A Novel Methodology of Working Capital Management for Large Public Constructions by Using Fuzzy S-curve Regression

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Abstract

In the contracting business, construction firms are generally more concerned with short-term financial strategies than the long-term ones. Working capital management is the central issue of all short-term financial concerns. More importantly, cash management is ultimate goal for achieving high liquidity and profitability. This study focuses on the cash portion of working capital management by using the concept of target cash balance. The aim is to develop a practical model for construction firms in Taiwan for rationalizing the amount of cash and current assets, which should be possessed in any point of time. The Fuzzy S-curve regression is introduced here for understanding the issues involved. Based on the project cash flow and progress payment records of an example project taken from Housing and Urban Development Bureau, Taiwan Provincial Government, this model is demonstrated and tentative conclusions concerning the model are given.

Key words: Fuzzy S-curve regression, working capital management, cash flow.

Introduction

In the present-day research of complex systems, such as engineering technology, environment and societal economy, become large in dimension and complexity so that the exact numerical data can not be obtained for the information of systems. To solve the problems arising from complex systems may become very inefficient or even impossible if using the traditional

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mathematical tools that are not constructed for dealing with high dimensionality models. Similarly, the traditional least square regression may not be applicable when dealing with curve fitting problems. In the past twenty years, some approaches containing fuzzy information have been attracting increasingly attention, as proposed in the literature [1-4].

Tanaka et al. [1] develop a fuzzy linear regression model by using linear programming techniques in 1982. In 1988, Diamond [2] resembled traditional least squares regression to establish fuzzy linear least squares models. Ruoning [3-4] considered the rationality of metric definition and discussed the problem for least squares fitting of fuzzy-value data, which are expressed as fuzzy numbers, and to develop an S-shaped curve regression model for fitting this type of data.

The characteristics of large public constructions are large-scaled, time-limited, high cost, and complex-technical etc., and there are many uncertain factors in which. Therefore, to perform this kinda project is difficult, especially for program and dispatch of working capital. In order to overcome the problem of controlling projects, the S-curves are widely used. They are valuable to project management in reporting current status and prediction the future of projects. Therefore, the S-type distribution is very suitable in regression on construction management and social economy etc. However, as far as we know, the working capital management for large public constructions by using fuzzy S-curve regression remains unresolved yet.

This study is organized as follows. First, classic S-curve theory is recalled. Then, based on fuzzy set theory and fuzzy inference engine as well as center of gravity defuzzification, a fuzzy S-curve is obtained for curve fitting problems. Finally, a numerical example with simulations is given to demonstrate the methodology, and the conclusions are drawn.

Classic S-Curve Theory

In biology and social economy, an S-shaped cure is often used to reflect the phenomena. It means that the trend of growth gets slow first and finally saturation rapidly. In practical problem of constructions, contractors' budgets are often performed on an overall basis. Changes in strategies and mix of contracts are ver difficult to evaluate on such a basis [5]. Therefore, the principle of simulation with tools of computer was proposed to generate possible scenarios based on the specified strategies and the expected environment. The relationship between budgets and time limit for a project can be represented via S-curve fitting. A typical S-curve figure is shown in Fig. 1. The x-axis and y-axis denote project duration and complete progress, respectively.

Miskawi [6] proposed an S-curve equation which can be used in a variety of applications related to project control. The S-curve model is of the following form:

$$P = \frac{3^T}{2}\sin\left[\frac{\pi(1-T)}{2}\right]\sin(\pi T)\log\left(\frac{T+(1.5-T_p)}{T_p+T}\right) - 2T^3 + 3T^2 \tag{1}$$

where P denotes percentage completion of a project or an activity; T denotes time at any point of the duration of a project or an activity; T_P is shape factor.

Fig. 2 is plotted with various values of T_P between T = 0 and T = 100% duration and the envelope of curves for T_P = 0 and T_P = 100% in Eq. (1).

Here we suppose we can exactly get all observed data taking part in the problems, but, actually, we may not know exact values, rather some approximation. For this reason, the traditional fitting method may not be quite suitable. Before introducing fuzzy S-curve regression, we give some relative definitions and conclusions in the following.

Fuzzy Set Theory

Definition 1: Let R is a real number set. A fuzzy set $\tilde{A}$ on R is said to be a fuzzy number if the following conditions are satisfied:

(1) $\exists x_0 \in R$, such that $\mu_{\tilde{A}}(x_0) = 1$; and membership function $\mu_{\tilde{A}}(x)$ is piecewise continuous; and

(2) $\forall \alpha \in (0,1]$, $A_\alpha \equiv \{x \mid \mu_{\tilde{A}}(x) \geq \alpha, x \in R\}$ is a convex set on R.

Where x_0 is the mean value of $\tilde{A}$ and A_α is a crisp set. The convex set means that $\forall\, x \in [x_1, x_2],\quad f(x) \ge \min(f(x_1), f(x_2))$.

Definition 2: A fuzzy number $\tilde{A}$ is said to be bounded if supp($\tilde{A}$) $\equiv \{x \mid \mu_{\tilde{A}}(x) > 0\}$ is a bounded set.

Where supp($\tilde{A}$) is a crisp set.

Evidently for any $\forall\, \alpha \in (0, \alpha]$ the α-level set, $\tilde{A}_\alpha$, will be expressed as a closed interval $[p, q]$. Based on the fuzzy extension principle [7], linear operations about closed intervals are obtained as follows:

Lemma 1: Let $[a, \mathrm{b}]$, $[d, \mathrm{e}]$ be closed intervals of real number. Then

$$[a, \mathrm{b}] + [d, \mathrm{e}] = [a + d, b + \mathrm{e}]; \quad [a, \mathrm{b}] - [d, \mathrm{e}] = [a - e, b - \mathrm{d}], \tag{2}$$

$$[a, \mathrm{b}] \cdot [d, \mathrm{e}] = [\min(ad, ae, bd, be), \max(ad, ae, bd, be)]; \tag{3}$$

$$[a, \mathrm{b}] / [d, \mathrm{e}] = [a, \mathrm{b}] \cdot [1/e, 1/d] = [\min(a/d, a/e, b/d, b/e), \max(a/d, a/e, b/d, b/e)]. \tag{4}$$

Remark 1: Given any operations which have commutative and associative characteristics, the operations of extension still have these characters.

From the theory of α-level described above and decomposition theorem [8], we have

$$(A * B)_\alpha \equiv A_\alpha * B_\alpha \tag{5}$$

$$A * B \equiv \bigcup_{\alpha \in (0,1]} (A * B)_\alpha \tag{6}$$

where $*$ denotes any arithmetic operation; A and B are fuzzy numbers and $A * B$ will be a fuzzy number.

Remark 2: Wang et al. [9] proposed that the resultant fuzzy number is the same type as the original fuzzy numbers after the operation of addition or subtraction.

Definition 3: Extended Operations for LR-Representation of Fuzzy Sets

A fuzzy number $\tilde{A}$ is LR-type, if there exists positive constants $\beta > 0$, $\gamma > 0$ and

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\dfrac{m-x}{\beta}\right) & \text{for } x \le m \\ R\left(\dfrac{x-m}{\gamma}\right) & \text{for } x \ge m \end{cases} \tag{7}$$

where m , a real number, is mean value of $\tilde{A}$; β, γ denote left spread and right spread, respectively; moreover, $\tilde{A}$ could be represented as $(m,\beta,\gamma)_{LR}$.

Lemma 2: Given two LR-type fuzzy numbers $\tilde{A}$ and $\tilde{B}$, we have

$$(m,\beta,\gamma)_{LR} + (n,\delta,\eta)_{LR} = (m+n, \beta+\delta, \gamma+\eta)_{LR} \tag{8}$$

$$(m,\beta,\gamma)_{LR} - (n,\delta,\eta)_{LR} = (m-n, \beta+\eta, \gamma+\delta)_{LR} \tag{9}$$

S-curve via Fuzzy Inference Engine

A fuzzy inference is described by a set of fuzzy IF-THEN rules in the following form:

Rule *i*: IF x_{11} is $\tilde{A}_{1i1}$, y_{11} is $\tilde{B}_{1i1}$ and $\cdots$ and x_{ng} is $\tilde{A}_{nig}$, y_{ng} is $\tilde{B}_{nig}$

$$\text{THEN } Y = a_{ik} x_i^k + b_{ik} \tag{10}$$

where *n* points $(x_1, y_1) \sim (x_n, y_n)$ and *k* order curve fitting is adopted and r is the number of IF-THEN rules for $i = 1, 2, \cdots, r$; $\tilde{A}_{nip}$ and $\tilde{B}_{nip}$ ($p = 1, 2, \cdots, g$) are the LR-type fuzzy sets, and $x_{11} \sim x_{ng}$ as well as $y_{11} \sim y_{ng}$ are the premise variables. Using the center of gravity defuzzification, product inference, and single fuzzifier, the final output is inferred as follows:

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} w_i(t)[a_{ik} x_i^k + b_{ik}]}{\sum_{i=1}^{r} w_i(t)} = \sum_{i=1}^{r} h_i(t)(a_{ik} x_i^k + b_{ik}) \tag{11}$$

It is assumed that $w_i(t) \ge 0$, $i = 1, 2, \cdots, r$; $\sum_{i=1}^{r} w_i(t) > 0$. Therefore, $h_i(t) \ge 0$ and $\sum_{i=1}^{r} h_i(t) = 1$ for all *t*.

Example

To illustrate the procedure of this fuzzy regression model, consider the following example project, which is taken from Housing and Urban Development Bureau, Taiwan Provincial Government. The data include seven metro bids of valuation and the mean scale and the mean .of time limit for a project are 2.7 billions and 6 years or so, respectively. From the data which are normalized and represented by percentage first, it is easily known that the first time of evaluation 4.5% of total work time. The observed data are given in table 1, where the data $M = (X_i, Y_i, u, v)$ are all triangular fuzzy numbers, and $u = 10\ \%$, $v = 10\ \%$ are the left and right spreads respectively. Fig.3 is the valuation data of seven metro bids and Fig. 4 is plotted by the proposed fuzzy regression.

Moreover, from Fig.4, the simulation results show that $C = -12.93T^6 + 42.07T^5 - 49.49T^4 + 23.07T^3 - 1.98T^2 + 0.25T$ by fuzzy regression approach, where C denotes the percentage of completeness (%), and T is time (%).

Conclusions

The least-squares method can usually be used to the problems of curve fitting, but when the observed data are not obtained exactly it may not be suitable. Therefore, we propose here a Fuzzy S-curve regression method here for understanding the issues involved. The aim finally is to develop a practical model for construction firms in Taiwan for rationalizing the amount of cash and current assets, which should be possessed in any point of time.

Furthermore, from the simulation results shown is Figs. 3-4, there are somehow discrete and delayed situation concerning the data. The progress of the former 30 % of time limit for a project is a bit slow. In addition, the first evaluation time of total work is 4.5% when a contractor proposes. It implies that we have to notice the delayed phenomenon of cash flow and maintain the liquidity of cash in some degree in the control and management of cash to verify the project can be finished smoothly and successfully.

References

[1] H. Tanaka, S Uejima and K. Asai, Linear regression analysis with fuzzy model, IEEE Trans. SMCB. 12 (1982) 903-907.

[2] P. Diamond, Fuzzy least squares, Inform. Sci. 46 (1988) 141-157.

[3] Xu Ruoning, A linear regression model in fuzzy environment, Adv. Modelling Simulation 27 (1991) 31-40.

[4] Xu Ruoning, S – curve regression model in fuzzy environment, Fuzzy sets and Syst. 90 (1997) 317-326.

[5] Kaka, A., P., "Contractors' financial budgeting using computer simulation," *Construction Management and Economics,* 12, pp.113-124, 1994.

[6] Miskawi, Z., "An S-curve equation for project control," *Construction Management and Economics,* 7, pp.115-124, 1989.

[7] L. A. Zadeh, Fuzzy sets, Inform. and Control 8 (1965) 338-353.

[8] G. J. Klir, B. Yuan, Fuzzy sets and fuzzy logic theory and applications, Prentic-Hall, Englewood Cliffs, NJ, 1995.

[9] Wen-June Wang and C.H. Chiu, "Entropy variation on the fuzzy numbers with arithmetic operations," *Fuzzy Sets and Systems*, vol.103, no.3, pp. 443-456, May 1999.

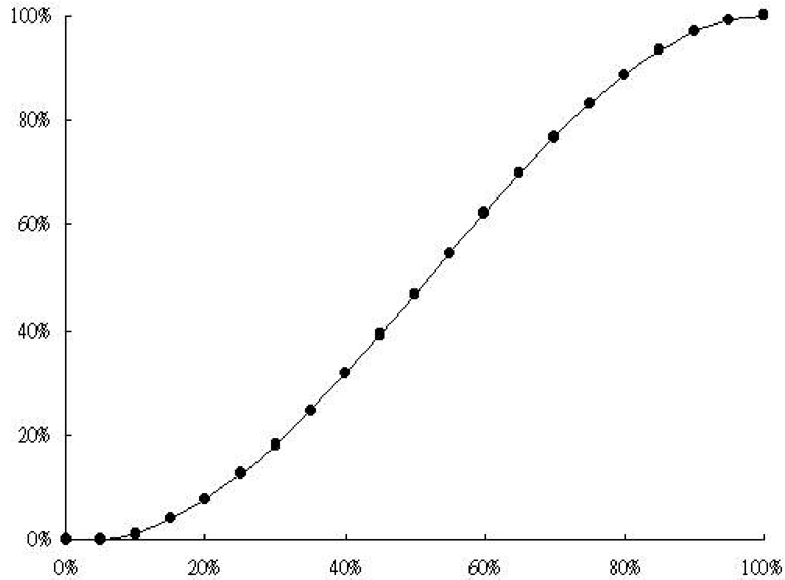

Fig.1. Typical S-curve figure

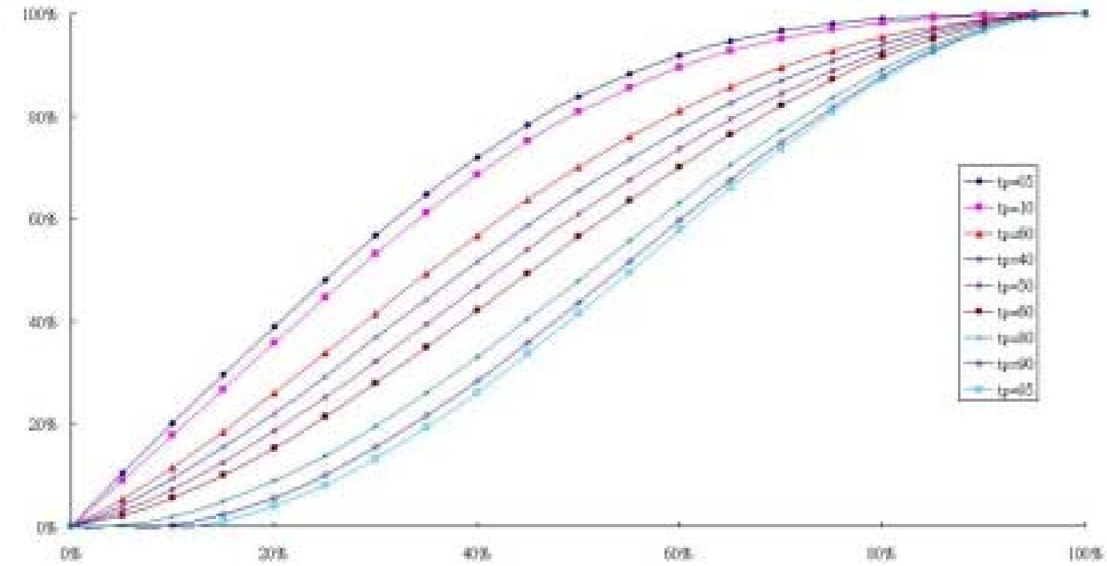

Fig.2. Miskawi S-curve model

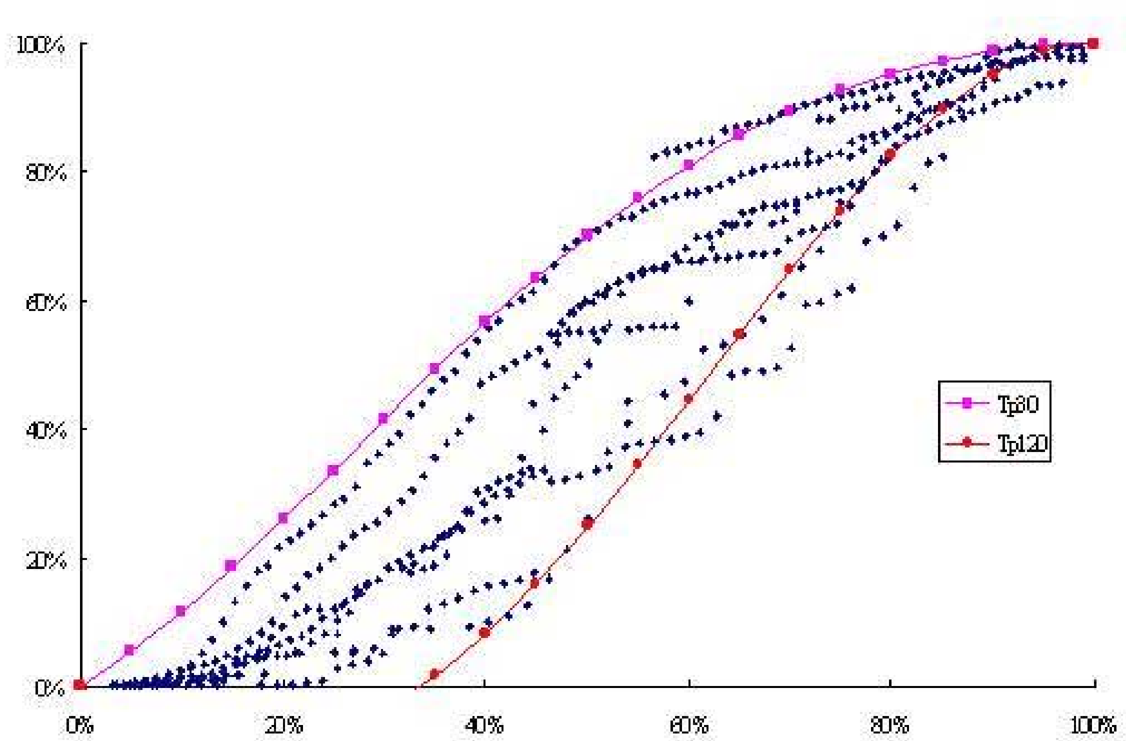

Fig.3 The valuation data of seven metro bids

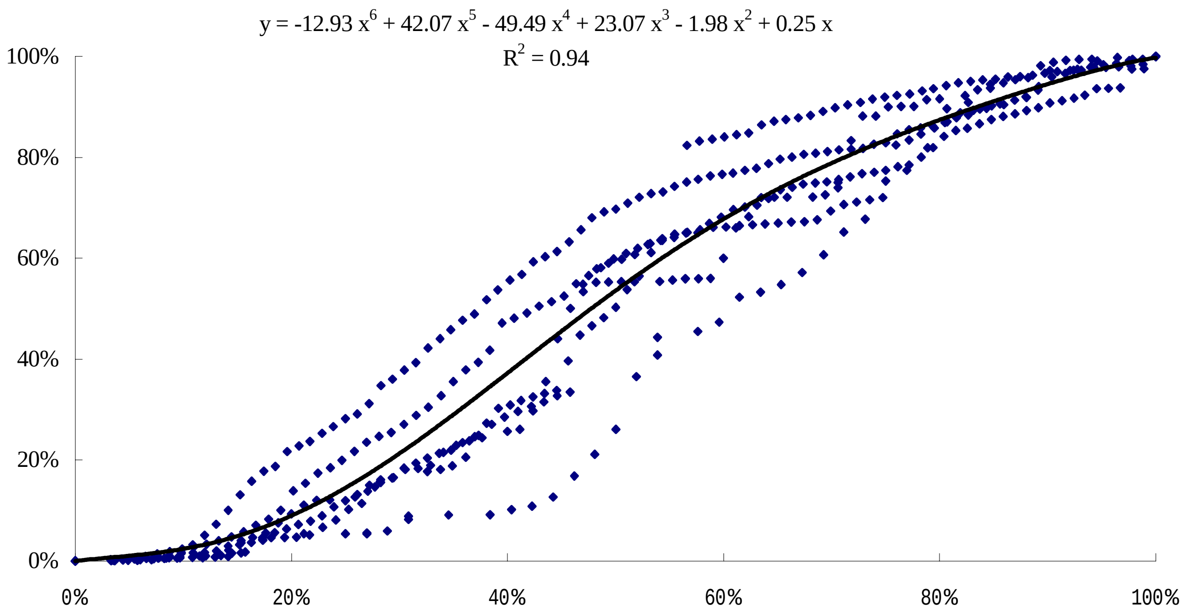

Fig.4 An S-curve by fuzzy regression method.