Performance of hybrid Turbo coding scheme for optical ppm communication systems

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Abstract – In this paper, the performance of hybrid Turbo codes over an optical PPM communication channel employing an avalanche photodiode (APD) detector is analyzed and simulated. Hybrid Turbo codes belong to a class of concatenated codes with the parallel concatenation of an interleaved convolutional code with a serially concatenated convolutional code. The simulations are carried out with Gaussian approximation model of APD output statistics. The results show that in less than six iterations, better bit-error rates (BER) can be obtained than reported in the literature corresponding to additive white Gaussian noise (AWGN) channels. For example, for -90dBW of received laser power, all the bits are received correctly after the third iteration.

Key-Words: - Turbo Codes, Hybrid Turbo Codes, Optical Pulse Position Modulation

1 Introduction
Turbo codes, also known as parallel concatenated convolutional codes (PCCC’s), offer excellent error performance near the Shannon capacity limit with reasonable decoding complexity [1]. Concatenated coding schemes are effective in dealing with bursts and random errors and at rates less than channel capacity, probability of error decreases exponentially, while decoding complexity increases only algebraically. High coding gains can be obtained with concatenated codes and with Turbo codes very high coding gains are achieved.

Turbo encoder comprises of two or more recursive systematic convolutional (RSC) encoders, which may be identical or different, that are preceded by different interleavers and whose output bits are concatenated in parallel. A Turbo decoder consists of the corresponding decoders with a de-interleaver in between to modify error patterns in the received sequence. It employs iterative decoding [2],[3]. The output of each component decoder contains three terms: systematic information generated by the code information bit, extrinsic information generated by the code parity bit and a priori information generated by the other decoder. The extrinsic information is exchanged between the two component decoders. It has been confirmed by computer simulation that by increasing the number of iterations, a bit-error rate (BER) of $10^{-4}$ can be obtained at an information bit signal-to-noise ratio $E_b/N_0$ as low as -0.15dB [4].

Using the same components, namely convolutional encoders and interleavers, serially concatenated convolutional codes (SCCC’s) have been shown to yield performances comparable, and in some cases of large photon counts, superior to Turbo codes [5]. Another possible concatenated code is hybrid concatenation of convolutional codes (HCCC’s). In this paper, we consider, as an example of HCCC, only the parallel
concatenation of a convolutional code with a serially concatenated convolutional code. This particular HCCC is referred as the hybrid Turbo codes in this paper.

These concatenated coding schemes use a suboptimum decoding process based on the iterations of the maximum a posteriori (MAP) algorithm [2] applied to each constituent code. A soft-input, soft-output (SISO) MAP module described in [3] was used.

As a modulation format for optical communications, ON-OFF keying (OOK) and pulse position modulation (PPM) have primarily been employed [6],[7]. Since PPM has better power efficiency than OOK, PPM has been chosen as the modulation scheme in this paper. It was shown in [8] that Turbo codes can provide substantial coding gain with a reasonable system complexity for optical PPM systems. This paper aims at exploring the potential of hybrid Turbo-coded optical PPM systems. It is assumed that the optical channel is an intensity modulated (IM) channel and that the received optical signal is detected using a direct-detection (DD) scheme. It is also further assumed that the receiver is thermal noise limited. This paper is organized as follows. System model and structures for hybrid Turbo encoder and decoder are described in Section II. In Section III, a receiver for an optical PPM system using Avalanche Photodiode (APD) is modeled. In Section IV, hybrid Turbo decoding algorithms are described. Simulation results are presented in Section V and conclusions are drawn in Section VI.

2 SYSTEM MODEL

2.1 System Description

The block diagram of a hybrid Turbo-coded optical PPM communication system is shown in Fig. 1. The encoder consists of two parallel branches. The upper branch consists of an interleaver ($\pi_1$) followed by a parallel encoder ($C_p$) and is called as the parallel branch. The lower branch contains an outer encoder ($C_o$), another interleaver ($\pi_2$) and an inner encoder ($C_i$) and is called as the serial branch. Thus, the encoder it is composed of three concatenated codes: the parallel code, the outer code and the inner code. The code rates for the three codes may or may not be the same. The component encoders may use either nonrecursive systematic convolutional (NSC) coders or RSC coders. But it is shown through analytical calculations in [9] that to achieve the highest interleaving gain for HCCC, we should select the component codes as follows. 1. A recursive inner encoder, 2. a recursive parallel encoder and 3. an outer encoder that can be either nonrecursive or recursive but that should have large minimum Hamming distance. The details of these component coders used in the simulations are presented in Section II(B).

The information bit sequence is fed into both the branches. The output from these two branches is then concatenated in parallel form before being fed to the optical modulator. The modulator employs binary PPM (BPPM) scheme. The transmitter then sends a pulse sequence in one of two time slots to represent the given bit. The slot represents the duration of each symbol in the $M$-ary PPM signaling format which has $M$ slots (here, $M=2$); i.e., the laser is pulsed on at the first chip position corresponding to the given symbol.

The BPPM symbols are then intensity modulated and transmitted through a fiber optic channel. At the receiver a direct-detection scheme is used to detect the optical signal. The output of the APD is demodulated and the soft outputs extracted are fed to the hybrid Turbo-decoder, here after referred to as the decoder, unless otherwise specified. The decoder uses three SISO modules to decode the hybrid codes. The SISO modules are based on MAP algorithms. The SISO module is a four-port device that accepts at the input the sequences of probability distributions $P(c|I)$, $P(u|I)$ and outputs the sequences of probability distributions $P(c|O)$ and $P(u|O)$
based on its inputs and on its knowledge of the trellis section (or code in general). Here \( P(c_i) \) and \( P(u_{ij}) \) represent the sequence of a priori probability (APP) distributions for the output and input symbols respectively. A detailed description of the MAP decoding process is given in Section IV. The decoder uses interleaving and de-interleaving functions to pass the extrinsic information from one SISO block to another in the right form which are connected in a feed forward and feedback manner, as shown in Fig. 1.

2.2 Component Structures

The structures of the component encoders used in the simulations are shown in Fig. 2. A rate 1/2 RSC parallel encoder is chosen with generator polynomial, \( G_p(D) = [1, 1+D^2/1+D+D^2] \), or in octal form (1,5/7). It is shown in Fig. 2(a). Fig. 2(b) shows a rate 1/2 NSC outer encoder with generator polynomial \( G_o(D) = [1+D+D^2, 1+D^2] \), or in octal (7,5). A rate 2/3 RSC inner encoder has been chosen with generator polynomial

\[
G_i(D) = \begin{bmatrix}
1 & 0 & 5/7 \\
0 & 1 & 3/7
\end{bmatrix}, \quad \text{or in octal }
\begin{bmatrix}
1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 &
where $G$ is the average APD gain, $k$ is the ionization ratio and $\Gamma$ is the gamma function. Averaging over the exact number of absorbed photons, we obtain

$$p_w(m|\bar{n}) = \sum_{n=1}^{m} p(m/n) \frac{-n}{n!} e^{-\bar{n}}$$  \hspace{1cm} (4)

as the distribution of APD output electrons generated over $T_s$ seconds given the mean number of absorbed photons in that interval. If no photons are absorbed, then no electrons can be generated according to this model. Conversely, no electrons are generated only if no photons have been absorbed. Hence $p(m = 0|\bar{n}) = p(n = 0|\bar{n}) = e^{-\bar{n}}$.

The pdf in Equation (4) has been approximated by Webb [13] and provides much simpler expression for the density of $m$ in response to the mean number of photons absorbed:

$$p_w(m|\bar{n}) = \frac{\exp\left(-\frac{(m-G\bar{n})^2}{2nG^2F(1+\frac{m-G\bar{n}}{nGF/(F-1)})}\right)}{\sqrt{2\pi nG^2F(1+\frac{m-G\bar{n}}{nGF/(F-1)})^{3/2}}}$$

where $F$ is the excess noise factor given by

$$F = kG + (2-1/G)(1-k)$$  \hspace{1cm} (6)

As shown in Fig. 3, the APD output is the sum of the charge due to Webb-distributed secondary electron emissions and Gaussian-distributed amplifier thermal noise. The total charge is then integrated over each slot time $T_s$. The APD output then consists of two soft outputs, each a continuously distributed random variable independent of the other soft output. Since the slot statistic is a random variable consisting of the sum of independent Webb and Gaussian random variables, its density function is the convolution of the individual densities, and may be written as

$$p(x|\bar{n}) = \sum_{m=0}^{\infty} \phi(x,\mu_m,\sigma^2) p_w(m|\bar{n})$$  \hspace{1cm} (7)

where $\phi(x,\mu_m,\sigma^2)$ is the Gaussian density function with the mean $\mu_m = mq + I_sT_s$ and the variance $\sigma^2 = \left(2qI_s + \frac{4\kappa T}{R}\right)B\bar{T}_s^2$, as given in [14]. Here $\kappa$ is Boltzmann’s constant, $T$ is the equivalent noise temperature, $B$ is the single-sided noise bandwidth, $R$ is the receiver load resistance, and $I_s$ is the APD surface leakage current. Note that the APD surface leakage current is not multiplied by the APD gain and is modeled here as a constant DC current. The APD dark current on the other hand, is multiplied by the APD gain. In an optical fiber medium there will be negligible background noise and so $\bar{n}_b = 0$.

However computation of Equation (7) is tedious even though it is accurate. To simplify calculations, it is easy to model the density of the APD output electron charge as Gaussian with mean $qG\bar{n}$ and variance $q^2G^2F\bar{n}$. Then the slot statistic consisting of the sum of APD output electrons and amplifier thermal noise is also Gaussian [10], and has mean $\mu = qG\bar{n} + I_sT_s$ and variance $\sigma^2 = \left(2q^2G^2F\bar{n} + qI_sT_s + \frac{4\kappa TT_s}{R}\right)B\bar{T}_s^2$.

4 Hybrid Turbo Decoding

An iterative decoding algorithm for decoding hybrid concatenated convolutional codes is given in [9]. Each SISO block in the decoder has two inputs and two outputs (where there is only one output is shown, it is assumed that the other port is not used). The input labeled $\lambda(c;l)$ represents the reliability of the unconstrained output symbols of the encoder, while that labeled $\lambda(u;l)$ represents the unconstrained input symbols.
of the encoder. Similarly, the outputs represent the same quantities conditioned to the code constraint. The SISO APP module updates reliability of both the input and output symbols based on the code constraints. Both the outputs of the SISO, i.e., \( \lambda(c;O) \) and \( \lambda(u;O) \), directly generate the “extrinsic” information required for iterative decoding. Although this structure was originally designed as a turbo decoder for a binary AWGN channel, part of its beauty is that each SISO module need not know anything about the channel in order to operate – its computation is determined completely by the a priori distributions \( \lambda(c;I) \) and \( \lambda(u;I) \). We use the same definition for \( \lambda \) as given in [9]. Thus with a given a priori probability distributions, the Turbo decoder operates in the usual way.

To properly initialize the Turbo decoder, we must determine the a priori probability of code words for each constituent code. We will now develop the theory for inner SISO decoder and same can be extended to parallel SISO decoder. Let \( c = (c_0, c_1, c_2) \) denote a possible output from the inner encoder. As in [3], we define \( P(c;I) = \{ P_k(c;I) \} \) for \( k = 1,2,\ldots,N \), where

\[
P_k(c;I) = P[\text{codeword is } c \text{ at time } k \text{ | all soft counts from all symbols }]
\]

(8)

Note that all soft counts in the conditioning the Equation (8) are irrelevant to computing \( P_k(c;I) \) except those that represent the \( k \)th constituent codeword bits in the PPM symbol. Since binary PPM is employed, the component bits, say \( x_{1p}, x_{2p} \) and \( x_{3p} \), from the \( k \)th constituent codeword are always grouped into three separate PPM symbol epochs \( r, s \) and \( t \). Let \( x_{1p} \) be the \( r \)th bit of the \( r \)th PPM symbol, let \( x_{2p} \) be the \( m \)th bit of the \( s \)th PPM symbol, let \( x_{3p} \) be the \( n \)th bit of the \( t \)th symbol, and also let \( x = (x_0, x_1), y = (y_0, y_1) \) and \( z = (z_0, z_1) \) denote the three sets of soft counts from the three transmitted PPM symbols. Then it follows from the independence of the soft counts from one PPM symbol to the next:

\[
P_k(c;I) = P_k(c_0, c_1, c_2;I)
\]

(9)

\[
= P(\text{bit } l \text{ of } r \text{th PPM symbol is } c_0 \mid x) \\
\times P(\text{bit } m \text{ of } s \text{th PPM symbol is } c_1 \mid y) \\
\times P(\text{bit } n \text{ of } t \text{th PPM symbol is } c_2 \mid z)
\]

(10)

\[
= P(a_l = c_0 \mid x) P(a_m = c_1 \mid y) P(a_n = c_2 \mid z)
\]

(11)

Let us denote \( a = \{a_i\}, a_i \in \{0,1\}, i \in \{l,m,n\} \), then \( P(a_i) \) represents a priori probability of each slot containing a pulse, where the \( i \) denotes the output soft counts and it can be computed using Bayes rule [10],[15] and is given by the following expression

\[
P(a_i) = \frac{L_a}{\sum_{j=0}^{I} L_j}
\]

(12)

where we use \( a \) as both a PPM symbol and an index to a slot, and where \( L_j \) is the likelihood ratio of the \( j \)th slot containing a signal pulse given the soft count in that slot, i.e., the ratio of Equation (5) with \( \bar{a} \) equal to mark photons as given in Equation (2a) to Equation (5) with \( \bar{a} \) equal to space photons as given in Equation (2b). Equation (12) may be used in Equation (10) to compute the a priori probabilities needed for the SISO modules.

Next we briefly describe the SISO algorithm (used for parallel, inner and outer convolutional codes) based on the trellis section shown in Fig. 4 for a generic code \( E \) with input symbol \( u \) and output symbol \( c \). Consider an inner code with \( p_1 \) input bits and \( q_1 \) output bits taking values \( \{0,1\} \), a parallel code with \( p_2 \) input bits and \( q_2 \) output bits taking values \( \{0,1\} \), and an outer code with \( p_3 \) input bits and \( q_3 \) output bits taking values \( \{0,1\} \). Let the input symbol to the convolutional code \( u_i(e) \) represent the input bits \( u_k(e), i = 1,2,\ldots,p_m \) on a trellis at time \( k \) \( (m = 1 \) for the inner code, \( m = 2 \) for the parallel code and \( m = 3 \) for the outer code), and let the output symbol of the
convolutional code \( c_k(e) \) represent the output bits \( c_k(e); \ i = 1,2,\ldots,q_m \). Let \( s \) represent the state of the encoder and for an encoder with memory \( v_m, s \in \{0,1,\ldots,2^{v_m} - 1\} \). Let the total number of trellis steps for an encoder be \( n \) (obviously the value of \( n \) can be different for the inner, outer and parallel codes).

As given in [3], the decoding process involves the computation of forward (\( \alpha \)) and backward (\( \beta \)) recursions. These values are used in the computation of extrinsic bit information. The algorithms for the inner and the parallel codes are given in [3],[9].

5 Simulation Results

The simulations are carried out for a rate 1/4 HCCC encoder structure with component codes as shown in Fig. 2. Binary pulse position modulation scheme is used. The channel is assumed to be suitable for IM/DD scheme. A typical APD with the following parameters is used for simulations: \( \nu = 1.935 \times 10^{14} \) Hz, \( G = 100 \), \( k = 0.007 \), \( \eta = 0.6 \), \( M_e = 100 \), \( I_b = 0.1 \) nA, \( I_s = 10 \) nA, \( T_s = 2 \times 10^{-8} \) seconds, \( R = 146650 \) \( \Omega \), \( T = 300^{\circ} \)K and \( B = 1/2T_s = 2.5 \times 10^7 \) Hz. The received laser power, \( P_w \) is varied from -75 dBW to -100 dBW. Fig. 5 shows the bit-error rate versus number of iterations with received laser power as a parameter. A total of 10240 bits were transmitted to obtain these results. The results show that the decoder converges to zero errors with in the first six iterations for received laser powers less than -90 dBW. This is far superior to the additive white Gaussian noise (AWGN) channel where the convergence to a BER of \( 10^{-6} \) takes place after ten iterations for an \( E_b/N_0 = 0.3 \) dB [9]. There is an oscillatory behavior in the error pattern for \( P_w \) less than -90 dBW. There is a burst in the error pattern after every six iterations which then converges back to zero errors in the next 3 to 4 iterations. A similar kind of behavior has been reported for Turbo codes in [16]. Above -95 dBW, the decoder is not converging to low BERs, but is also maintaining a large number of errors. From the results, we can use six iterations as stopping criterion for decision making for \( P_w \) less than -90 dBW.

6 Conclusions

In this paper, we have explored the possibility of using the hybrid Turbo codes for optical channels. We have considered HCCC encoded optical communication system with APD noise as well as thermal noise. The performance of hybrid Turbo codes over an optical PPM communication channel employing an avalanche photodiode (APD) detector is analyzed. Bit error rate (BER) as the performance measure has been used. The proposed system employing hybrid Turbo codes is shown to perform better than the existing schemes. The decoder converges to zero errors within six iterations for the transmit powers used for the simulations. The behavior of the error pattern is oscillatory with periodic bursts. An important conclusion is that if the number of iterations is kept fewer than 6, we can achieve arbitrarily low error rates for a given transmit power. For example, the stopping criterion can be kept at the sixth iteration for \( P_w \) less than -90 dBW to achieve BER close to zero. The results in this paper can be easily extended to optical CDMA systems.

References


Fig. 1. Hybrid Turbo-coded optical PPM communication system.

Fig. 2(a). Rate 1/2 RSC parallel encoder with $G_p(D) = [1, 1+D^2/1+D+D^2]$. 
Fig. 2(b). Rate 1/2 NSC outer encoder with $G_o(D) = [1+D+D^2, 1+D^2]$.

Fig. 2(c). Rate 2/3 RSC inner encoder with $G_i(D) = \begin{bmatrix} 1 & 0 & \frac{1+D^2}{1+D+D^2} \\ 0 & 1 & \frac{1+D}{1+D+D^2} \end{bmatrix}$.
Fig. 3. APD noise model

Fig. 4. The trellis section for the code $E$. 
Fig. 5. Simulation results for HCCC encoded optical communication system for different levels of received laser powers