A Robust Disturbance Observer for Multivariable Systems

XINKAI CHEN*, GUISHENG Zhai**
* Department of Intelligent Systems
Kinki University
930 Nishimitani, Uchita, Naga-Gun, Wakayama 649-6493
JAPAN

** Faculty of Systems Engineering
Wakayama University
930 Sakaedani, Wakayama 640-8510
JAPAN

Abstract: - This paper discusses the disturbance estimation problem for continuous-time multivariable dynamical systems with arbitrarily relative degrees. The disturbances, which are assumed bounded, refer to the combination of the external disturbances, the nonlinearities and the model uncertainties of the system. First, the disturbance observer is proposed for multivariable minimum phase systems. Then, the proposed formulation is extended to nonminimum phase systems, where the nonminimum phase systems are approximated by minimum phase systems based on the least square approximation technique. The estimation error of the disturbances is controlled by the design parameters. Further, the accuracy of the estimation also depends on the frequency of the disturbance signal for nonminimum phase systems.

Key-Words: - Disturbance observer, multivariable systems, relative degree, nonminimum phase dynamical systems, least square approximation.

1 Introduction
In the practical control systems, the presence of disturbances, nonlinearities and model uncertainties is inevitable. It might be argued that if these uncertainties can be estimated, the control problems of the systems with uncertainties may become easy to deal with. For example, the state observer and the controller with disturbance cancellation functions can be easily constructed by using the estimated disturbances. Therefore, the problem of designing a disturbance observer for the systems with uncertainties and nonlinearities has been a topic of considerable interest recently [1,3,4,6].

For single disturbance single output (SDSO) minimum phase systems, disturbance observer techniques [3] have been popularly applied in the design of tracking controllers for motion control systems. This procedure closes an inner loop around the controlled plant to reject disturbances and force the input-output character of this inner loop to approximate a “nominal” plant model at low frequencies. Tuning of the loop is accomplished through adjustment of a low pass filter. Since the plant approximates a nominal model at low frequencies, overall closed loop dynamics are usually well known and feedforward techniques are often applied [3].

From the point of view of robust control, the desirable properties of variable structure control systems are well documented in [4,6]. It should be pointed out that the equivalent control method is very effective for estimating the unknown parts of a plant with known relative degrees [4,6]. For SDSO minimum phase dynamical systems with arbitrarily relative degrees, a new formulation, which is motivated by the variable structure control techniques, is proposed in [1] to estimate the disturbance even though the upper and lower bounds of the disturbance are unknown.

It should be pointed out that the above mentioned results are restricted to SDSO minimum phase systems. And no result has been reported until now about the disturbance observation problems for multivariable systems even though most of the practical control systems are multivariable systems, such as the robot systems, etc. Further, no results has been reported until now about the disturbance observation problems for nonminimum phase systems even though many practical control systems are nonminimum phase, such
as the vibration and position control systems for the flexible arms, the vibration suppression control systems for DC or SR motors, etc. Since the inverse system (from the output to the disturbance) of a nonminimum phase system is unstable, the disturbance observer formulations proposed in [1,3] for minimum phase systems cannot be extended to nonminimum phase systems.

This paper tries to approximate the nonminimum phase systems by minimum phase systems based on the least square approximation method, i.e. inverse. In this paper, we consider the disturbance observation problem for multivariable systems with arbitrarily relative degrees. First, the proposed formulation in [1] is extended to multivariable minimum phase systems (with respect to the relation between the disturbance and the output). Then, the formulation is extended to nonminimum phase multivariable systems. For this purpose, first, nonminimum phase systems are approximated by minimum phase systems based on the least square approximation method; then, the formulation for multivariable minimum phase systems is applied.

This paper is organized as follows. Section 2 gives the problem formulation. In section 3, the disturbance observer for multivariable minimum phase systems is formulated. In section 4, the disturbance observer is extended to multivariable nonminimum phase systems. Section 5 concludes this paper.

2 Problem Formulation
Consider the uncertain dynamical system described by

\[ A(s)y(t) = K(s)v(t) \]  

with

\[ A(s) = \begin{bmatrix} s^n + a_{11}(s) & \cdots & a_{1p}(s) \\ \vdots & \ddots & \vdots \\ a_{p1}(s) & \cdots & s^n + a_{pp}(s) \end{bmatrix}, \]

\[ K(s) = \begin{bmatrix} k_{11}(s) & \cdots & k_{1m}(s) \\ \vdots & \ddots & \vdots \\ k_{p1}(s) & \cdots & k_{pm}(s) \end{bmatrix}, \]

and

\[ y(t) = \begin{bmatrix} y^{(1)}(t) \\ \vdots \\ y^{(m)}(t) \end{bmatrix}, \quad v(t) = \begin{bmatrix} v^{(1)}(t) \\ \vdots \\ v^{(m)}(t) \end{bmatrix}. \]

where \( y^{(i)}(t) \) ( \( i = 1, \ldots, p \) ) are the outputs; \( v^{(i)}(t) \) ( \( i = 1, \ldots, m \) ) are the unknown signals composed of the disturbances, the model uncertainties and the nonlinear parts of the system; \( p, m \) and \( n_i \) ( \( i = 1, \ldots, p \) ) are known positive integers; \( a_{ij}(s) \) and \( k_{ij}(s) \) are known at most \( (n_i - 1) \)-th order polynomials; \( s \) denotes the differential operator.

For simplicity, we call the signals \( v^{(i)}(t) \) ( \( i = 1, \ldots, m \) ) the disturbances of the system.

In this paper, the following assumptions are made.

(A1) \( p \geq m \).

(A2) The poles of the system (1) are stable.

(A3) The signals \( v^{(i)}(t) \) are bounded. However, the upper bound of \( |v^{(i)}(t)| \) is unknown. The disturbances \( v^{(i)}(t) \) are piecewise differentiable, and their first order derivatives (at the undifferentiable points, we mean the right- and left-hand derivatives) are bounded.

Remark 1: Assumption (A1) is essential to solve the problem. From Assumptions (A2) and (A3), it can be seen that \( y(t) \) is bounded. The boundedness of the signal \( y(t) \) is employed in the proposed algorithm.

For simplicity, it is assumed that \( m = p \) in the following analysis of the paper.

Let \( k(s) = \det\{K(s)\} \) and \( q = \deg\{k(s)\} \). Suppose

\[ k(s) = k_0 s^q + k_1 s^{q-1} + \cdots + k_q, \]

where \( k_0 \neq 0 \). It can be easily concluded that

\[ q \leq n_1 - 1 + n_2 - 1 + \cdots + n_p - 1 = \sum_{i=1}^p n_i - p. \]

The aim of this research is to estimate the disturbances by using the measurement of the outputs even though \( k(s) \) may not be a Hurwitz polynomial. Please, leave two blank lines between successive sections as here.

3 Disturbance observer for multivariable minimum phase systems
In this section, we make the following assumption about \( k(s) \).

(A4): \( k(s) \) is a Hurwitz polynomial.

To begin with, multiplying the both sides of (1) by \( \text{adj}(K(s)) \) yields

\[ \left\{\text{adj}[K(s)]\right\}A(s)y(t) = k(s)\begin{bmatrix} v^{(1)}(t) \\ \vdots \\ v^{(m)}(t) \end{bmatrix}. \]  

Now, rewrite equation (6) as

\[ \text{adj}[K(s)]A(s)y(t) = k(s)\begin{bmatrix} v^{(1)}(t) \\ \vdots \\ v^{(m)}(t) \end{bmatrix}. \]
where, in the i-th equation, $\sigma^{(i)}$ is a row vector whose entries are constants, $\Phi^{(i)}(s)$ is a row vector whose entries are at most $(l_i - 1)$-th order polynomials of $s$. Because $A(s)$ and $K(s)$ are known, $\sigma^{(i)}$, $\Phi^{(i)}(s)$ and $k(s)$ can be calculated. Here, $l_i - q$ can be regarded as the “relative degree” (with respect to the relation between $\sigma^{(i)}(y(t))$ and $v^{(i)}(t)$) of the i-th equation in (7).

For simplicity, let $l_i - q = \eta_i$. From now on, we will estimate $v^{(i)}(t)$ based on the i-th equation in (7) by using the formulation proposed in [1]. By introducing an $l_i$-th order monic Hurwitz polynomial

$$f^{(i)}(s) = \frac{1}{k_0}k(s)(s + \lambda_i)^{\eta_i},$$

the i-th equation in (7) can be rewritten as

$$\dot{z}^{(i)}(t) + \lambda z^{(i)}(t) = L^{(i)}(y(t)) + \frac{k_0}{(s + \lambda_i)^{\eta_i}} v^{(i)}(t),$$

where $\lambda_i$ is a positive constant; $z^{(i)}(t)$ and $L^{(i)}(y(t))$ are defined as

$$z^{(i)}(t) = \sigma^{(i)} y(t),$$

$$L^{(i)}(y(t)) = (s + \lambda_i)\left\{\frac{f^{(i)}(s) - \frac{1}{k_0}k(s)}{f^{(i)}(s)} \{\sigma^{(i)} y(t)\} + \frac{\Phi^{(i)}(s)}{f^{(i)}(s)} y(t)\right\}.$$

**Remark 2:** It should be pointed out that $z^{(i)}(t)$ and $L^{(i)}(y(t))$ are computable signals.

By assumption (A2), it can be seen that

$$\frac{1}{(s + \lambda_i)^{\eta_i}} v^{(i)}(t)$$

are also bounded for any non-negative integer $j$, i.e. there exist positive constants $C^{(i)}_j > 0$ such that

$$\frac{1}{(s + \lambda_i)^{\eta_i}} v^{(i)}(t) \leq C^{(i)}_j,$$

where $C^{(i)}_j > 0$ are unknown.

By mimicking the formulation in [1], we have the next theorem to estimate the signal $v^{(i)}(t)$ based on equation (9).

**Theorem 1.** Construct the differential equations

$$\dot{z}^{(i)}(t) + \lambda z^{(i)}(t) = L^{(i)}(y(t)) + k(s)w^{(i)}(t),$$

$$\dot{w}^{(i)}_{\mu - 1}(t) + \lambda w^{(i)}_{\mu - 1}(t) = w^{(i)}_{\mu}(t),$$

where $z^{(i)}(t)$ (with $\dot{z}^{(i)}(t_0) = z^{(i)}(t_0)$) and $w^{(i)}_{\mu - 1}(t)$ (with $\dot{w}^{(i)}_{\mu - 1}(t_0) = 0$) $(1 < \mu \leq \eta_i)$ are the variables which can be obtained by respectively solving (10) and (11), $w^{(i)}_1(t)$ and $w^{(i)}_{\mu}(t)$ are the inputs described by

$$w^{(i)}_1(t) = \hat{C}^{(i)}_{\eta_i - 1}(t) - \frac{k_0}{k_0} \left\{z^{(i)}(t) - z^{(i)}(t_0) \right\} + \delta^{(i)}_{\eta_i - 1},$$

$$w^{(i)}_{\mu}(t) = \hat{C}^{(i)}_{\eta_i - \mu}(t) - \frac{k_0}{k_0} \left\{\dot{z}^{(i)}(t) - \dot{z}^{(i)}(t_0) \right\} + \delta^{(i)}_{\eta_i - \mu}.$$

and $\delta^{(i)}_{\eta_i - \mu}$ are small positive design parameters; $\hat{C}^{(i)}_{\eta_i - \mu}(t)$ are updated by the following adaptive algorithms

$$\hat{C}^{(i)}_{\eta_i - \mu}(t) = \begin{cases} 0 & \text{if } |z^{(i)}(t) - z^{(i)}(t_0)| > 2\delta^{(i)}_{\eta_i - \mu}, \\ \frac{k_0}{k_0} \left\{w^{(i)}_{\mu - 1}(t) - \dot{w}^{(i)}_{\mu - 1}(t) \right\} + \delta^{(i)}_{\eta_i - \mu} & \text{otherwise} \end{cases}$$

$\hat{C}^{(i)}_{\eta_i - \mu}(t_0)$ $(1 \leq \mu \leq \eta_i)$ can be chosen as any small positive constants, $\hat{C}^{(i)}_{\eta_i - \mu}$ are positive constants for $i = 1, \cdots, p$. Then, all the generated signals $\dot{z}^{(i)}(t)$, $\dot{w}^{(i)}_{\mu - 1}(t)$ $(1 < \mu \leq \eta_i)$, $w^{(i)}_1(t)$ $(1 \leq \mu \leq \eta_i)$ and $\hat{C}^{(i)}_{\eta_i - \mu}(t)$ $(1 \leq \mu \leq \eta_i)$ are uniformly bounded for $i = 1, \cdots, p$. Further, when $\sum_{j=1}^\mu \delta^{(i)}_j$ is very small, $w^{(i)}_{\mu}(t)$ are the corresponding approximate estimates of $\frac{1}{(s + \lambda_i)^{\eta_i - \mu}} v^{(i)}(t)$ for $1 \leq \mu \leq \eta_i$ as $t$ is large enough, i.e. there exist $T^{(i)}_{\mu_i} > t_0$ and $e^{(i)}_{\mu_i} (\delta^{(i)}_1, \cdots, \delta^{(i)}_\mu) > 0$ such that

$$\frac{1}{(s + \lambda_i)^{\eta_i - \mu}} v^{(i)}(t) - w^{(i)}_{\mu}(t) < e^{(i)}_{\mu_i} (\delta^{(i)}_1, \cdots, \delta^{(i)}_\mu)$$

for all $t > T^{(i)}_{\mu_i}$, where $e^{(i)}_{\mu_i} (\delta^{(i)}_1, \cdots, \delta^{(i)}_\mu) \to 0$ as $\sum_{j=1}^\mu \delta^{(i)}_j \to 0$. Particularly, $w^{(i)}_{\eta_i}(t)$ are the corresponding estimates of the disturbances.
Proof: See [1].

Remark 3: The choice of the parameters is same as that in [1]. For the $i$-th equation in (7), it may not be a minimal realization about the relation between $x^{(i)}y(t)$ and $v^{(i)}(t)$. However, the relative degree $\eta_i$ is definite. So, the estimation step is definite. By observing the proposed algorithm, it can be concluded that it is not the essential problem whether the $i$-th equation in (7) is a minimal realization or not.

4 Extension to nonminimum phase multivariable

For nonminimum phase systems, we will find minimum phase systems to approximate them. In the following, the least square approximation method is employed to get an approximate inverse system.

4.1 Inverse systems

To begin with, we express $k(s)$ as

$$k(s) = k_0k_1(s)k_2(s),$$

where $\kappa_1(s)$ is a $u$-th order monic polynomial with no root lying in the left half plane, $\kappa_2(s)$ is an $(q-u)$-th order monic Hurwitz polynomial.

In the following, an inverse system for $\kappa_1(s)$ is derived based on the least square approximation method.

Now, we express $\kappa_2(s)$ as

$$\kappa_2(s) = (s - \varphi_1)\cdots(s - \varphi_r).$$

where $\tau + 2i = \varphi_i$ ($i = 1, \cdots, \tau$) are real numbers satisfying $\varphi_i \geq 0$; $\rho_j$ ($j = 1, \cdots, \tau$) are complex numbers satisfying $\text{Re}(\rho_j) \geq 0$; $\rho_j^*$ denotes the corresponding complex conjugate of $\rho_j$ for $j = 1, \cdots, \tau$.

First, we consider the approximate inverse system of $s - \rho$, where $\rho \in \mathbb{C}$ ($\mathbb{C}$ denotes the set of complex numbers), $\text{Re}(\rho) \geq 0$. The problem is to find a polynomial $c(s)$ defined as

$$c(s) = s^r + c_{r-1}s^{r-1} + \cdots + c_1s + c_0$$

such that the absolute values of the coefficients of the polynomial $(s - \rho)c(s) - (s + \beta)^r$ are as small as possible, where $\beta \in \mathbb{C}$ with $\text{Re}(\beta) > 0$ and $r$ is a positive integer. $\beta$ and $\gamma$ can be assigned in advance. Let

$$(s - \rho)c(s) - (s + \beta)^r = e_0s^r + e_1s^{r-1} + \cdots + e_{r-1}s + e_r.$$

Thus, the considered problem is that we try to find the polynomial $c(s)$ minimizing the following criterion

$$J = \sum_{i=0}^r |e_i|^2.$$  \hspace{1cm} (16)

Define

$$s + \beta)^r = s^{r+1} + l_1s^r + \cdots + l_{r+1}s + l_{r+1},$$

where $l_k$ are real numbers satisfying $0 \leq l_k \leq 1$. For nonminimum phase systems, we will find coefficients of the polynomial $c(s)$ such that the absolute values of the coefficients of the polynomial $(s - \rho)c(s) - (s + \beta)^r$ are as small as possible, where $\beta \in \mathbb{C}$ with $\text{Re}(\beta) > 0$ and $r$ is a positive integer. $\beta$ and $\gamma$ can be assigned in advance. Let

$$(s - \rho)c(s) - (s + \beta)^r = e_0s^r + e_1s^{r-1} + \cdots + e_{r-1}s + e_r.$$
\( \{s + \beta \}(s + \beta^*) \) is the approximate of 
\((s - \rho)\)s - \rho^* e(s) e^*(s) \), where \( e^*(s) \) is defined as 
\[ e^*(s) = s^\tau + c_1 s^\tau-1 + \cdots + c_{\tau}s + c_\tau. \] 
(25)

Now, we consider all the factors of \( \kappa_i(s) \).

Introduce the next polynomial 
\[ \xi(s) = \left\{ \prod_{i=1}^\tau (s + \chi_i) \right\} \left\{ \prod_{j=1}^\tau (s + \beta_j)(s + \beta^*_j) \right\} \] 
(26)

where \( \gamma \) is a positive integer, \( \chi_i \) (\( i = 1, \cdots, \tau \)) are positive real numbers, \( \beta_j \) (\( j = 1, \cdots, \tau \)) are complex numbers. Let 
\[ (s + \chi_i)^{\tau_i} = s^{\tau_i} + g_i s^{\tau_i-1} + \cdots + g_{i,\gamma_i} s + g_{i,\gamma_i+1}, \] 
(27)
\[ (s + \beta_j)^{\tau_j} = s^{\tau_j} + l_{j,\gamma_j} s^{\tau_j-1} + \cdots + l_{j,\gamma_j+1}, \] 
(28)

Further, we introduce the following polynomials 
\[ \theta_j(s) = s^{\tau_j} + \theta_{j,\gamma_j}s^{\tau_j-1} + \cdots + \theta_{j,\gamma_j+1}, \] 
(29)
\[ \hat{\theta}_j(s) = s^{\tau_j}, \] 
(30)

The coefficients of \( \theta_j(s) \) and \( \hat{\theta}_j(s) \) are determined by 
\[ \theta_i = (N_i^T N_i)^{-1} N_i^T g_i, \quad \hat{\theta}_j = (K_j^* K_j)^{-1} N_j^T l_j, \] 
(31)

where the matrices \( N_i, \theta_i, g_i, \) and \( K_j, \hat{\theta}_j, l_j \), for \( i = 1, \cdots, \tau \) and \( j = 1, \cdots, \tau \) are similarly defined as those in (17)-(19).

Define 
\[ \zeta(s) = \left\{ \prod_{i=1}^\tau \theta_i(s) \right\} \left\{ \prod_{j=1}^\tau \hat{\theta}_j(s) \theta_j(s) \right\} \] 
(32)

and 
\[ \bar{v}^{(i)}(t) = \frac{\kappa_i(s) \zeta(s)}{\xi(s)} v^{(i)}(t). \] 
(33)

**Remark 5:** It should be pointed out that \( \xi(s) \) is a \((\gamma + 1)\)th order monic Hurwitz polynomial with real coefficients, and \( \zeta(s) \) is a \( \gamma \)th order monic polynomial with real coefficients.

We have the next theorem to state the relation between \( v^{(i)}(t) \) and \( \bar{v}^{(i)}(t) \).

**Theorem 2.** For the signal \( \bar{v}^{(i)}(t) \) generated in (33), it gives 
\[ v^{(i)}(t) - \bar{v}^{(i)}(t) \to 0 \] 
as \( t \to \infty \).

**Proof:** Since \( v^{(i)}(t) - \bar{v}^{(i)}(t) \) and \( \frac{d}{dt} \left( v^{(i)}(t) - \bar{v}^{(i)}(t) \right) \) are bounded, it is well known that, if the \( L_2 \) norm \[ \| v^{(i)}(t) - \bar{v}^{(i)}(t) \|_2 \] exists, then \( v^{(i)}(t) - \bar{v}^{(i)}(t) \to 0 \) as \( t \to \infty \). Thus, the remaining task is to prove that 
\[ \int_0^\infty \left( v^{(i)}(t) - \bar{v}^{(i)}(t) \right)^2 dt \] exists. It is well known that it is equivalent to consider the convergence of the following \( H_2 \) norm 
\[ \frac{1}{\xi(s)} \int_0^\infty \left[ (\zeta(j\omega) - \kappa_i(j\omega)\zeta(j\omega)) \right]^2 \omega \] 
(35)

where \( V^{(i)}(s) \) is the Laplace transform of \( v^{(i)}(t) \).

Since \( v^{(i)}(t) \) is uniformly bounded, it can be seen that, if the integral 
\[ \int_0^\infty \left[ (\zeta(j\omega) - \kappa_i(j\omega)\zeta(j\omega)) \right]^2 \omega d\omega \] 
(36)
is convergent, then (35) is also convergent.

On the other hand, the convergence of the integral (36) can be easily checked by using the fact that 
\[ \frac{\zeta(s) - \kappa_i(s)\zeta(s)}{\xi(s)} \] is strictly proper. Thus, the theorem is proved.

**Remark 6:** For the imaginary parts \( \text{Im}(\beta_j) \) of the parameters \( \beta_j \) (\( j = 1, \cdots, \tau \)), they should be chosen as 
\( \text{Im}(\beta_j) = -\text{Im}(\rho_j) \). The parameters \( \gamma, \chi_i \) (\( i = 1, \cdots, \tau \)) and \( \text{Re}(\beta_j) \) (\( j = 1, \cdots, \tau \)) should minimize the integral (36).

**Remark 7:** If \( \kappa_i(s) \) contains only one unstable root, say \( \phi_i \) (Thus, \( \phi_i \) is a positive real number). By Lemma 1, the integral (36) gives 
\[ \frac{\phi_i^2 - 1}{\phi_i^{2(\gamma + 1)} - 1} \] 
(37)
with \( \Delta_\gamma = \begin{cases} 1 \quad \text{as } \gamma = 4z + 1 \\ -1 \quad \text{as } \gamma = 4z + 3 \quad \text{or } 4z \end{cases} \)
for any non-negative integer. Thus, \( \gamma \) and \( \chi_i \) can be determined such that (37) is minimal.

**Remark 8:** If \( \kappa_i(s) \) contains only two unstable complex conjugate roots \( \rho \) and \( \rho^* \), we express...
\[
\left\{ (s + \beta)(s + \beta^*) \right\}^{-1} - (s - \rho)(s - \rho^*)c(s)c^*(s) \\
= (s + \beta^*)^{-1} \left\{ (s + \beta)(s + \beta^*)^{-1} - (s + \rho)c(s) \right\} \\
+ (s - \rho)c(s) \left\{ (s + \beta^*)^{-1} - (s - \rho^*)c^*(s) \right\}
\]
\[
= \left| \frac{\rho}{\rho^2 + 1} \right|^2 \left\{ (s + \beta + \rho^*)^{-1} \left\{ \rho x + \ldots + \rho s + 1 \right\} \right\} \\
+ \left\{ (s + \beta)^{-1} - \left| \frac{\rho}{\rho^2 + 1} \right|^2 \left\{ (s + \beta + \rho^*)^{-1} \left\{ \rho x + \ldots + \rho s + 1 \right\} \right\} \right\}.
\]

Since \(\text{Im}(\beta) = -\text{Im}(\rho)\), the integral (37) becomes a function of \(\gamma\) and \(\text{Re}(\beta)\). Then \(\gamma\) and \(\text{Re}(\beta)\) can be determined such that the integral (37) is minimal.

**Remark 9:** For general \(\kappa_i(s)\) in (14), the parameters \(\gamma, \chi_i (i = 1, \ldots, \tau)\) and \(\text{Re}(\beta_j) (j = 1, \ldots, 1)\) can be similarly determined such that the integral (36) is minimal. In general, the parameter \(\gamma\) should not be chosen very large (if there are many solutions of \(\gamma\) ), since a very large \(\gamma\) may result in complicated computation, slow and long transients, etc.

**Remark 10:** The difference between \(v^{(i)}(t)\) and \(v^{(i)}(t)\) also depends on the frequency of the disturbance signal \(v^{(i)}(t)\).

### 4.2 The disturbance observer for nonminimum phase systems

To begin with, multiplying the both sides of (1) with \(\zeta(s) \cdot \text{adj}(K(s))\) yields
\[
\zeta(s) \cdot \text{adj}(K(s)) A(s) y(t) = k_0 \kappa_2(s) \zeta(s) \left[ v^{(i)}(t) \ldots v^{(i)}(t) \right]^T
\]
(38)

By the definition of \(v^{(i)}(t)\), it can be easily seen that \(v^{(i)}(t)\) and their first order derivatives are bounded. Since \(\kappa_2(s) \zeta(s)\) is a Hurwitz polynomial, the signals \(v^{(i)}(t)\) can be estimated by a procedure stated in Theorem 1. Further, by Theorem 2, it can be seen that the estimate of \(v^{(i)}(t)\) can be approximately regarded as the estimate of \(v^{(i)}(t)\) as \(t\) is large enough.

**Remark 11:** It should be pointed out that the relative degree does not change during the introduction of inverse system. Thus, the number of the estimating steps has no relation with the parameter \(\gamma\).

### 5 Conclusion

In this paper, the robust disturbance observer proposed in [1] is extended to multivariable systems with arbitrarily relative degrees. The term “disturbance” refers to the combination of the model uncertainties, the nonlinear parts of the system, the external disturbances, etc. The disturbance is assumed bounded, but the bounds are unknown. First, the disturbance observer is formulated for multivariable minimum phase systems with arbitrarily relative degrees. Then, the proposed formulation is extended to nonminimum phase systems, where the nonminimum phase systems are approximated by minimum phase systems based on the least square approximation technique. The estimation error of the disturbance can be controlled by choosing the design parameters. Further, the accuracy of the estimation of the disturbance also depends on the frequency of the disturbance for nonminimum phase systems.

A state observer can be easily generated by employing the estimate of the disturbance for multivariable dynamical systems. However, until now, the state observer can be constructed only for the minimum phase systems with relative degree one [4]. Further, the estimated disturbances can be applied to a controller to cancel the influence generated by the disturbances.

References: