

# Modeling and Control of SMA Actuator

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**Abstract:** - The article describes several methods of modeling of hysteresis which is present in new biologically inspired actuators, mainly in actuators based on shape memory alloys. Also there are shortly described practical results obtained in construction and control of an actuator based on wires made from shape memory alloy.

**Key-Words:** - Smart actuators, Shape memory alloy, Muscle wire, Hysteresis, Hysteresis modeling, Non-linear system control.

## 1 Introduction

Until now the most common way to create motion in robotics is to use electrical motors of various forms. Less common is usage of pneumatic or hydraulic actuators e.g. pneumatic cylinders. Thus electrical, pneumatic and hydraulic motors are considered to be classical actuators. These types of actuators hardly resemble natural actuators – biological muscles observed in nature. Muscles are ubiquitous actuators which are used by insects animals and human beings. That is why biologically inspired actuators are in focus of robotic research for several years [1]. Biologically inspired actuators – artificial muscles (AM) are nowadays based on technologies which include electromagnetics, mechano-chemical polymers, electromechanical polymers, piezoelectric materials, magnetostrictive materials, shape memory alloys (SMA) and polymers, electrostatics and pneumatics (Mc-Kibben muscles). The following table shows brief comparison AM based on some technologies [2].

Actuator type	Max. Strain (%)	Max Pressure (Mpa)	Max. Efficiency (%)
Natural Human Muscle	>40	0.35	>35
SMA	>5	>200	<10
Piezoelectric ceramic	0.2	110	90
Electrostrictive polymer silicone	32	0.2	90

Table 1. Comparison of human muscle and AM

Natural muscle has outstanding overall performance which is not reached by AM yet. Nevertheless AM outperform classical actuators in many criteria e.g power to weight ratio, but they are difficult to control. The main reason is complex and nonlinear behavior of AM. The following article deals with some aspects of modeling and control of position of SMA in form of muscle wire (MW).

## 2 Basic models of SMA

Developing a mathematical model that captures the behavior of MW is extremely complicated and challenging problem. The complete model must capture the dominant nonlinear effects that occur during MW control. The most important of them is hysteresis. The most common SMA NiTiNol consists of nearly equal amounts of nickel and titanium atoms. Memory wire made of NiTiNol in an assembly shown in Fig. 1. exhibits temperature - length hysteresis shown in Fig.. 2.

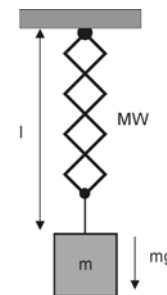


Fig.1. Wire of SMA loaded by constant force.

When heated from cold temperature MW changes its structure from martensite to austenite and becomes

shorter. When cooled from high temperature it changes its structure from austenite to martensite and becomes longer [3].

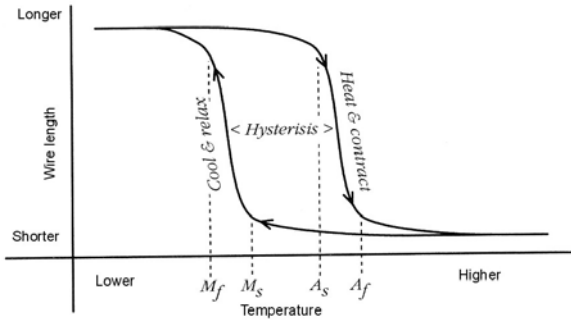


Fig.2. Typical main hysteresis loop of MW in assembly from Fig.1.

The hysteresis of the MW is modeled by several ways. The most often mentioned model is Ikuta's model described in [4]. Ikuta's model uses exponential functions to calculate relation between martensite and austenite volume fraction and temperature in SMA. After calculation of SMA composition simple linear relations are used to calculate SMA strain. Extremely simple calculation of hysteresis is used in [5]. For increasing temperature the following law is used

$$\begin{aligned} y &= y_0 & \text{for } u \leq A_s \\ y &= .5y_0[\cos K_A(u - A_s) + 1] & \text{for } A_s < u < A_f \\ y &= 0 & \text{for } u \geq A_f \end{aligned} \quad (1)$$

For decreasing temperature they use

$$\begin{aligned} y &= y_0 & \text{for } u \leq M_f \\ y &= .5y_0[\cos K_M(u - M_f) + 1] & \text{for } M_f < u < M_s \\ y &= 0 & \text{for } u \geq M_s \end{aligned} \quad (2)$$

Where  $y$  represents length,  $u$  represents temperature, and  $K_A = \pi/(A_f - A_s)$ ,  $K_M = \pi/(M_s - M_f)$ .

These and other simple methods model only the main hysteresis loop and are good models when the actuator works in on-of mode i.e. is fully heated or fully cooled only. During continuous position control minor hysteresis loops occur (see Fig.12) and must be considered in control system design.

## 2.1. Preisach model

For more realistic modeling of hysteresis which respects minor hysteresis loops there is recommended Preisach hysteresis operator [6]. Preisach model was originally developed to describe physical mechanism of

magnetization. Concise representation of Preisach operator is expressed in form

$$y = \iint_{\alpha \geq \beta} \mu(\alpha, \beta) \gamma_{\alpha\beta} d\alpha d\beta \quad (3)$$

The operator (3) is calculated in practice according to Fig. 3. as a sum of simple relay operators.

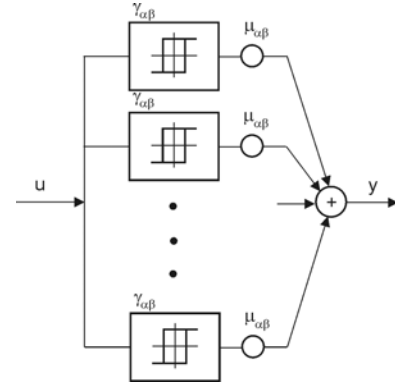


Fig.3. Modeling scheme of Preisach operator.

Individual elementary hystereses in Preisach operator are of relay type depicted in Fig.4.

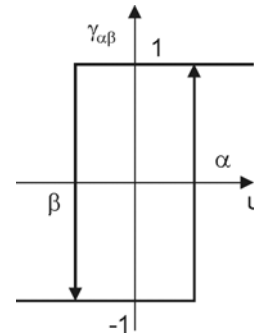


Fig.4. Elementary hysteresis

These elements can be described as systems in discrete state-variable form

$$\begin{aligned} y(k) &= 1 & \text{for } u(k) > \alpha \\ y(k) &= y(k-1) & \text{for } \beta \leq u(k) \leq \alpha \\ y(k) &= -1 & \text{for } u(k) < \beta \end{aligned} \quad (4)$$

where  $k$  is integer ... discrete time.

The disadvantage of this representation of hysteresis is that great number of elementary relays must be used for calculation. The following figure shows results of hysteresis modeling according to described principle. Altogether 231 elementary relays are used in the example with  $\alpha$  and  $\beta$  evenly spread in interval  $-1$  and  $1$  with weights  $\gamma = 1$ .

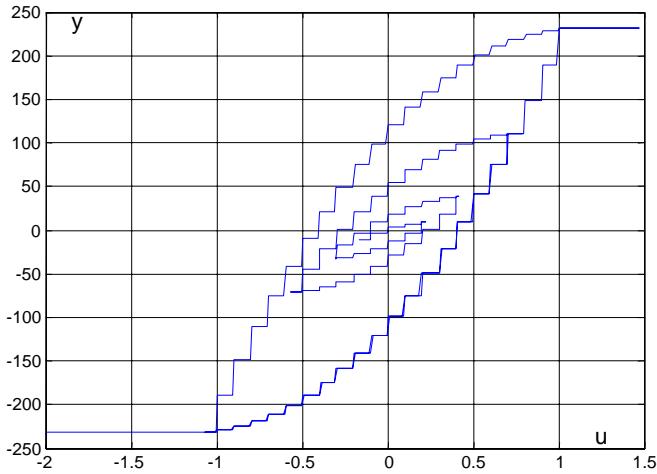


Fig.5. Hysteresis modeled according to Fig.3. with 231 elementary relays.

$$u_+ = \frac{y(k-1)}{K} + h$$

$$u_- = \frac{y(k-1)}{K} - h$$

and  $k$  is discrete time.

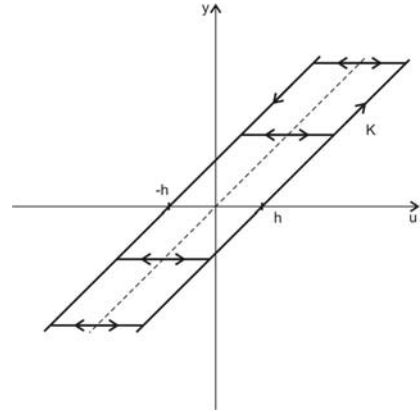


Fig.7. Elementary backlash operator.

## 2.2 Prandtl – Ishlinskii model

Another variant of hysteresis modeling is described in [7]. This model of hysteresis uses similar scheme as Preisach model but its basic elements are not relays but backlash operators see Fig.6. and Fig.7.

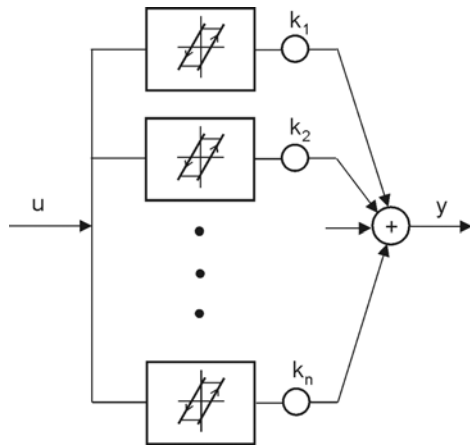


Fig.6. Prandtl-Ishlinskii model of hysteresis.

The elementary backlash operator is described in discrete state-variable form as follows.

$$\begin{aligned}
 y(k) &= K(u(k) - h) && \text{for } u(k) > u_+ \\
 y(k) &= K(u(k) + h) && \text{for } u(k) < u_- \\
 y(k) &= y(k-1) && \text{otherwise}
 \end{aligned} \quad (5)$$

where

The following figure shows results of “summation” of several elementary backlash operators. Total hysteresis operator is created by sum of  $N$  elementary backlash operators with parameters  $h$  and  $K$  arranged to reach total hysteresis with  $h = 1$  and  $K = 1$ . Summation weights are  $k_i = 1$  for all elements.

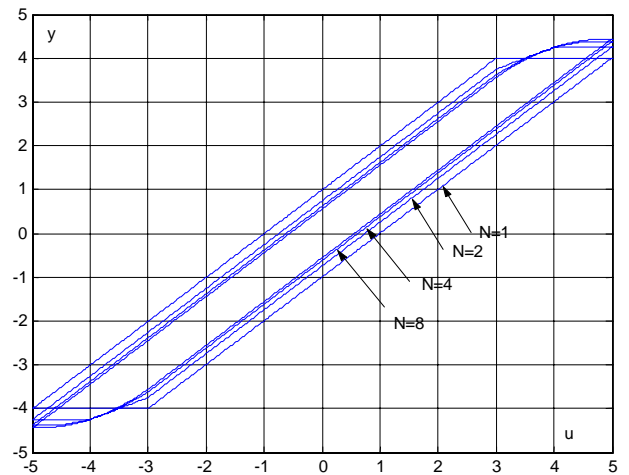


Fig.8. Hysteresis modeled by help of Prandtl-Ishlinskii operator.

One can observe smooth behaviour of the model for relatively low number of basic elements. This leads to faster calculation of total hysteresis and thus faster simulation of hysteresis. The following figure shows inner loops of the hysteresis performed by model using 8 basic elements.

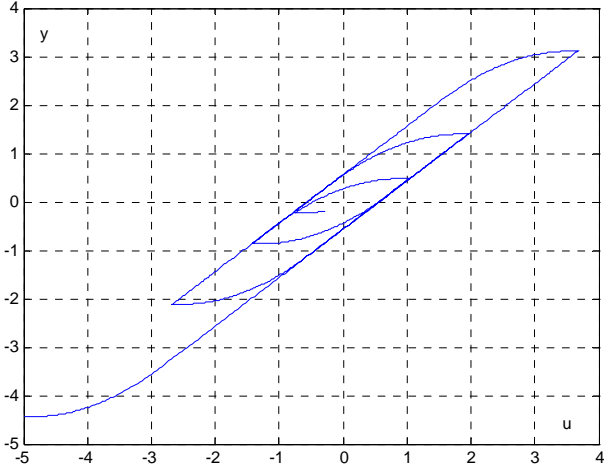


Fig.9. Hysteresis by help of Prandtl-Ishlinskii model using 8 basic elements.

Disadvantage of Prandtl-Ishlinskii classical approach is absence of saturation in final model of hysteresis. That is why we proposed to use basic elements of the model in form of backlash with saturation, see the following figure.

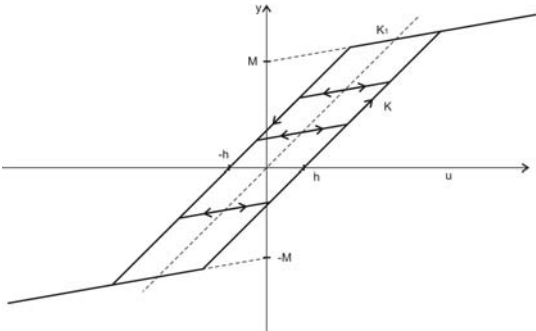


Fig.10. Elementary operator – backlash with saturation.

State space description of this operator in discrete form is

$$\begin{aligned}
 y(k) &= K_1 u(k) + M && \text{for } u > u_m \\
 y(k) &= K_1 u(k) - M && \text{for } u < -u_m \\
 y(k) &= K(u(k) - h) && \text{for } u > u_+ \\
 y(k) &= K(u(k) + h) && \text{for } u < u_- \\
 y(k) &= y(k-1) + K_1(u(k) - u(k-1)) && \text{otherwise}
 \end{aligned} \quad (6)$$

where  $k$  is discrete time and

$$u_m = \frac{Kh + M}{K - K_1}$$

$$u_+ = \frac{y(k-1) - K_1 u(k-1) + Kh}{K - K_1}$$

$$u_- = \frac{y(k-1) - K_1 u(k-1) - Kh}{K - K_1}$$

$M, K, K_1, h$  are parameters meaning of which is shown in Fig. 10.

Hysteresis using 8 basic elements of this type is shown in the following figure

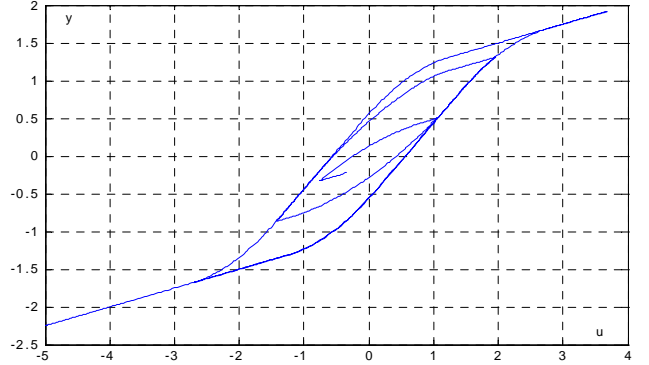


Fig.11. Hysteresis modeling by help of 8 modified basic elements.

Both Prandtl-Ishlinskii models of hysteresis are not only easily calculated but they can be also easily inverted. This property can be used for design of the controller which enables continuous change of the length of the wire. Real hysteresis characteristics measured on MW Flexinol 150 LT is shown in the following figure, where  $\epsilon$  is the relative strain.

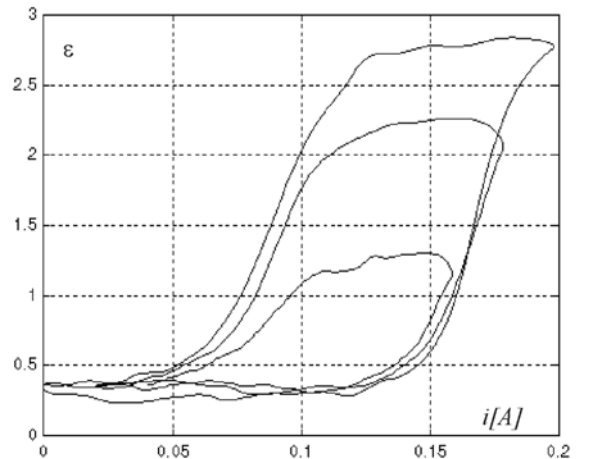


Fig. 12. Real hysteresis measured on MW.

Parameters of both Preisach and Prandtl-Ishlinskii models can be tuned to represent real hysteresis by help of least mean squares method. Especially Prandtl-Ishlinskii is apt for this method of tuning because of lower number of parameters.

### 2.3. Dynamical model of SMA

Models discussed until now characterize approximately relation between temperature of MW and its length in assembly shown in Fig.1. Unfortunately the temperature of the wire cannot be measured easily and thus its direct control is not possible. The only accessible manipulated variable is electrical current by which one can change temperature of the wire. Relation between electrical current  $i$  and temperature of the wire  $\vartheta$  is represented by differential equation

$$C \frac{d\vartheta}{dt} = Ri^2 - hA(\vartheta - \vartheta_a) \quad (7)$$

where  $C$  is thermal capacity of the wire,  $R$  is the electrical resistance, and  $\vartheta_a$  is the ambient temperature. The equation can be transferred to transfer function form and the complete dynamical model of the MW can be represented by the following block scheme

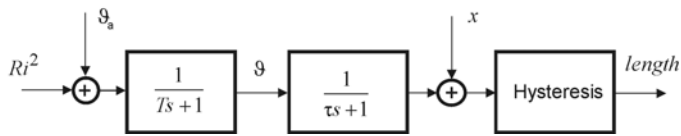


Fig.13. Basic dynamical model of the MW.

where  $T$  represents basic time constant of the wire,  $\tau$  represents un-modeled fast dynamics,  $x$  represents mainly external disturbances and  $\vartheta_a$  is ambient temperature.

### 3 MW control and SMA actuator

Because SMA wire can develop only contractive force any drive based on use of the wires must be equipped with bias force mechanisms, which provides force opposing the contraction. A bias force can be provided by many means e.g. by a spring, a weight pulled by gravity, even an opposing SMA wire (so called antagonistic drive). We have designed two kind of actuators. The first one uses force of gravity as the opposing force (see Fig.1). The second actuator based on principle of antagonistic MW is depicted on Fig.14. Actuator consists of two MW Flexinol 150 LT, length of each wire is 250 mm. Each wire is controlled by a pulse width modulated amplifier (PWM). The PWM amplifiers are controlled by PI controller designed on the basis of MW model developed in the previous subchapter. The wires are coupled with output shaft element, a ball bearing. Position of the output shaft is measured by potentiometer providing feedback signal to the controller.

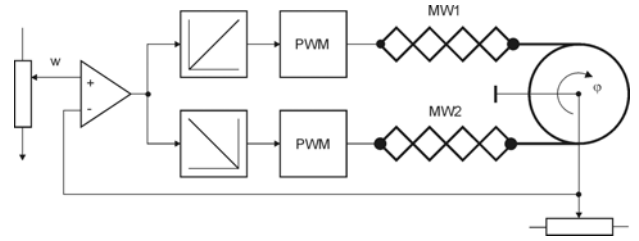


Fig.14. Scheme of antagonistic MW actuator.

Electric current of MW is limited by a current limiter with  $I_{max}$  which affects function of PWM amplifiers directly. The following figures show some achieved results.

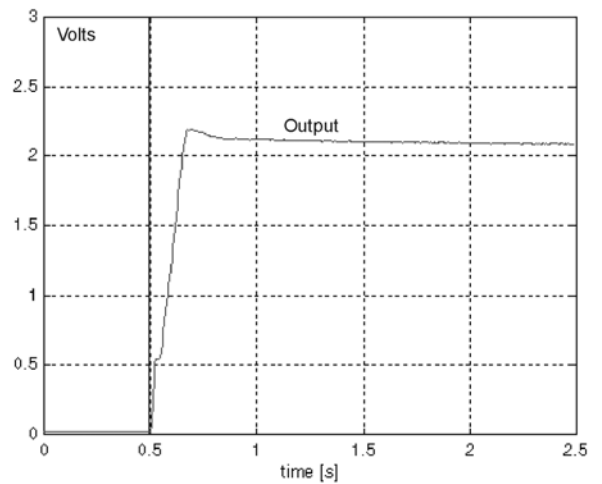


Fig.15. Step response of the SMA actuator.

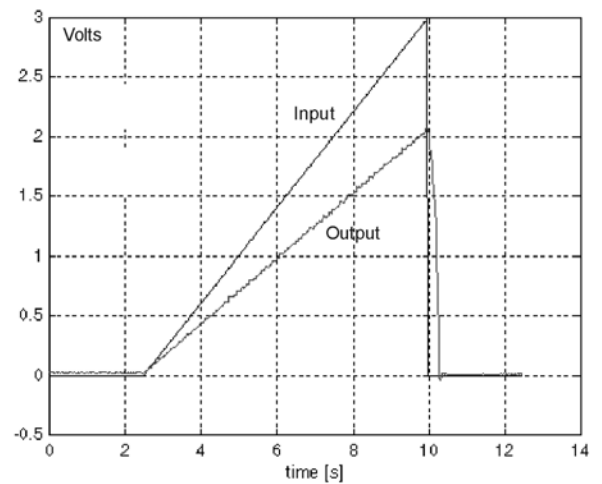


Fig.16. Ramp response of the SMA actuator.

PI controller with sampling time 25ms developed by help of dynamical model with hysteresis is used in all experiments. Only voltage proportional to desired value and voltage of feedback sensor are shown in the figures.

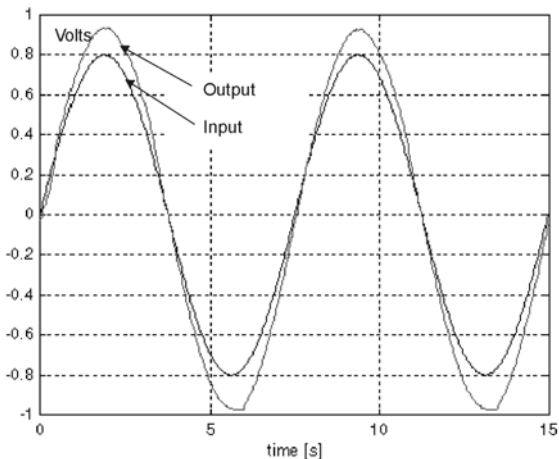


Fig.17. Response of the SMA actuator to sinusoidal input.

Simple on-off control of MW actuators (fully stretched-fully contracted) was used for control of miniature six-legged robot which is shown in the following figure.

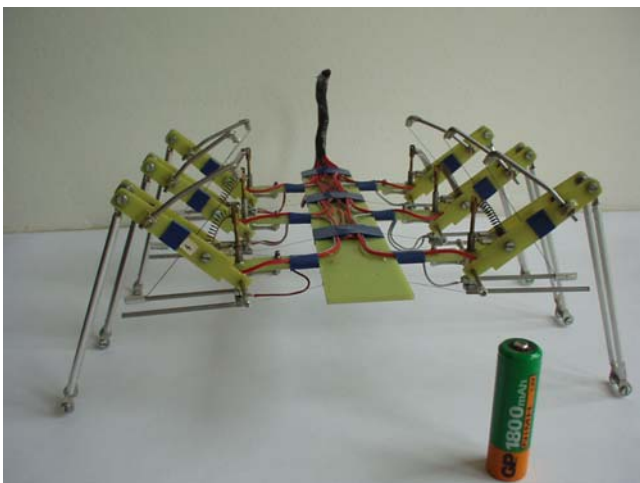


Fig.18. Six legged robot controlled by SMA

very simple equipment. Usage of hysteresis inversion included in the controller should improve the performance in the future.

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## 4 Conclusion

Extreme power to weight ratio, high reliability and no need of maintenance, makes drives based on SMA very attractive. Application of SMA in continuous position control is hindered by highly non-linear behavior of the SMA material. This phenomenon makes research in SMA actuators a challenging problem. Reasonable modeling of SMA actuators enables development of more effective controllers. The modified Prandtl-Ishlinskii model of hysteresis was developed and tuned to be a reasonable model of real hysteresis in a MW. Results described in the article show that relatively good performance of a SMA actuator can be reached with