A Fuzzy Approach to Uncertainty Principle in Quantum Theory

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Abstract- Uncertainty principle is one of the important bases of the quantum theory. Major concept of this principle is very difficult to understand specially with classical view. In this paper, a new approach to this drawback, using additive fuzzy system theory, is presented. The theory is very suitable to estimation of system behavior which is uncertain inherently. The proposed approach will be explained on approximation of electron path, and the simulation results verify its validity.

Key-Words: Quantum theory, Uncertainty principle, Fuzzy logic.

1 Introduction
Since its inception, the theory of quantum mechanics has advanced in a variety of ways and in many disciplines. Applications of this theory can be found, for example, in atomic technology, chemistry, optics and microelectronics. However, the concepts of this theory are very far from a mind with classical view. In fact it is extremely difficult for a newcomer to the field or for somebody who wants to apply quantum mechanics to his problems to properly recognize these concepts. The most important problem of this theory is the absence of a conventional language for describing the phenomena. On the other hand, there is a gap between classical logic and quantum concepts.

In the recent decades, the theory of fuzzy logic has matured in a wide collection of concepts and techniques for dealing with complex phenomena that do not lend themselves to analysis by classical methods based on bivalent logic. In conventional logic, for instance, a statement can be true or false and nothing in between. Precision of this resulting assumes that the parameters of a model present exactly either our perception of the modeled phenomena or the feature of the real system that has been modeled. Precision also implies that the model contains no ambiguities. However, in many cases this precision is invalid and phenomena contains some uncertainty. The fuzzy theory is an appropriate tool for describing these phenomenon because this theory provides a strict mathematical framework in which vague conceptual phenomena can be precisely studied. Therefore, fuzzy theory can be merged with quantum theory for describing its concepts which are difficult to explain by conventional logic.

The goal of this paper is to help to close the gap between quantum theory concepts and conventional language. The main matter of this discussion is the uncertainty principle. This principle is one of the most important bases of quantum theory and explains the behavior of subatomic particles. Since understanding of this principle is very difficult by classical view and conventional logic, in this paper, a new approach to this principle, based on fuzzy theory, is presented. This approach gives a regular view to the reader for recognizing the uncertainty principle. This paper is divided into five sections. In the next section, the uncertainty principle will be explained. The tools of fuzzy analysis will be described in section III. Usage of these tools in uncertainty principle will be presented in section IV and also a deep view of this principle will be achieved through this
analysis. Section V contains a brief conclusion and some important remarks.

2 UNCERTAINTY PRINCIPLE
One of the most important bases of quantum theory is the uncertainty principle. This principle, which is established by Werner Heisenberg in 1924 [1], governs all things in the Universe. This principle implies that the position and momentum of a particle cannot be measured without error instantaneously.

Now, consider a measuring equipment that observes a particle as shown in Fig. 1.

![Fig. 1. Observation of a particle by a measuring equipment.](image)

Observation of this particle needs at least one photon to be sent from equipment toward the particle. Then the photon will be reflected from the particle to equipment and the particle is observed. Thus, “observation” is the interaction between this photon and particle.

In this observation, the error of position measurement is equal to the wavelength, \( \lambda \), of photon. Therefore, achieving small error in position leads to small wavelengths. On the other hand, Plank equation explains that the following relation exists between the wavelength and the energy of a particle:

\[
E \propto \frac{1}{\lambda}
\]  

(1)

Now, if the wavelength of photon tends to zero, its energy will be infinite. Therefore, the observed particle will be moved out by this large energy as shown in Fig. 2, and measuring its momentum will be done with error.

![Fig. 2. Interaction between a particle and a photon.](image)

In a similar example, Heisenberg presented an inequality between the uncertainty of momentum and position measurement, in the following from

\[
\Delta X \Delta P \geq \frac{h}{2\pi}
\]  

(2)

where \( h \) is the Plank constant. Therefore, instantaneous and errorless measuring of position and momentum of a particle is impossible. This inequality is the base of the following analysis by fuzzy theory.

3 FUZZY FUNCTION APPROXIMATION
A fuzzy system is a set of fuzzy rules that maps inputs to outputs. So it defines a function \( f : X \to Y \). The rules are if-then rules in the form of “If \( X \) is \( A \), then \( Y \) is \( B \)” where \( A \) and \( B \) are fuzzy sets that contain members to some degree. A fuzzy system approximates a function by covering its graph with fuzzy patches and averaging patches that overlap. Additive fuzzy systems are a convenient tool for estimating a function in the fuzzy system categories. An additive fuzzy system can uniformly approximate continuous or measurable functions [2], [3].

Each fuzzy rule defines a fuzzy patch or a cartesian product \( A \times B \) as shown in Fig. 3 [2]. The fuzzy system covers the graph of a function with fuzzy patches. Uncertain fuzzy sets give a large patch while, small or more certain fuzzy sets give small patches. Additive fuzzy systems fire all rules in parallel and average the scale output sets to give the output fuzzy set \( B \) as in Fig. 4 [2], [4].
In this figure, $W_1, W_2, \ldots$ and $W_n$ are the weighting functions that scale the importance of fuzzy sets compared to others. Defuzzification with correlation product inference gives the output $y$ for given input $x$:

$$y = \frac{\sum W_i a_i(x) V_i c_y}{\sum W_i a_i(x) V_i} \quad (3)$$

where $V_i$ is the volume of the $i$-th output set $B_i$, $c_y$ is the center of this set and $a_i(x)$ is the $i$-th membership of input $x$. This equation allows to approximate a function by some measured data.

4 APPROXIMATING THE PATH OF AN ELECTRON

In this section the results of additive fuzzy systems are applied to path of an electron. The path of an electron is the basic example of uncertainty principle. Fig. 5 shows a typical path of an electron in a magnetic field. This path is defined in the position-momentum plane because the uncertainty principle is applied to these parameters. In this figure, the amount of position and momentum are expressed in terms of normalized values for simplicity. This assumption does not affect the generality of the following analysis.

Some experiments have been accomplished on this electron and some data has been achieved. These data use the relation (2) and have some uncertainty. According to these data, fuzzy sets of this path are shown in Fig. 6. It is assumed that the fuzzy sets have no overlap on one axis. It means that next the observation is performed out of uncertainty boundaries of previous observation. These fuzzy sets in an additive fuzzy system can estimate the desired function that corresponds to the path of an electron.
Fig. 7 shows an approximation to this path based on additive fuzzy system approximation. As already expressed, the following algorithm is employed in this approximation.

- Some samples of electron path are observed. In these observations, the uncertainty of position or momentum is known.
- Fuzzy sets of an axis whose uncertainty is known are established.
- Minimum uncertainty of other quantity is calculated based on equation 2.
- Fuzzy sets of the latter quantity are established.
- Approximation of electron path is performed by additive fuzzy theory.

Fig. 8 shows the effect of the Plank constant on the estimation. The error in the estimation grows with increasing the Plank constant. This is an important result in uncertainty principle.
As is seen in these figures, increasing the Plank constant leads to poor approximation. This effect has already been predicted by quantum theory but here it is converted to a simple approximation problem. This problem is solved by additive fuzzy logic theory without having any difficulty.

5 CONCLUSION
Additive fuzzy systems are suitable tools for estimating the quantities which are joined with uncertainty. Uncertainty principle is the base of quantum theory and governs to every thing in the nature. Concept of this principle is very difficult in conventional physics. In this paper, an additive fuzzy system has been proposed to simply express the uncertainty principle. By this additive fuzzy system, the path of an electron is approximated successfully. Simulation results show that uncertainty principle can be converted to a simple approximation problem. Also the effect of the Plank constant is shown on the estimated electron path that verifies the ability of an additive fuzzy system estimator.

References: