Analysis of 1/f Nature of Sea-Wave Noise Data

TOLGA ESAT ÖZKURT1   TAYFUN AKGÜL2
1Computer Science Department
2Department of Electronics and Communications Engineering
Istanbul Technical University
34469 Maslak, Istanbul
TURKEY

Abstract: - In this paper we give some examples of 1/f processes and show the (nearly) 1/f behavior of sea-wave noise data. Wavelet-based method is used as a tool to analyze the power-law relationship that is observed in the wavelet coefficients’ variance progression.

Keywords: - 1/f processes, self-similarity, wavelet-based analysis.

1 Introduction

1/f processes are a family of fractal signals that show statistically self-similar behavior. There are many natural and man-made phenomena that exhibit 1/f behavior, i.e., physiological heart-rate records, economical time-series, rainfall data, variations in traffic flow, electromagnetic fluctuations [1]. From our experiments, we observe that nearly 1/f behavior is also seen for sea-wave noise data.

Our approach is based on estimating the spectral exponent ($\gamma$) and/or the Hurst parameter ($H$), which are called the self-similarity parameters, using a wavelet decomposition method.

2 Background

In this section, we provide the background information on 1/f processes and wavelet-based analysis method.

2.1 1/f Processes

Self-similar time series are related with the following power law relation [1]:

$$S_x(\omega) \approx \frac{\sigma_x^2}{|\omega|^\gamma}$$

(1)

where $S_x(\omega)$ is the power spectrum of the time series $x(t)$, $\sigma_x^2$ is the variance and $\omega$ is the angular frequency. Such signals have the scaling form as:

$$x(t) = a^{-d/t} x(at)$$

(2)

where $d$ denotes statistical equivalence and $a$ is a positive real constant. This means 1/f processes have the same statistical behavior against different dilations of the time axis and hence, different scales.

2.2 Wavelet-based Analysis of 1/f Noise

Wavelet transform is an effective tool to analyze 1/f processes. This efficiency especially comes from its suitability with nonstationarity while any other linear transformations such as the Fourier transform can just deal with stationary processes. The wavelet transform of a signal with an orthonormal wavelet basis $\psi(t)$ can be expressed as:

$$x(t) = \sum_n \sum_m x_n^m \psi_n^m(t)$$

(3)

where $n$ is the translation and $m$ is the dilation indices.

Wavelet coefficients $x_n^m$ are weakly correlated and zero mean random variables that have variances:

$$\text{var} x_n^m = \sigma_x^2 2^{-m \gamma}$$

(4)

Taking the logarithm of both sides, the spectral exponent $\gamma$ can be estimated by a linear-fit algorithm.

3 Algorithm

In our algorithm after applying the wavelet-based method and obtaining the wavelet coefficients, we estimate the variances of these coefficients for various scales. Then, we check the variance progression scale-to-scale to decide whether the data is exhibiting nearly 1/f behavior. This is determined by a linear-fit error, which is calculated by summing the squares of the differences of the predicted and the real scale values. Then, this error is normalized with the total number of scales. If this error is small, and hence, the data segment is accepted as nearly 1/f, we estimate the spectral exponent $\gamma$ by this slope. Note that for smaller scales, the number of
coefficients is not enough to estimate the variance reliably. Therefore, in our linear-fit algorithm, we choose the ones that have more than 40 coefficients for a scale to be statistically acceptable to estimate its variance.

4 Examples

Mathematical characteristics of 1/f processes can be represented through fractional Gaussian noises (fGn), which are stationary Gaussian statistically self-similar signals, and their integrated forms fractional Brownian motions (fBm). Both are well-known and synthesizable fractal time series. For fBm, the relationship between $\gamma$ and $H$ is $\gamma = 2H - 1$, while for fGn, it is $\gamma = 2H - 1$. It can be shown that $H$ is less than unity and greater than zero [2]. Therefore they can be models of 1/f processes only when $\gamma$ is between -1 and 3, which is the general case.

In Fig. 1 and Fig. 2, two simulation examples for fBm with $\gamma = 1.8$ and fGn with $\gamma = 0.8$ are given. Note that they are synthesized by the so-called spectral synthesis method [3].

In Fig. 4, we present daily wind speed data, which is recorded in a meteorological station in the Republic of Ireland where 1/f noise structure observed. (We only take the first 1024 days since the beginning of 1961.) The slope of the variance progression of the wavelet coefficients is estimated as 0.7024 with a linear-fit error 0.0238.

In Fig. 5, weekly Dow Jones Industrial Average is given for the first 1024 days from the beginning of 1989. It has also 1/f noise structure with the spectral exponent 1.7776 and the linear-fit error 0.0709.

By exploiting 1/f behavior of various signals similar to our examples, different signal processing applications can be achieved. For instance, in [4], using 1/f behavior of heart rate variability, a modeling and simulation method is presented. In [5], by using 1/f nature of speech residual signal, a speech quality enhancement approach is proposed. In [6], a novel deconvolution method is developed for systems with 1/f type input signals.

\[\begin{array}{|c|c|c|}
\hline
\text{Figure No} & \text{Nearly 1/f?} & \text{Estimated spectral exponent} \\
\hline
1 & yes & 1.7677 \\
2 & yes & 0.7618 \\
3 & yes & 0.6107 \\
4 & yes & 0.7024 \\
5 & yes & 1.7776 \\
6 & yes & 1.6287 \\
7 & yes & 2.0388 \\
8 & yes & 1.9681 \\
\hline
\end{array}\]

Table 1

1 Recorded in Children Hospital, University of Pittsburgh.

\[\begin{array}{|c|c|c|}
\hline
\text{Figure No} & \text{Nearly 1/f?} & \text{Estimated spectral exponent} & \text{Linear-fit error} \\
\hline
1 & yes & 1.7677 & 0.0027 \\
2 & yes & 0.7618 & 0.0254 \\
3 & yes & 0.6107 & 0.1706 \\
4 & yes & 0.7024 & 0.0238 \\
5 & yes & 1.7776 & 0.0709 \\
6 & yes & 1.6287 & 0.4642 \\
7 & yes & 2.0388 & 0.0869 \\
8 & yes & 1.9681 & 0.4208 \\
\hline
\end{array}\]

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\hline
\end{array}\]

\[\begin{array}{|c|c|c|}
\hline
2 \text{ http://lib.stat.cmu.edu/datasets/wind.desc} \\
3 \text{ http://fisher.osu.edu/fin/resources_data/data/dow1900.txt} \\
\hline
\end{array}\]
5 Experiments

We use the sea-wave noise data which is recorded by the shore of Pacific ocean. The water–pressure sensor, namely hydrophone is located approximately 10 m. below the sea level and 1 m. above the sea-bed to record the sound of the sea-waves that reach the shore. We process only three different segments of this data. Each segment has a length of 8192 which corresponds a duration of 0.16 seconds.

For each segment the wavelet-based analysis is applied. Results are shown in Figures 6-8. It is observed that each segment exhibits $1/f$ behavior with slightly different slope values.
Figure 8. (a) Another segment of sea-wave noise data. (b) Variances of the wavelet coefficients. The slope of the variance progression is estimated as 1.9681 with a linear-fit error 0.4208.

6 Conclusion

We have used our wavelet-based method to analyze the sea-wave noise data and observed that it has $1/f$ noise structure (when it is free from any man-made disturbance, i.e., fishing, etc.) Values of estimated spectral exponents for each data are given in Table 1.

By this analysis, we show that a single parameter can help detecting the sudden and/or long term disturbances in the environment.

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References