Performance Investigation of a Broadband OFDM System Employing Spatial Diversity and Turbo Coding

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Abstract: Diversity is a powerful communication receiver technique that provides wireless link improvement at relatively low cost. Furthermore, there are a wide range of diversity implementations, many of which are very practical and provide significant link improvement with little added cost. In this paper a well-known diversity technique, such as maximal ratio combining (MRC) is, will be investigated. This scheme will be applied in systems utilising several well known (convolutional coding-CC) and recently presented (turbo coding-TC) techniques used to combat the impact of selective fading on the OFDM system performance. Also the adaptivity technique through bit loading has been considered for a further improvement in system performance.

We will demonstrate that the turbo coding scheme, especially when combined with MRC, provides sufficiently robust performance, rendering it a potential candidate for next generation broadband wireless communication systems, with the aim of providing extremely high data rate, bandwidth efficient signalling and improved quality of service (QoS).


1 Introduction
Orthogonal Frequency Division Multiplexing (OFDM) [1] has become an important method of digital transmission in several wire line and wireless applications such as xDSL, digital audio broadcasting (DAB), digital terrestrial television and wireless LANs. The reasons for such wide use are: high spectral efficiency, large flexibility of matching to the transmission channel properties and relatively easy high-speed modem implementation as compared with sophisticated high bit-rate serial transmission supporting the same data rate in the same bandwidth.

Forward error correction (FEC) can be incorporated with OFDM to improve the bit error rate (BER) performance. In the IEEE 802.11a standard, convolutional coding and Viterbi decoding have shown to yield low BER in wireless data applications.

In order to further improve the performance of convolutional coding, several adaptive modulation schemes have been recently suggested [2, 3, 4]. Turbo codes [5] have also drawn much interest particularly since they provide greater BER performance and flexibility in controlling bandwidth utilisation and QoS.

In our simulation studies we have compared the OFDM BFWA system performance when the following means were applied:
- Convolutional coding (CC) and Viterbi decoding,
- Adaptive modulation with CC (AM-CC),
- Turbo coding (TC),
- Maximal ratio combining with turbo coding (MRC-TM).

2 System Model
An OFDM signal [6], as depicted in Fig. 1, consists of a sum of subcarriers, which can be generated by applying the inverse Fourier transform. Modulation can be based on either phase shift keying (PSK) or quadrature amplitude modulation (QAM).

The subcarriers spacing is a multiple of $1/\tau = 1/NT$ (N = number of subcarriers and T is the sampling time), thus they will be linearly independent or orthogonal where their sidebands overlap, while still allowing reception without interference from adjacent carriers.

The OFDM signal’s orthogonality can be easily maintained by applying inverse fast Fourier transform (IFFT, 128 size for our simulations) to
the vector $m_j$ to transform from frequency-domain to time-domain. The only obstacle to use the variant of Fourier transform for generating the OFDM symbol is the non-periodical nature of the time domain signal.

Therefore, inserting cyclic prefix (CP), Fig. 2, that comprises the last $L$ length of an active OFDM symbol makes the symbol appear periodically at the receiver. The addition of a CP also creates a guard band around individual OFDM symbols to allow for delay spread from the previous symbol which greatly reduce inter symbol interference (ISI) and inter channel interference (ICI) [1].

At the receiver, the reverse process is performed. The OFDM signal is sampled into discrete-time sequence and the CP is removed from the OFDM symbol, Fig. 3. Assuming that the CP completely eliminates ISI and ICI, the time-domain sequence is converted to frequency-domain by the OFDM demodulator (FFT) where the in-phase and quadrature component are generated.

**3 Channel Model**

In this study an indoor line-of-sight (LOS) model was used in the 17GHz band with a coherence time of 1 ms and a Doppler spectrum of 96 Hz (speed of 6 Km/h). The channel’s main wideband parameters are 20.72MHz for coherence bandwidth, 101ns maximum delay ($T_{MAX}$) and 17.5ns for the RMS delay spread. The impulse response was generated assuming Wide-Sense Stationary and Uncorrelated Scattering (WSSUS). In addition to that, the channels path loss exponent is 1.68 and its K-factor equal to 9. Finally during our simulations, perfect timing and frequency synchronization were assumed.

**4 Bit Loading Algorithms**

In an OFDM system, it is possible to adjust the modulation level for each subcarrier individually. This technique, known as bit loading, is also referred to as adaptive modulation. In this approach several bits are allocated to subcarriers with a high signal-to-noise ratio (SNR), whereas on subcarriers with low SNR only few or no bits at all are transmitted. Loading algorithms, which mainly differ in their optimisation criteria and computational load perform bit allocation. Three of them are presented herein.

**4.1 The Chow-Cioffi Algorithm**

This algorithm consists of three main sections. First, an optimal system performance margin, $\gamma_{\text{margin}}$, is approximately found, then convergence is guaranteed with a sub optimal loop, and finally the energy distribution is adjusted accordingly on a subcarrier-by-subcarrier basis [2].
4.2 The Fischer-Huber Algorithm
Based on knowledge of the subcarriers noise variances, the Fischer-Huber algorithm, depicted in [3], first calculates the bit rate \( R_i \) using the following equation:

\[
R_i = \frac{(RT + \log(N_i))}{D} - \log(N_i)
\]

(1)

where \( RT \) is the target bit rate, \( \log(N_i) \) is the log of the noise variances and \( D \) is the number of carriers. In the second step, \( R_i \) is quantized to \( R_{Qi} \), and the quantization error \( \Delta R_i = R_i - R_{Qi} \) is determined. The third part of the algorithm is adopted from [2]. If \( \sum R_{Qi} < (>) R_T \), the rate of the channel with the largest (smallest) \( \Delta R_i \) is incremented (decremented). The algorithm stops if \( \sum R_{Qi} = R_T \).

4.3 The Piazzo Algorithm
The Piazzo algorithm, illustrated in [4], is based on knowledge of channel’s frequency-domain attenuation vector. Consequently, all the bits are initially assigned to the highest modulation type (64QAM). Then a series of bit reallocations is performed, reducing the transmit power upon each reallocation, thus leading to an optimum bit allocation in each subcarrier.

5 Turbo Codes
Since its introduction in 1993 [5], turbo codes (TC) have attracted tremendous interest due to its remarkable performance. TC is also known as parallel-concatenated convolutional code (PCCC). The code [7] employs two identical recursive systematic convolutional (RSC) encoders connected in parallel, with an interleaver preceding the second RSC, as depicted in Fig. 4. Depending on the code rate desired, the parity bits from the two constituent encoders are punctured before transmission.

The TC data can be decoded by using either Log-MAP or SOVA algorithms [8, 9]. The two decoders are arranged serially in a simplified turbo decoder structure. The OFDM demodulator’s outputs are de-multiplexed into three streams of M data, while the punctured bits are replaced with zeros, and are then fed to the constituent decoder.

6 Maximal Ratio Combining
In this method first proposed by Kahl [10], the signals from all of the \( M \) branches are weighted according to their individual signal voltage to noise power ratios and then summed. Figure shows a block diagram of the technique. Here, the individual signals must be co phased before being summed (unlike selection diversity) which generally requires an individual receiver and phasing circuit for each antenna element.

Maximal ratio combining produces an output SNR equal to the sum of the individual SNR’s, as will be explained hereunder. Thus, it has the advantage of producing an output with an
acceptable SNR even when none of the individual signals are themselves acceptable. This technique gives the best statistical reduction of fading of any known linear diversity combiner.

### 6.1 Derivation of MRC Improvement

In maximal ratio combining, the voltage signals $r_i$ from each of the $M$ diversity branches are co phased to provide coherent voltage addition and are individually weighted to provide optimal SNR. If each branch has gain $G_i$, then the resulting signal envelope applied to the detector is

$$ r_M = \sum_{i=1}^{M} G_i r_i $$

(2)

Assuming that each branch has the same average noise power $N$, the total noise power $N_T$ applied to the detector is simply the weighted sum of the noise in each branch. Thus

$$ N_T = N \sum_{i=1}^{M} G_i^2 $$

(3)

which results in an SNR applied to the detector, $\gamma_M$, given by

$$ \gamma_M = \frac{r_M^2}{2N_T} $$

(4)

Using Chebychev’s inequality [11], $\gamma_M$ is maximized when $G_i = \frac{r_i}{N}$, which leads to

$$ \gamma_M = \frac{1}{2N} \left( \frac{r_i^2}{N} \right) = \frac{1}{2} \sum_{i=1}^{M} \frac{r_i^2}{N} = \sum_{i=1}^{M} \gamma_i $$

(5)

Thus, the SNR out of the diversity combiner (Fig. 6) is simply the sum of the SNRs in each branch.

The value for $\gamma_i$ is $\frac{r_i^2}{2N}$, where $r_i$ is equal to $r(t)$. The received signal envelope for a fading mobile radio signal can be modeled from two independent Gaussian random variables $T_c$ and $T_s$, each having zero mean and equal variance $\sigma^2$. That is,

$$ \gamma_i = \frac{1}{2N} \frac{r_i^2}{N} = \frac{1}{2N} \left( T_c^2 + T_s^2 \right) $$

(6)

Hence $\gamma_M$ is a Chi-square distribution of $2M$ Gaussian random variables with variance $\sigma^2 / (2N) = \Gamma / 2$. The resulting pdf for $\gamma_M$ can be shown to be

$$ p(\gamma_M) = \frac{\gamma_M^{M-1} e^{-\gamma_M / \Gamma}}{\Gamma^M (M-1)!} \quad \text{for } \gamma_M \geq 0 $$

(7)

The probability that $\gamma_M$ is less than some SNR threshold $\gamma$ is

$$ P_r \{ \gamma_M \leq \gamma \} = \int_{0}^{\gamma} p(\gamma_M) \, d\gamma_M $n

$$ = 1 - e^{-\gamma / \Gamma} \sum_{k=1}^{M} \left( \frac{\gamma / \Gamma}{(k-1)!} \right) $$

(8)

The probability distribution for maximal ratio combining derives from (8). It follows directly from (5) that the average SNR, $\gamma_M$, is simply the sum of the individual $\gamma_i$ from each branch. In other words,

$$ \gamma_M = \sum_{i=1}^{M} \gamma_i = \sum_{i=1}^{M} \Gamma = M \Gamma $$

(9)

The control algorithms for setting the gains and phases for maximal ratio combining receivers are similar to those required in equalizers and RAKE receivers. Fig. 6 illustrate maximal ratio combining structures. Maximal ratio combining can be applied to virtually any diversity application, although often at much greater cost and complexity than other diversity techniques.

### 7 Simulation Results

Intensive simulation studies have been performed using the MATLAB simulation package for all the OFDM systems considered above. Below we will present selected BER vs. SNR curves for all the implemented scenarios.

The parameters applied for the OFDM system and channel model are the following:

- The sampling frequency equals to 50MHz,
- The channel estimation was performed using the two long preambles precedent the OFDM frame.
- Each frame had a variable number of OFDM symbols, varying from 18 to 100,
- The OFDM burst consisted of 10 frames.
- The number of active OFDM carriers is 100,
- The number of nominal OFDM carriers is 128,
- The number of the cyclic prefix is 22 carriers; hence the total length of an OFDM symbol is 150 (128 + 22).
- The average number of bits carries carried by each subcarrier is $m = 1, 2, 4, 6$ - in average BPSK, 4 QAM, 16QAM or 64QAM has been applied.
- The total number of transmitted bits is $10^5$, since the target BER is $10^{-3}$.

Fig. 7 and Fig. 8 present the BER vs. SNR curves using CC with and without applying adaptivity, which is, as mentioned above, implemented by three bit-loading algorithms - Cioffi, Fischer and
Piazzo. Clearly the system incorporating AMCC has better performance, with all three algorithms, than the simple CC one. This improvement is interpreted, in terms of dB, in a gain of about 2dB for low data bit rates (Fig. 7) and 3.8dB for high ones (Fig. 8). In both cases these values are achieved by using the Piazzo algorithm.

The next step involved the implementation of a turbo coder. The encoder was a PCCC one, as describes in the literature [7], using a random interleaver. For the decoding process two decoding algorithms were used, Log-Map and SOVA, whose performance is depicted in Fig. 9. It can be observed that the Log-Map algorithm outperforms the SOVA one in all modulation schemes offering a step up of 0.7dB in average, which is in accordance with the theory [9].

Fig. 10 presents the direct comparison of TC and AMCC using the Piazzo algorithm. Studying the above plot, we can see that both TC algorithms outperform AMCC in either of the bit rate cases. The offered gain is about 2.22dB (Log-Map) for 13.34Mbps. For 53.34Mbps the gain falls to 0.88dB using again the Log-Map decoding algorithm.

Fig. 7: BER vs. SNR using CC and three adaptation algorithms at 13.34Mbps.

Fig. 8: BER vs. SNR using Convolutional coding and three adaptation algorithms at 53.34Mbps.

Fig. 9: BER vs. SNR using both the turbo - decoding algorithms, Log-Map and SOVA.

Fig. 10: BER vs. SNR using TC and AMCC at 13.34Mbps and 53.34Mbps.

Fig. 11: BER vs. SNR applying maximal ratio combining to the turbo coded system.

Fig. 11 brings out the performance of both the decoding algorithms, and for all the mapping
schemes, when MRC is applied to the system. Again the Log Map algorithm precedes the SOVA one, with a smaller gain this time, varying from 0.5 to 0.85dB.

Concluding with our simulations [12], we compared the TC systems performance with and without the MRC diversity technique being utilized. Fig. 12 and Fig. 13 illustrate this comparison. More precisely the combination of MRC and TC offers an improvement in performance by almost 2.1dB and 1.85dB using the Log Map and SOVA decoding algorithms respectively.

Generally, form Fig. 7 and Fig. 8 one concludes that the bit loading scheme results in better performance compared to CC. The gain was as much as 3.8dB at the error rate of $10^{-3}$.

The system with TC offered improvement performance compared to CC or even to AMCC in all data rate cases, an improvement, which reached 2.2dB. Similar upgrading, compared to TC, was offer by the MRC-TC system that utilized a diversity technique.

Thus, it can be asserted that TC and especially MRC-TC has a strong potential for use in OFDM packet transmission. Further simulations will follow in order to examine other diversity techniques and environment scenarios.

8 Conclusion
The plots shown in the previous section are a small but representative part of the simulation results achieved in our research. The have been achieved for the system with different PSK and QAM modulations and coding schemes.

Reference: