Abstract: We start by stating a mathematical programming problem for the optimisation of the operation of an automatic self-production power system. We discuss the optimisation problem in order to design an algorithm for our self-production power system. We present a case studied based in a real prices from Portuguese power market. Finally, we report that our approach displays good results for a small self production prototype.

Key Words: Power Systems, Resource Scheduling, Non-Linear Optimisation.

1 Introduction

The paper reports the development of an optimised automatic self-production system. The system considered in the paper consists of one unit for electrical energy control and one power unit, turbine/generator set. We address an optimisation problem for an automatic self-production power system in the Portuguese power system market. This problem is a mix-integer mathematical programming problem similar to an economic problem of dispatching production to a market. The objective function is the profit, that is, the good minus the production operational cost. The determination of the production operational cost can be obtained by experimental approach through the set history analysis. In the present work, a quadratic cost function is used to illustrate the algorithm. The feasible set is defined by the technical constrains. The algorithm for our optimum problem can be described in two procedures, a procedure for the decision of production and a procedure for the optimal production level. The procedure for the production decision starts by calculating the value of power that leads to lower cost per unit of production. The procedure for the optimal production level determines the optimal power level.

Several authors recently addressed this issue, namely by incorporating into the decision process transmission losses [1] and the initial state conditions [2]. Also several mathematical approaches were used to obtain results, e.g. Markovian approaches [1].

2 Mathematical Problem

By considering a cost function associated with the turbine/generator set, an optimisation algorithm can be used to determine optimal time to start the generator and optimal production at any time. In the present work, the economical dispatch algorithm used is a function of the electrical energy cost per unit and determines the starting time of the generator, as well as the optimal value of the active power associated with the production at all times. Schematically the system can be represented as it is shown in fig. 2.1.

![System studied](image)

Fig. 2.1 – System studied

The determination of the production cost function is not a goal of this work, since it can be obtained by experimental approach through the set history analysis. Therefore to illustrate the present algorithm a quadratic cost function is used (1).
\[ C(u, P) = u(\alpha + \beta P + \frac{\gamma}{2} P^2) \]  \hspace{1cm} (1)

In (1), \( u \) is the Boolean decision variable that identifies if the generator is on, \( u=1 \), or off, \( u=0 \), \( P \) is the power associated with the energy conversion and \( \alpha, \beta, \gamma \) are the turbine and generator cost operational parameters. The feasible set is defined by the technical constrains. The determination of the feasible depends of what one wants to impose as constraints. Therefore only technical limits are used to illustrate the present algorithm (2).

\[ uP \leq P \leq u\bar{P} \]  \hspace{1cm} (2)

In (2), \( \lambda \) is the price market of energy in the market, \( \bar{P} \) the maximum power value and \( \underline{P} \) the minimum power value. Hence the problem can be stated as maximising the profit subjected to the constraint (3), that is

\[ \text{max} \; \lambda P - u(\alpha + \beta P + \frac{\gamma}{2} P^2) \]  \hspace{1cm} (3)

s.t.

\[ uP \leq P \leq u\bar{P} \]

This problem is a mix-integer mathematical programming problem similar to an economic problem of dispatching production to a market. The problem is solved by the use of Karrush Kuhn Tucker conditions.

### 3 Algorithm

The decision algorithm solves the maximisation problem with constraints obtained by considering technical limits (3). The algorithm can be described in two procedures, i.e. a procedure for the production decision and a procedure for the optimal production level. The procedure for the production decision starts by calculating the value of power that leads to lower cost per unit of production, problem (4).

\[ \text{min} \; \frac{C(P^*)}{P^*} \]  \hspace{1cm} (4)

s.t.

\[ P \leq \bar{P} \]

\[ P \geq \underline{P} \]

By solving the problem (4), the optimal power solution \( (P^*) \) can be obtained by (5)

\[ P^* \equiv \begin{cases} \frac{2\alpha}{\gamma} & \text{se} \; P \leq \sqrt{\frac{2\alpha}{\gamma}} \leq \bar{P} \\ \bar{P} & \text{se} \; \sqrt{\frac{2\alpha}{\gamma}} > \bar{P} \\ \underline{P} & \text{se} \; \sqrt{\frac{2\alpha}{\gamma}} < \underline{P} \end{cases} \]

The optimal cost production \( (\lambda_{ef}) \), considered as described in (6), points to the lower production cost per unit of production \( (P^*) \) as one can see in fig. 3.1

\[ \lambda_{ef} = \frac{C(P^*)}{P^*} \]

If the costs \( (\lambda_{ef}) \) is lower than the price of the energy from the market distribution supplier, \( (\lambda) \), there is an advantage to self-produce.

![Fig. 3.1. – Cost per production unit](image-url)
The optimal production value \( P^o \) can be calculated by (7)

\[
\max \lambda P - (\alpha + \beta P + \frac{\gamma}{2} P^2) \quad (7)
\]

subject to

\[
P \leq P \leq \bar{P}
\]

Therefore the solution is (8)

\[
P^o \equiv \begin{cases} \frac{\lambda - \beta}{\gamma} & \text{if } \frac{\hat{c} - \hat{a}}{\alpha} > \bar{P} \\ \bar{P} & \text{if } \frac{\hat{c} - \hat{a}}{\alpha} < P \\ P & \text{else} \end{cases} \quad (8)
\]

The full decision algorithm to maximise profit and to determine the optimal power value is presented in fig. 3.2.

4 Case Study

The energy price considered in this work is extracted from Portuguese energy market and considers three types of prices. The small time demand price, is the price for a consumer with a period less than 1000 hours per year of demand. Medium time demand price, is the price for a consumer with a period between 1000 hours and 5000 hours per year of demand. Large time demand prices, is the price for a consumer with a period of more than 5000 hour per year of demand. In the tri cost structure there are three different periods, empty hours, full hours and peaks. For installations with several labouring hours, the bi and tri cost structures [3] can be very advantageous, enabling a managing so that the consumption can be transferred to lower cost hours. The taken power is the higher value of average active power in a 15-minute interruptible gap. The arising of this value is due for example to the entrance in service simultaneously of a production line. In order to avoid this problem by foreseeing the entrance in service of a process part that will demand high start-up power, it could be more profitable to connect the alternator to the power grid in order to produce part of the required power, so that the one requested to the power grid shouldn’t be so high.

The present case study is based on the block diagram of fig. 4.1. The demand block represents the studied installation’s electric energy consumption during the day. This energy can be supplied by the distributor’s electrical grid, which is considered to be infinite, and it is represented in the diagram by the electrical grid block. The control block analyses permanently the buying price of energy, and, as it is shown in

\[
\begin{align*}
\text{Begin} \\
& P^o = \sqrt[\gamma]{\frac{2\alpha}{\gamma}} \\
& \text{if } P^o \geq P \quad \text{then} \quad P^o = \bar{P} \\
& \quad \text{else if } P^o < P \quad \text{then} \quad P^o = P \\
& \quad \text{end if} \\
& \lambda_{ref} = \frac{C(P^o)}{P^o} \\
& \text{if } \lambda > \lambda_{ref} \quad \text{then} \quad u = 1 \\
& \quad \text{begin 1} \\
& \quad \quad P^o = \frac{\lambda - \beta}{\gamma} \\
& \quad \quad \text{if } P^o \geq \bar{P} \quad \text{then} \quad P^o = \bar{P} \\
& \quad \quad \quad \text{else if } P^o < P \quad \text{then} \quad P^o = P \\
& \quad \quad \quad \text{end if} \\
& \quad \quad \text{begin 2} \\
& \quad \quad \quad \text{else} \quad u = 0 \quad P^o = 0 \\
& \quad \quad \quad \quad \text{end if} \\
& \text{end}
\end{align*}
\]

Fig. 3.2. – Decision algorithm
section 3, decides when the self-production starts, and it’s optimum value.
When the generator group, represented by the self-production of energy block, gets to the paralleling conditions, the control of production of energy block gives synchronism order, and the parallel with the grid is accomplished, starting to supply to the installation the optimum value of power determined by the optimisation algorithm.
If there is an agreement with the distributor, when the optimum value of power produced by the generator group is superior to the installation’s consumption, the surplus can be sold to the distributor.

In the present work, an industrial installation with a power capacity of 2.1 MW equipped with a steam turbine and a three-phase alternator with a maximum technical limit of 700 kW and minimum of 10 kW was considered.
The value of the consumed power varied randomly in the gap comprehended the half and the maximum value of hired power for the installation. The cost function used for the case studied belongs to a group installed in a labouring unit.
Based on the production cost’s historical reports, a trendline was drawn from which resulted the parameters \( \alpha=1.4, \beta=6.10, \gamma=0.01 \).
The cost structure applied to the user was the same one applied to clients with hired power above 2 MW [3], considering that the electric energy delivery was made during the wet period, between November 1\(^{st}\) and April 30\(^{th}\), winter legal hour [3] and large time demand prices, that is, an annual consumption superior to 5000 hours.

Fig. 4.2 represents the studied installation’s consumption of electric energy over a 24 hour period.

In fig. 4.3 it’s shown the price of electric energy over 24 hours of a given weekday. This cost structure results from de fact that the installation has a consumption of electric energy superior to 5000 hours in a year, therefor large time demand prices, and from the fact of being in the wet period comprehended between November 1\(^{st}\) and April 30\(^{th}\), accordingly with the table 1 presented in appendix.

Fig. 4.4 represents the installation’s consumption of electric energy and the energy consumed by the group. The difference
between both results from the energy obtained from the distributor.

![Energy importation chart](image)

**Fig. 4.4 – Energy importation**

The chart in fig. 4.5 represents the percentage of energy bought from the distributor and the energy self-produced by the installation.

![Self-production vs energy importation chart](image)

**Fig. 4.5 – Self-production vs energy importation**

## 5 Conclusion

In the present work an optimisation problem was addressed to support the self-production decision and the value of power production at all times. A decision algorithm was proposed to achieve the optimal power value and to maximise profit. The case studied is supported in the Portuguese Power Market and it shows that if the system allows self-production it is advantageous to apply a solution like the one presented in this paper. It was shown that in the considered situation it was profitable to self-produced 24 % of the total energy demand. The system proposed can save substantially in the energy bill and contracted power availability.

Further work is currently taking place to improve the system capabilities, namely by considering more technical and economical constrains.

### References:


## Appendix

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