A Novel Decoupled Iterative Method for Deep-Submicron MOSFET RF Circuit Simulation

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Abstract: - In this paper, we solve the MOSFET RF circuit ordinary differential equations with the waveform relaxation method, monotone iterative method, and Runge-Kutta method. With the monotone iterative method, we prove each decoupled and transformed circuit equation converges monotonically. This method provides an alternative in the time domain numerical solution of MOSFET RF circuit equations.

Key Words: - Nonlinear Circuit Model, MOSFET Device, ODE, Monotone Iterative Method

1 Introduction

Numerical methods for the radio frequency (RF) circuit provide an efficient alternative in the development of integrated RF components, such as filter, low-noise amplifier, mixer, and power amplifier. The MOSFET devices used for design such integrated RF circuit becomes a tendency because of the successful experience in digital circuit design. Due to the unusually high linearity of MOSFETs at high frequencies, these active devices have been of great interests for RF and wireless applications in the recent years. The common approach to analyze the inter-modulation distortion and two-tone characteristics for an MOSFETs is to solve a set of equivalent circuit ordinary differential equations (ODEs) in the frequency domain. The harmonic balanced method is a standard approach for solving such RF problems [1][2]. However, this frequency domain approach has its merits and limitations in studying the physical properties of MOSFET with time variations. Another approach to the analysis of electrical characteristics for a MOSFET RF circuit is to solve a set of equivalent circuit ODEs in time domain. The time domain results are then further calculated with fast Fourier transformation (FFT) for obtaining its spectrum. However, the discretized ODEs in circuit simulation are often solved with conventional Newton’s iterative (NI) method. The well-known HSPICE circuit simulator is right adopted the NI method and its variants in its numerical kernel. It is well-known the NI method is a local method; in general, it has a quadratic convergence property in a sufficiently small neighborhood of the exact solution, and hence it encounters convergence problem for practical engineering applications. These properties have their limitation and should be carefully verified in the practical engineering application.

In this work, we apply this method to simulate MOSFET characteristics with exploiting the basic nonlinear property in the equivalent circuit model. By considering the Kirchhoff’s current
law for each node, the circuit governing equations are formulated in terms of the nodal voltages \((V_G, V_D, V_S, V_{DX}, V_{SX}, V_{GX})\). The circuit model is decoupled into several independent ODEs with the waveform relaxation [3] decoupling scheme. The basic idea of the decoupled method for circuit simulation is similar to the well-known Gummel’s decoupling method for device simulation [5]; it is that the circuit equations are solved sequentially [4]-[5]. In the circuit model, the first equation is solved for \(V_G^{(g+1)}\) given the previous states \(V_X^{(g)}\), \(X = D, S, GX, SX, DX\), respectively. For the second equation is solved for \(V_D^{(g+1)}\) given \(V_X^{(g)}\), \(X = G, S, GX, SX, DX\), respectively. We have the same procedure for other ODEs. Each decoupled ODE is transformed and solved with Runge-Kutta (RK) method and the monotone iterative method. We prove this approach converges monotonically for all decoupled circuit equations. It means that we can solve the circuit ODEs with arbitrary initial guesses. Numerical results for various devices have been reported to demonstrate the robustness of the method in this paper. This paper is organized as follows. In Sec. 2, we state the ODE model. Sec. 3 is the monotone iterative method for the decoupled ODEs. For each decoupled equation, we prove the convergence property by using monotone iterative method. A computational procedure is also introduced in this section. Sec. 4 shows some computational results and some discussions. Sec. 5 draws the conclusions and suggests future works.

2 A MOSFET RF Circuit Model

As shown in Fig. 1, based on the node current flow conservation (the well-known Kirchhoff’s current law) and utilize the EKV large signal equivalent circuit model (Fig. 2) for the MOSFET device [6], the complete simulation model can be formulated with nodal equations. The system of node equations for time dependent MOSFET circuit is a set of nonlinear coupled ODEs. At nodes B, D, S, and GX we have the following equations (1)-(4), respectively [7].

\[
(C_{gs} + C_{gs0})(\frac{dV_S}{dt} - \frac{dV_G}{dt}) + \frac{(C_{gd} + C_{gd0})(dV_D}{dt} - \frac{dV_G}{dt}) + \frac{(C_{gb} + C_{gb0})(dV_B}{dt} - \frac{dV_G}{dt}) + \frac{V_{GX} - V_G}{R_G} = 0,
\]

\[
-I_{DS} - I_{DB} + (C_{gd} + C_{gd0})(\frac{dV_G}{dt} - \frac{dV_D}{dt}) + \frac{V_{DX} - V_D}{R_D} = 0,
\]

\[
I_{DS} + (C_{gs} + C_{gs0})(\frac{dV_G}{dt} - \frac{dV_S}{dt}) + \frac{C_{bs}(dV_B}{dt} - \frac{dV_S}{dt}) + \frac{V_{SX} - V_S}{R_S} = 0,
\]

\[
C_{GX}(\frac{dV_{IN}}{dt} - \frac{dV_{GX}}{dt}) + \frac{V_G - V_{GX}}{R_G} + \frac{V_{IN} - V_{GX}}{R_{IN}} = 0
\]

Figure 1 : A typical MOSFET RF circuit.
The DC solution as the starting point to compute the ODEs in the large-scale time domain directly [7]. Firstly, under the steady state condition, we find convergent simulation technique to solve the system model can be found in [6].

We propose here a decoupled and globally convergent nonlinear ODEs, due to the exponential dependence of current and capacitance models. The equations as follows:

\[ \frac{V_S - V_{SX}}{R_S} + \frac{-V_{SX}}{R_{SX}} = 0 \]  
\[ \frac{V_D - V_{DX}}{R_D} + \frac{V_{DD} - V_{DX}}{R_{DD}} + \frac{-V_{DX}}{R_{OUT}} = 0 \]  

The Eqs. (1)-(4) are the ODEs, and the Eqs. (5) and (6) are the algebraic equations. These equations are subject to proper initial values at time \( t = 0 \) for all unknowns to be solved. All currents \( I \) and capacitances \( C \) above are nonlinear functions of unknown variables. These nonlinear terms are

There are 4 coupled ODEs with the nonlinear current and capacitance models have to be solved and the unknowns to be calculated in the system of ODEs are \( V_G, V_D, V_S \), and \( V_{GX} \), respectively. We note that the system consists of strongly coupled nonlinear ODEs, due to the exponential dependence of current and capacitance models. The model parameters above for the EKV MOSFET model can be found in [6].

\[ \frac{dV}{dt} = f(V, t), \quad V(0) = V_0, \quad t \in [0, T], \]  

where \( T > 0, f : \mathbb{R}^D \times [0, T] \rightarrow \mathbb{R}^D, V_0 = [V_{1,0}, V_{2,0}, \ldots, V_{D,0}] \in \mathbb{R}^D \) is the initial vector of \( V \), and \( V_0 = [V_1(t), V_2(t), \ldots, V_D(t)] \in \mathbb{R}^D \) is the solution vector. The system can be written as follows,

\[
\begin{cases}
\frac{d}{dt} V_1 = f_1(V_1, V_2, \ldots, V_D, t), & V_1(0) = V_{1,0} \\
\frac{d}{dt} V_2 = f_2(V_1, V_2, \ldots, V_D, t), & V_2(0) = V_{2,0} \\
\vdots \\
\frac{d}{dt} V_D = f_D(V_1, V_2, \ldots, V_D, t), & V_D(0) = V_{D,0}
\end{cases}
\]

The WR method for solving (7) is a continuous-time iterative method. Therefore, given a function which approximates the solution, it calculates a new approximation along the whole time-interval of interest. Clearly, it differs from most standard iterative techniques in that its iterates are functions in time instead of scalar value. The iteration formula is chosen in such a way that one avoids having to solve a large system ODEs. A particularly simple, but often very effective iteration scheme is written below. It maps the old
iterate $V^{(n-1)}$.

\[
\begin{align*}
\frac{d}{dt}V_1^{(n)} &= f_1(V_1^{(n)}, V_2^{(n-1)}, \ldots, V_d^{(n-1)}, t), V_1^{(n)}(0) = V_{1,0} \\
\frac{d}{dt}V_2^{(n)} &= f_2(V_1^{(n)}, V_3^{(n-1)}, \ldots, t), V_2^{(n)}(0) = V_{2,0} \\
&\vdots \\
\frac{d}{dt}V_d^{(n)} &= f_d(V_1^{(n-1)}, \ldots, V_d^{(n)}, t), V_d^{(n)}(0) = V_{D,0}
\end{align*}
\]

It is obvious similar to the Gauss-Seidel method for iteratively solving linear and nonlinear systems of algebraic equations, so-call the Gauss-Seidel waveform relaxation scheme. It converts the task of solving a differential equation in terms of algebraic equations, so-call the Gauss-Seidel waveform relaxation scheme. It converts the task of solving a differential equation in $D$ variables into the task of solving a sequence of differential equations in a single variable. A closely related iteration is the Jacobi waveform relation scheme, the iteration formula is given by,

\[
\begin{align*}
\frac{d}{dt}V_1^{(n)} &= f_1(V_1^{(n-1)}, V_2^{(n-1)}, \ldots, t), V_1^{(n)}(0) = V_{1,0} \\
\frac{d}{dt}V_2^{(n)} &= f_2(V_1^{(n-1)}, V_3^{(n-1)}, \ldots, t), V_2^{(n)}(0) = V_{2,0} \\
&\vdots \\
\frac{d}{dt}V_d^{(n)} &= f_d(V_1^{(n-1)}, \ldots, V_d^{(n)}, t), V_d^{(n)}(0) = V_{D,0}
\end{align*}
\]

For a given specified time step $t$ and the previous calculated results, the decoupling algorithm solves the circuit equations sequentially, for instance, the $V_G$ in Eq. (1) is solved for given the previous results $(V_D, V_S, V_{GX}, V_{DX}, V_{SX})$. The $V_D$ in Eq. (2) is solved for newer given $V_G$ and $(V_S, V_{GX}, V_{DX}, V_{SX})$. The $V_S$ in Eq. (3) is solved for newer given $(V_G, V_D)$ and $(V_{GX}, V_{DX}, V_{SX})$. We have similar procedure for other unknowns.

Each decoupled ODE is solved with the MI algorithm. To clarify the MI algorithm for the numerical solution of the decoupled nonlinear ODEs, we write the above decoupled ODEs as the following form

\[
\frac{dV_X^{(g)}}{dt} = f(V_X^{(g)}, t), V_X^{(g)}(0) = V_X^{(0)},
\]

where $V_X^{(g)}$ is the unknowns to be solved, $g$ is the decoupling index $g = 0, 1, 2, \ldots$. We note that the $f$ is the collection of the nonlinear functions and $f \in C[I \times \mathbb{R}, \mathbb{R}]$ and $I = [0, T]$. For a fixed index $g$ and $X$, because the upper and lower solutions, $\bar{V}_X^{(g)}$ and $\underline{V}_X^{(g)}$, exist in the circuit and $\bar{V}_X^{(g)} \geq \underline{V}_X^{(g)}$, we can prove the solution existence in the set $\Omega = \{(t, V_X^{(g)}) \mid \bar{V}_X^{(g)}(0) \geq V_X^{(g)}(0) \geq \underline{V}_X^{(g)}(0), \forall t \in I\}$ for each decoupled circuit ODE.

**Theorem 1** Let $\bar{V}_X^{(g)}$ and $\underline{V}_X^{(g)}$ are the upper and lower solutions of Eq. (11) in $C^1[I \times \mathbb{R}, \mathbb{R}]$ such that $\bar{V}_X^{(g)}(0) \geq \underline{V}_X^{(g)}(0)$ in the time interval $I$ and $f \in C[I \times \mathbb{R}, \mathbb{R}]$. Then there exists a solution $V_X^{(g)}$ of Eq. (11) such that $\bar{V}_X^{(g)}(0) \geq V_X^{(g)}(0) \geq \underline{V}_X^{(g)}(0)$.

**Proof 1** It is a direct result with the continuous property of $f$, here the comparison theorem is applied [5].

**Remark 1** We note that for each decoupled ODE, the nonlinear function $f$ is nonincreasing function of the unknown $V_X^{(g)}$ and the upper and lower solutions $\bar{V}_X^{(g)}(0)$ and $\underline{V}_X^{(g)}(0)$ of Eq. (11) in $I$ can be found. We can further prove there exists a unique solution $V_X^{(g)}$ of Eq. (11) in $I$ and $\bar{V}_X^{(g)}(0) \geq V_X^{(g)}(0) \geq \underline{V}_X^{(g)}(0)$.

We see that the Theorem 1 provides an existence result of the problem, and we now describe a monotone constructive method for the computer simulation of the circuit ODEs. The constructed sequences will converge to the solution of Eq. (11) for all decoupled ODEs in the circuit simulation. In this condition, instead of original nonlinear ODE to be solved, a transformed ODE is solved with such as the RK method. Now we state the main result for the solution of each decoupled MOSFET circuit ODEs.

**Theorem 2** Let the $f \in C[I \times \mathbb{R}, \mathbb{R}]$, $\bar{V}_X^{(g)}(0)$ and $\underline{V}_X^{(g)}(0)$ are the upper and lower solutions of Eq. (11) in $I$. Because $f(t, V_X^{(g)}) - f(t, \bar{V}_X^{(g)}) \geq -\lambda(V_X^{(g)} - \bar{V}_X^{(g)})$, $\bar{V}_X^{(g)}(0) \geq V_X^{(g)}(0) \geq \bar{V}_X^{(g)}(0)$ and $\lambda > 0$, sequences $V_X^{(g)} \xrightarrow{\text{unif.}} \bar{V}_X^{(g)}$ and $V_X^{(g)} \xrightarrow{\text{unif.}} \underline{V}_X^{(g)}$ as $n \to \infty$ monotonically in $I$.

**Proof 2** The proof was done in [5].
**Theorem 3** For decoupled ODEs, the nonlinear function $f$ is nonincreasing in $V^{(g)}_X$ and $f(t, V^{(g)}_{X_1}) - f(t, V^{(g)}_{X_2}) \geq -\lambda (V^{(g)}_{X_1} - V^{(g)}_{X_2})$, $\forall V^{(g)}_{X_1} \geq V^{(g)}_{X_2}$. Thus $\{V^{(g)}_{X_n}\}_{n=1}^{\infty}$ and $\{V^{(g)}_{X_n}\}_{n=1}^{\infty}$ converge uniformly and monotonically to the unique solution $V^{(g)}_X$ of Eq. (11).

**Proof 3** By using Theorem 2 and note the nonincreasing property of $f$, the result is followed directly.

### 4 Results and Discussion

As shown in Fig. 1, the input signal $V_{IN}$ is the DC bias and the expression of the two-tone input signal $V_{m}$ is as follows:

$$V_{in} = V_m \sin(2\pi f_1 t) + V_m \sin(2\pi f_2 t), \quad (12)$$

The input two-tone signal has an amplitude $V_m = 0.005$ V, and fundamental frequencies $f_1$ and $f_2$ are 1.71 and 1.89 GHz, respectively. Fig. 3-4 shows our simulation results and the HSPICE results in time domain, respectively. We also can find the difference in the Figs. 3 and 4 at the time between 2.5 ns and 3.5 ns, the HSPICE results show some non-smooth calculation, but our calculation does not have this phenomenon. With the time domain results, we calculate the spectra of the output power by the FFT. Fig. 5 shows the corresponding spectra with our time domain results. We can find the 3rd-order intermodulation (IM3) products at $2f_1 - f_2$ and $2f_2 - f_1$ clearly [7]. Fig. 6 shows the corresponding spectra with HSPICE time domain result. However, it is difficult to identify the two IM3 products.

Fig. 7 show the convergence property of our simulation kernel. Fig. 7(a) shows the inner loop iteration versus the maximum norm error at the beginning of the simulation. This figure shows the inner loop iteration has $n \log(n)$ convergence where $n$ is the number of iterations. The Fig. 7(b) shows the outer loop iteration convergence property in different time period. As we can see the speed of convergence is almost around $n^2$. 

![Figure 3: Our time domain results.](image3)

![Figure 4: The HSPICE time domain outputs.](image4)

![Figure 5: Our output power in frequency domain.](image5)
Figure 6: The HSPICE output power in frequency domain.

5 Conclusions

A novel RF circuit simulation method based on the waveform relaxation and monotone iterative methods has been proposed. With monotone iterative technique, we have proved each decoupled circuit ODE converges monotonically. The proposed method here is an alternative in the numerical solution of electric circuit equations. Computational results have been reported in this work to demonstrate the robustness of the method. This method not only provides a computational technique for the time domain solution of circuit ODEs but also can be generalized for circuit simulation including more transistors. Furthermore, this method can be parallelized and systematically extended to simulate such as RF IC and system-on-a-chip large-scale VLSI circuit.

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References


