Active Noise Control for a One-dimensional Acoustic Duct Using Feedback Control Techniques: Modelling and Simulation

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Abstract:
The modelling and simulation of the Active Noise Control (ANC) using the fixed feedback control for a hard-wall one-dimensional acoustic duct system is discussed. The state-space models as well as the transfer function description are derived based on physical principles. The coupled dynamic between the cancelling loudspeaker and the acoustic duct is explicitly expressed in these models. Besides that, the basic and advanced ANC design problems can be formulated into a set of standard control problems. Simulation tests using a simple lag compensator show a bright potential of using feedback control techniques in the ANC design.

Key words: Active noise control, feedback control, loudspeaker, acoustic duct

1. Introduction

The traditional approaches for acoustic noise cancellation often use enclosures or barriers to attenuate the undesired noise. However, these approaches become costly and ineffective when they need to deal with the low-frequency noise [3], [6], [12], such as the low-frequency noise in a ventilation system ranges from 50 Hz to 500 Hz which includes the throb and rumble of turbulent airflow and the roar, hum, buzz and whine of fans, pumps, and chillers, etc. In order to cancel/reduce the low frequency noise, the active approaches which are usually referred to as Active Noise Control (ANC) techniques have become more and more used in our daily life [6], [12].

Based on the signal superposition principle, the ANC system introduces an anti-noise wave through an appropriate set of secondary sources, which usually includes a set of loudspeakers and a set of microphones/signal-generators. These secondary sources can be interconnected through an electronic system using some specific signal processing algorithms [6], [12]. The design of ANC systems can be generally classified as feedforward, feedback and hybrid approaches with respect to different system structures [12], [13]. Under different system structure, different signal processing algorithms can be used, such as adaptive IIR/FIR-based LMS/FXLMS methods [12], adaptive or fixed feedback control methods (e.g., pole placement [11] and \( H_\infty \) control [15]). The efficiency of different ANC systems depends on specific problems and systems.

From the control point of view, the design of an ANC system is a typical control design problem [6], [8], [11], [14], [17]. Comparing with other methods, the ANC design using feedback control techniques has the following benefits:

- Only the residual is measured and used to control the cancelling loudspeaker, such that the "secondary path feedback to reference" problem [12] can be avoid;
- The transient signal suppression as well as steady-state suppression could be controlled through the selection of damping ratio and integral control [5];
- The robustness of the designed system can be systematically analyzed and coped using the existing robust control theory [7];
- The global attenuation can be explicitly explored using the state space models [9], [10].

Adaptive algorithms are often used for ANC design [6], [12], however, the convergency of the adaptive algorithm (such as LMS and its extensions) and the stability of the designed system can not be guaranteed in advance [20]. The design of a fixed controller for an ANC system becomes more interesting comparing with an adaptive controller with respect to the facts that: (i) the designed ANC algorithm is relatively simple and can be easily implemented; and (ii) the information about system’s stability and performance becomes predictable. Some pioneer work can be found in this direction. For example, A.J. Hull et al proposed a global ANC design using the pole placement method for an one-dimensional acoustic duct in [11], after they developed a state space model for the acoustic duct in [10], [9]. However, this design is under a precondition that the considered system model is controllable and observable. This condition can be easily invalid if we select a different number of considered acoustic modes (e.g., 6-modes) under the same physical condition. Besides that, the strong dynamic coupling between the cancelling loudspeaker and acoustic duct was not considered. This coupling phenomenon was discussed later in [2] for a loudspeaker compensation. V. Toochinda et al discussed the "hybrid ANC" (one loudspeaker, one reference and one error microphone) design using the \( H_\infty \) and QFT techniques when they formulated the design into a single-input two-output feedback control problem [18]. Under the same idea, they also discussed the effects of microphone and loudspeaker locations to the ANC performance [17]. However, the coupling dynamic between the loudspeaker and the acoustic duct was not considered in their work. K. A. Morris developed a model of the one-dimensional acoustic duct in [14], [15], which has a non-constant termination impedance and consists of coupled P- and ODE...
equations. He formulated the ANC problem as an $H_{\infty}$-optimization problem and discussed the global reduction using a performance point besides the loudspeaker point and microphone point [15].

In this paper, the modelling and simulation of the ANC design using the fixed feedback control for a hard-wall one-dimensional acoustic duct system is discussed. The state-space models as well as the transfer function description are derived for the considered system. Based on feedback control framework, the basic and global ANC design problems then can be formulated as some standard control problems. A lag compensator is designed for the considered system, and simulation results show a bright potential of using feedback control techniques in the ANC design.

The rest of this paper is organized as: Section 2 introduces the considered system; Section 3 derives the state space models as well as transfer functions based on physical principles; Section 4 formulates the ANC design into a set of control design problems; Section 5 shows simulation tests under different situations; Finally, we conclude the paper in Section 6.

2. A Benchmark System

Consider an acoustic duct which is constructed by the PVC plastic and its diameter is significantly smaller than its length. At one end of the duct one loudspeaker is installed to act as the primary noise source. Another loudspeaker which acts as the secondary acoustic resource and one microphone which measures the attenuated residual are used in the considered system. The measured residual goes into an anti-aliasing filter after passing the microphone’s amplifier circuit. The filtered signal feeds into the microprocessor/PC which acts as the ANC controller (running some specific algorithm). The output signal of the controller goes through a reconstruction filter and then the amplifier circuit for loudspeaker, so as to drive the cancelling loudspeaker to generator a proper anti-noise signal. From the practical point of view, the duct should be sealed properly at all openings (microphone ports, sound sources, etc.) in order to achieve low background SNR level. The microphone must mounted flush with the inside wall of the tube and isolated from the tube (to minimize the sensitivity to vibration).

In the physical system, the loudspeaker is fixed at some position and microphone can locate at several different locations. However, in our simulation models, we can shift both locations at any possible positions along the duct.

3. Physical Modelling

In the following, the modelling of the loudspeaker and the acoustic duct is considered with respect to their dominant roles in the ANC design.

3.1 Model of the Loudspeaker

The basic structure of a typical low-frequency loudspeaker can be found in [4]. The voice coil of a typical low-frequency loudspeaker is placed on a former, which makes a mechanical connection to the diaphragm. There are two suspensions: an outer suspension connecting the diaphragm to the frame, and an inner suspension connecting the coil former to the frame. The pair of suspensions create a strong stiffness against “rocking” or “wobbling” of the diaphragm-coil assembly. At the low displacement of the diaphragm-coil assembly, the dynamic can be approximated using a linear model. At the high displacement some parameters, such as suspension and the force-factor, of the model will vary with displacement. In the following, we focus on a linear model with the coupled dynamics from the rear enclosure and the front acoustic duct. The nonlinear situation can be approximated using a set of linear models operating at different displacement/frequency regions.

A principal diagram of the loudspeaker is shown in Fig.2, and system parameters, values and variables used in the considered system are listed in the following table.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assembly mass</td>
<td>$m_a$</td>
<td>$2.5 \times 10^{-3}$</td>
<td>Kg</td>
</tr>
<tr>
<td>Viscous friction</td>
<td>$f_s$</td>
<td>5.2</td>
<td>N/m</td>
</tr>
<tr>
<td>Suspension stiffness</td>
<td>$k_s$</td>
<td>$12 \times 10^3$</td>
<td>N/m</td>
</tr>
<tr>
<td>Voice coil resistance</td>
<td>$R_s$</td>
<td>4</td>
<td>Ohm</td>
</tr>
<tr>
<td>Voice coil inductance</td>
<td>$L_s$</td>
<td>1.1</td>
<td>mH</td>
</tr>
<tr>
<td>Force factor</td>
<td>$Bl$</td>
<td>3.9</td>
<td>N/An</td>
</tr>
<tr>
<td>Effective radius</td>
<td>$r_s$</td>
<td>0.07</td>
<td>m</td>
</tr>
<tr>
<td>Assem. displacement</td>
<td>$x(t)$</td>
<td>variable</td>
<td>m</td>
</tr>
<tr>
<td>Assem. velocity</td>
<td>$\dot{x}(t)$</td>
<td>variable</td>
<td>m/sec</td>
</tr>
<tr>
<td>Assem. acceleration</td>
<td>$a_s(t)$</td>
<td>variable</td>
<td>m/sec^2</td>
</tr>
<tr>
<td>EMF voltage</td>
<td>$u_{em}(t)$</td>
<td>variable</td>
<td>Volt</td>
</tr>
</tbody>
</table>

Table 1 Modelling parameters of a loudspeaker
From the analysis of the electrical part, there is

$$i_s(t) = -\frac{R_s}{L_s}i_s(t) - \frac{B_l}{L_s}\dot{x}(t) + \frac{1}{L_s}u_{in}(t). \tag{1}$$

The mechanical part can be modelled as a single-degree of freedom (SDF) mechanical system as shown in Fig.2. Following the Newton Principles, there is

$$F_s(t) - F_{ext}(t) = m_s\ddot{x}(t) + f_s\dot{x}(t) + k_s x(t), \tag{2}$$

where $F_s(t)$ is the force generated by the electrical part following the rule $F_s(t) = Bli_s(t)$, and $F_{ext}$ represents all the external forces acting on the assembly. If the loudspeaker is used in the free air, then $F_{ext}(t) \equiv 0$. If the loudspeaker is mounted in a cavity and faced a duct in front, then, there is

$$F_{ext}(t) = F_{rear}(t) + F_{front}(t).$$

In our benchmark, the cancelling loudspeaker is mounted in a closed-box (without any ventilation hole) in order to avoid the acoustic short-circuit [1]. If all dimensions of the cabinet are small compared to the considered signal’s wavelengths, the air pressure, represented as $p_r(t)$, can be approximated as some constant throughout the cavity [4], i.e., there is

$$p_r(t) = \rho c^2 \frac{V_r(t)}{V_0}$$

where $V_r(t)$ represents the volume into the cavity and $V_0$ represents the volume capacity of the cavity.

Since $V_r(t) = -S_d\dot{x}(t)$, then, there is

$$p_r(t) = \rho c^2 \frac{V_r(t)}{V_0} = -\rho c^2 \frac{S_d}{V_0} x(t) \tag{3}$$

Therefore, the force $F_{rear}$ working on the diaphragm generated by the air pressure inside the rear enclosure

$$F_{rear} = \rho c^2 \frac{S_d}{V_0} x(t). \tag{4}$$

Denote the air pressure in front of the loudspeaker as $p(x_s, t)$, then, the front external force $F_{front}$ can be calculated as

$$F_{front}(t) = p(x_s, t)S_d. \tag{5}$$

Combining (1), (2), (4) and (5), a state space model for the loudspeaker with the coupled dynamic between the loudspeaker and acoustic duct can be obtained as

$$\begin{cases}
\dot{X}_{spk}(t) = A_{spk}X_{spk}(t) + B_{spk}U_{spk}(t) \\
Y_{spk}(t) = C_{spk}X_{spk}(t)
\end{cases} \tag{6}$$

where

$$A_{spk} = \begin{bmatrix}
\frac{-B_l}{m_s} & 0 & \frac{B_l}{m_s} \\
0 & \frac{-k_s}{m_s} & 0 \\
\frac{-f_s}{m_s} & \frac{\rho c^2 s^2}{m_s} & \frac{-f_s}{m_s}
\end{bmatrix},$$

$$B_{spk} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix},$$

$$C_{spk} = \begin{bmatrix}
1 & 0 & 0
\end{bmatrix}.$$
by a linear second-order PDE:
\[ \frac{\partial^2 u(x,t)}{\partial t^2} - c^2 \frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{\delta(x)p(t)}{\rho} \right) - \frac{\partial}{\partial t} \left( \frac{\delta(x-x_s)M(t)}{\rho S} \right) \]
\[ (8) \]

(8) One end at location \( x = L \) is modelled by the partially reflective boundary condition [9], [10], i.e.,
\[ c \frac{\partial u(x,t)}{\partial x} = -K \frac{\partial u(L,t)}{\partial t} \]
\[ (9) \]
where \( K \) is the complex impedance of the terminal end.
There are \( K \neq 0 \pm 0i \), \( 1 \pm 0i \), \( \infty \). \( K \) can be determined through experimental modal analysis [9], [10].

The duct at \( x = 0 \) is modelled as a totally reflective end, i.e.,
\[ \frac{\partial u(0,t)}{\partial x} = 0. \]
\[ (10) \]

The acoustic pressure inside the duct is related to \( u(x,t) \) as
\[ P(x,t) = -\rho c^2 \frac{\partial u(x,t)}{\partial x}. \]
\[ (11) \]

Equation (4), (9), (10) and (11) consist of the model of the acoustic duct dynamic. Using the separation of variable method [9], [10], \( u(x,t) \) can be expressed as
\[ u(x,t) = g(t) + \sum_{n=-\infty}^{n=\infty} a_n(t) \varphi_n(x), \]
where \( g(t) \) and \( a_n(t) \) are called state variables, \( \varphi_n(x) \) has the form:
\[ \varphi_n(x) = e^{\lambda_n x} + e^{-\lambda_n x}, \]
\[ n = 0, \pm 1, \pm 2, \ldots \] and \( \lambda_n = \frac{\pm 2L}{2L} \log \frac{1}{1 + K} - \frac{n \pi i}{L}, \]
\[ n = 0, \pm 1, \pm 2, \ldots \] where \( a_n(t) \) satisfies the ODE:
\[ \dot{a_n}(t) = c \lambda_n a_n(t) - \frac{1}{2 \pi \lambda_n L \rho} p(t) + \frac{1}{4 \pi \lambda_n^2 L \rho S} \int_0^L \frac{\partial u(x,0)}{\partial t} \varphi_n(x) \, dx \]
\[ (14) \]
Substitute (11,12,14) into (4), (9), (10), (11), and rearrange these equations, finally a state space description of the acoustic dynamics can be obtained as:
\[ \begin{cases} \dot{X}_n(t) = A_n X_n(t) + B_a u_n(t) + B_p p(t) \\ y_n(t) = C_n X_n(t) \end{cases} \]
\[ (15) \]
where \( X_n(t) = [ \cdots a_{-1}(t) a_0(t) a_1(t) \cdots ]^T \) is the vector of modal wave amplitude [10]. The control input \( u_n(t) = \frac{\partial u(x,t)}{\partial t} \) corresponds the mass flow rate. The output \( y_n(t) = p(x_m,t) \) is the air pressure (measured by microphone) at location \( x_m \). System matrices are defined as
\[ A_n = diag(\lambda_n) = \begin{bmatrix} c \lambda_{-1} & 0 & 0 & \cdots \\ \cdots & 0 & c \lambda_0 & 0 & \cdots \\ 0 & 0 & c \lambda_1 & \cdots \end{bmatrix} \]
\[ B_a = \text{column\_vector}(\frac{1}{4 \pi \lambda_n^2 L \rho S} \frac{\partial \varphi_n(x)}{\partial x})_{n = 0, \pm 1, \cdots} \]
\[ B_p = \text{column\_vector}(\frac{1}{2 \pi \lambda_n L \rho} p_n)_{n = 0, \pm 1, \cdots} \]
\[ C_n = \text{row\_vector}(\rho c^2 \frac{\partial \varphi_n(x)\, dx}{dx})_{n = 0, \pm 1, \cdots} \]
\[ (16) \]

The imaginary part of \( c \lambda_n \) corresponds the \( n \)-th mode resonance frequency of the considered acoustic duct. When the terminal impedance \( K \) can be approximated by some constant value, this acoustic duct model is a complex-valued two-input one-output LTI system [10]. It is noticed that [17], [18] also developed a same model with real-valued coefficients.

This model (15) can also be represented by two transfer functions: one denoted as
\[ G_{duct1}(s) = \frac{n_{a1}(s)}{d_{a1}(s)} = C_a(sI - A_a)^{-1}B_p \]
\[ (18) \]
is from \( p(t) \) to \( y_a(t) \); another one denoted as
\[ G_{duct2}(s) = \frac{n_{a2}(s)}{d_{a2}(s)} = C_a(sI - A_a)^{-1}B_n \]
\[ (19) \]
is from \( u_a(t) \) to \( y_a(t) \). It can be noticed that there is \( d_{a1}(s) = d_{a2}(s) \approx d_{a1}(s) \). Using the data from Table 2, these two transfer functions of the considered system have the frequency property as shown in Fig.41.

When a bandlimited noise signal is used as the primary noise, the measured signal at \( x_m = 1.7m \) and its FFT analysis are shown in Fig.4. From the real time analysis, it can be observed that measured signal has 5 steps delay corresponding to input signal with respect to the time delay \( x_m/c/T_a = 1.7/343/0.001 \approx 5 \). From FFT analysis, the first three resonance frequencies (app. 88Hz, 176Hz, 264Hz) have effect to the input signal which is bandlimited with 0 ~ 250Hz.

It can be seen that the terminal impedance \( K \) plays a critical role in determine system parameters. In a strict
In order to check the global noise reduction situation, a feedback controller denoted as \( C(s) \) needs to be used by the ANC controller later. Furthermore, in the sense, \( K \) should be a complex and frequency-dependent parameter [1], [16], [15]. In order to model the acoustic duct more precisely, a multiple-model scheme should be built based on (15), where each model can operate within a specific frequency period at which the terminal impedance can be approximated by a constant value.

### 3.3 Block Diagram of the Whole System

The whole system model can be obtained by combining the loudspeaker model (6) and the acoustic model (15) together. There is the relationship:

\[
M(t) = \rho S_d \dot{x}(t) \Rightarrow u_{in}(t) = \frac{d(M(t))}{dt} = \rho S_d a_s(t). \tag{20}
\]

Therefore, the following two transfer functions which denote the output of the loudspeaker model as \( a_s(t) \) are used in order to get the total system diagram:

\[
G_{spk1}(s) = G_{spk1}(s) s, \quad G_{spk2}(s) = G_{spk2}(s) s, \tag{21}
\]

where \( G_{spk1}(s) \) and \( G_{spk2}(s) \) are defined in (7).

The pressures at two locations \( x_s \) and \( x_m \) need to be known, with respect to the fact that \( p(x_s, t) \) needs to feedback to the loudspeaker dynamic, and \( p(x_m, t) \) will be used by the ANC controller later. Furthermore, in order to check the global noise reduction situation, a movable performance point \( x_p \) [15] also needs to be defined. The following transfer functions are defined based on (18) and (19):

\[
\begin{align*}
G_{\text{duct}}^{s}(s) &= G_{\text{duct}}^{s}(s), & \text{when } x_m = x_s, \\
G_{\text{duct}}^{m}(s) &= G_{\text{duct}}^{m}(s), & \text{when } x_m = x_s, \\
G_{\text{duct}}(s) &= G_{\text{duct}}(s), & \text{TF from } x_s \text{ to } x_m \\
G_{\text{duct}}^{m}(s) &= G_{\text{duct}}^{m}(s), & \text{TF from } x_m \text{ to } x_p \tag{22}
\end{align*}
\]

From (16) it can be noticed that different \( x_s, x_m, \) and \( x_p \) can only affect the matrix \( C_n \). Therefore, \( G_{\text{duct}}^{m}(s) \) and \( C_{\text{duct}}^{m}(s) \) are complex constants related to \( x_s, x_m, \) and \( x_p \), respectively. In the following \( G_{\text{duct}}^{m}(x_p) \) is used to represented a complex variable which depends on the value \( x_p \) with \( 0 < x_p < L \).

The whole system block diagram is shown in Fig.6. The considered system model is an infinite dimensional system. However, this model can be truncated by taking several acoustic modes through selecting \( N \) with respect to the fact that ANC system is mainly used to deal with low frequency noise. The neglected modes can be regarded as modelling uncertainties.

### 4. ANC Feedback Design

The diagram using the feedback ANC framework for the acoustic duct system is shown in Fig.6. This diagram can be further simplified as shown in Fig.7, where

\[
\begin{align*}
G_p(s) &= \frac{G_{\text{duct}}^{s}(s)}{1 + \rho S_d G_{\text{duct}}^{s} G_{spk2}} = \frac{n_{a1}(s) d_{s}(s)}{a_s(s) d_{s}(s) - \rho S_d a_{s2}(s) n_{a2}(s)} \\
G_u(s) &= \frac{n_{a2}(s) d_{s}(s)}{\rho S_d G_{\text{duct}}^{s} G_{spk2}} = \frac{a_s(s) d_{s}(s) n_{a2}(s)}{\rho S_d a_{s2}(s) n_{a2}(s) n_{d2}(s)} \tag{23}
\end{align*}
\]

The closed-loop system from the primary noise input \( p(t) \) to the measured pressure \( p(x_m, t) \) using a negative feedback controller denoted as \( C(s) \) can be expressed as

\[
G_{cl}(s) = \frac{P(x_m, s)}{P(s)} = \frac{G_p(s) G_{\text{duct}}^{m}(s)}{1 + C(s) G_{\text{duct}}^{m}(s)} \tag{24}
\]

If the controller \( C(s) \) is denoted as \( C(s) = \frac{n_{c}(s)}{d_{c}(s)} \), (24) can be further expressed as

\[
G_{cl}(s) = \frac{n_{c}(s)}{d_{c}(s)} = \frac{n_{c}(s) G_{\text{duct}}^{m}(s)}{d_{c}(s)} \tag{25}
\]
The Basic ANC Design Problem [19] is to find two polynomials \( n_a(s) \) and \( d_a(s) \) such that
- (i) Polynomial \( d_a(s) \) defined in (25) is stable;
- (ii) There is \( |n_a(s)| \ll |d_a(s)| \) for \( \omega \in [0, B_{bw}] \); and
- (iii) The order of \( n_a(s) \) is not higher than the order of \( d_a(s) \).

Frequency period \([0, B_{bw}]\) is usually referred to as the ANC effective bandwidth. The bandwidth is determined by the location of the zero which is nearest to the imaginary axis from the right half \( s\)-plane. It should be noticed that the time delay caused by the secondary path under the feedback ANC framework usually reduces the ANC effective bandwidth [12].

If a controller can be find to satisfy above conditions, it only means that the noise reduction can be achieved at the measurement point \( x_m \). In order to achieve global noise reduction [11], [15], further conditions need to be satisfied.

The global ANC design problem [19] is to find two polynomials \( n_a(s) \) and \( d_a(s) \) such that
- Condition (i) and (iii) in the basic design problem should be satisfied; and
- (iv) There is \( |n_a(s)|G_{dcl}(x_p) \ll |d_a(s)| \) for all \( \omega \in [0, B_{bw}] \) as well as any \( x_p \in (0, L) \), where the complex \( G_{dcl}(x_p) \) can be calculated through (22).

The minimal reduction level for a specific frequency period corresponds to \( \|G_{dcl}(s)W_1(s)\|_{\infty} \), where \( W_1(s) \) is a weighting function [19]. It is obvious that the \( H_{\infty} \) control technique can be directly used to design an ANC system.

The \( H_{\infty} \) ANC design problem is to find a proper and real-rational function, denoted as \( C(s) \in R_{ss} \), such that
\[
\min_{C(s) \in R_{ss}} \|G_{dcl}(s)W_1(s)\|_{\infty} < 1, \quad (26)
\]
under the condition that the closed-loop system is internally stable, where \( W_1(s) \) is a weighting function. Some robust control techniques, such as \( H_{\infty} \) control and LMI method [7], could be good tools for the robust ANC design [15].

5. Simulation Tests

5.1 A Lag Compensator

A lag compensator is designed as the ANC controller for the considered system. The number of acoustic modes is selected as \( N = 10 \), which corresponds the fact that resonance frequencies will approximately be from 550 rad/sec up to 5500 rad/sec. The locations of loudspeaker and microphone are selected as \( x_s = x_m = 1.7m \). The designed lag compensator is
\[
C(s) = 10^{-0.0001s + 1} \quad \frac{0.1s + 1}{0.1s + 1} \quad (27)
\]
The part of poles and zeros nearing the imaginary axis, and the bode plot of the closed-loop system from \( u_{ref}(t) \) to \( p(x_m, t) \) are shown in Fig.8. It can be observed the closed-loop system is stable, and there are some down peaks in the bode plot, these phenomena are caused by the strong dynamic coupling between the loudspeaker and the acoustic duct [2].

5.2 Local noise Attenuation

Transfer functions from the primary \( p(t) \) to \( p(x_m, t) \) without any ANC and with the designed ANC are shown in the Fig.9. It can be observed that the noise attenuation can be achieved at most frequency points.

5.3 Global Noise Attenuation

Several performance points are selected as: \( x_p = 0.2m, 1.0m \) and \( 1.5m \), as well as \( 1.9m \), respectively. The transfer functions from the primary input to the observing points without ANC and with ANC are shown in the following Fig.10 and Fig.11, respectively. It can be ob-
observed that noise attenuations have been achieved at all points for considered frequency period. A largest attenuation is obtained at $x_p = 0.2m$ in the upstream and a least attenuation is obtained at $x_p = 1.9m$ in the downstream of the loudspeaker. It can be observed that in the upstream of the cancelling loudspeaker, especially between the primary noise and the microphone, a better noise attenuation can be achieved comparing with the case in the downstream. It means that the loudspeaker and microphone had better be put near the end of the duct in order to get a good attenuation inside the duct, this solution is consistent with [17]. Of course, the attenuation level also depends on the relative locations of the loudspeaker and microphone, which will be discussed in the following.

5.4 Loudspeaker and Microphone Locations

In the following we fix the loudspeaker’s position at $x_s = 1.7m$, and shift the microphone’s position. Firstly, we shift the microphone a little to the upstream, i.e., $x_m = 1.65m$. It can be checked that this controlled system is stable. From Fig.12 it can be observed that noise attenuation can still be achieved at most frequency points except a small area around 3260 rad/sec, even though this case is not as good as the case that $x_m = 1.7m$.

The microphone is further shifted to the left, i.e., $x_m = 1.5m$. From Fig.13 it can be observed that this controlled system becomes unstable. That problem is caused by the time delay between the loudspeaker and microphone. If the microphone is shifted further to the upstream, this stability problem always exists.

Once the microphone is shifted to the downstream, e.g., $x_m = 1.75m$, From Fig.14 it can be observed that
this controlled system is stable. From Fig.15 it can be observed that it is almost as good as the case $x_m = 1.7m$.

If the microphone is shifted further to the right, e.g., $x_m = 1.9m$. From Fig.16, it can be observed that this controlled system becomes unstable. It can be concluded that the microphone should be put as close as possible to the loudspeaker.

5.5 Robustness Test

In order to test the robustness of the designed system towards the modelling uncertainties, the number of considered modes $N$ is up to 30 from 10 where (27) is developed. From Fig.17 it can be observed that the controlled system is still stable and the noise attenuation is achieved as well.

However, if we use the lag-gain as 100 instead of 10 in (27), from Fig.18, it can be observed that when $N = 20$ the controlled system becomes unstable.

5.6 Real Time Simulation

A machine noise is recorded and filtered by a low-pass filter with the cutoff frequency at 250Hz. The filtered signal is used as the primary input noise $p(t)$. The lag compensator defined in (27) is used as the ANC controller, and design parameters are selected as $N = 10$, $x_s = x_m = 1.7m$. The measured signal and its FFT analysis are shown in the Fig.19. It can be observed that a good noise attenuation has been obtained within $[0, 250](Hz)$.

6. Conclusion

The ANC design using feedback control techniques for an one-dimensional acoustic duct system is discussed. The state space models as well as the transfer function expressions of the cancelling loudspeaker and the
Acknowledgement

The author would like to thank Dr. David L. Hicks from Aalborg University Esbjerg for constructive discussions.

1Physical tests of our system is undergoing in our Laboratory.

References