tau
\]
\[
\leq \gamma_2(\|e\|) + \int_{t_0}^t \gamma(\|x(\tau)\|)d\tau + \int_{t_0}^t \gamma(\eta)(\omega(e) - \omega(e_0))d\tau
\]
\[
\leq \gamma_2(\|e_0\|) + \int_{t_0}^t \gamma(\|x(\tau)\|)d\tau + \gamma(\eta)\omega(e_0)
\]
then for every $\varepsilon > 0$, we have $\|e\| \leq \varepsilon$ if and only if
\[
\gamma_2(\|e_0\|) + \gamma(\eta)\omega(e_0) \leq \gamma_1(\varepsilon)
\]
Since $\eta$ is a positive definite function of $e_0$, there exist $\beta(\varepsilon, t_0)$ so that (45) is verified for any $\|e_0\| \leq \beta$.

\[\text{ii. from (45), we can write}\]
\[
0 \leq \gamma_1(\|e\|) \leq \gamma_2(\|e_0\|) - \int_{t_0}^t \gamma(\|x(\tau)\|)d\tau + \gamma(\eta)\omega(e_0)
\]
and
\[
\lim_{t\to\infty} \int_{t_0}^t \gamma(\|x(\tau)\|)d\tau \leq \gamma(\|e_0\|) + \gamma(\eta)\omega(e_0) < \infty
\]
Considering the continuity of the solution and the property that $\gamma(.)$ is a continuous positive definite function, we then conclude that \[\lim_{t\to\infty} \gamma(\|e\|) = 0\]
and then \[\lim_{t\to\infty} \|e\| = 0\].

\section{A simulation example}

The robust performance of our proposed design is tested in this section on the tracking control of a two-link robot by computer simulation. The robotic system is assumed here to contain unknown parametric perturbations and external disturbance. A mixed Optimal/Robust tracking control can be then designed according to the proposed design procedure. Consider a two link manipulator described by:
\[
M(q) = \begin{bmatrix}
(m_1 + m_2)l_1^2 & m_2l_1l_2(s_1s_2 + c_1c_2) \\
 m_2l_1l_2(s_1s_2 + c_1c_2) & m_2l_2^2
\end{bmatrix}
\]
\[
C(q,\dot{q}) = m_2l_1l_2(c_1s_2 - s_1c_2) \begin{bmatrix}
0 & -\dot{q}_2 \\
 -\dot{q}_1 & 0
\end{bmatrix}
\]
\[
G(q) = \begin{bmatrix}
-(m_1 + m_2)l_1gs_1 \\
-ml_2gs_2
\end{bmatrix}
\]
Where the short-hand notations $c_1(.) = \cos(.)$ and $s_1(.) = \sin(.)$ are used. For the convenience of simulation, the nominal parameters are given as: $m_1 = 1$ (kg), $m_2 = 10$ (kg), $l_1 = 1$ (m), $l_2 = 1$ m and $g = 9.8$ (ms$^{-2}$).

The parameter perturbations are assumed to be of the following form
\[
\Delta M(q) = \begin{bmatrix}
-2 & 2(s_1s_2 + c_1c_2) \\
 -2(s_1s_2 + c_1c_2) & -2
\end{bmatrix}
\]
\[
\Delta C(q, \dot{q}) = 2(c_1s_2 - s_1c_2) \begin{bmatrix}
0 & -\dot{q}_2 \\
 -\dot{q}_1 & 0
\end{bmatrix}
\]
\[
\Delta G(q) = \begin{bmatrix}
2gs_1 \\
2gs_2
\end{bmatrix}
\]
which corresponds to a change of load from 10 (kg) to 2 (kg). The exogenous disturbances are supposed to be
\[
\tau_{d,}\text{ext} = \begin{bmatrix}
10 & 10
\end{bmatrix}^T \text{Nm}
\]
The control system is required to follow the desired trajectories
\[
q_1^d(t) = 0.55 + 1.45 \cos t \text{ (rad)}
\]
\[
q_2^d(t) = 1.1 + 0.9 \cos t \text{ (rad)}
\]
Simulation results are shown in Figs 1 to 6.

![Fig. 1: Angular position $q_1(t)$ and $q_1^d(t)$ with control (43)](image)

![Fig. 3: Angular position $q_3(t)$ and $q_3^d(t)$ with control (43)](image)
We conclude that the proposed mixed Optimal/Robust control (43) is suitable to overcome the effects due to the parameter perturbations and external disturbances to achieve the robust tracking performance in perturbative robotic control system.

6 Conclusions

A nonlinear mixed optimal/robust control has been proposed for optimal and robust tracking performance design of robotic systems under a class of parametric uncertainties and external disturbances. This nonlinear time-varying mixed optima/robust control tracking problem design must first solve a coupled nonlinear Ricatti equation. In order to avoid the difficulty of solving this Ricatti equation, an adequate state transformation and the property of skew symmetry of robotic systems have been employed to obtain an equivalent algebraic equation. A class of nonlinear time varying continuous feedback functions were introduced to solve the problem of robust control.

References