Abstract: – The investigation of spectrum overlay of a Spread Spectrum system on the existing narrowband FM broadcasting system is presented. The overlaid Spread Spectrum (SS) system is assumed to utilize direct sequence (DS) spreading, using maximal length pseudorandom sequences with long spreading codes. We studied the performance degradation of the analog FM system, expressed by the audio frequency signal-to-noise ratio (AF SNR), due to the interference produced by the SS signal for various types of spreading scenarios and for different carrier frequency differences ($\Delta f$).

Key-words: – Spread Spectrum, Analog FM, AWGN, Interference, AF SNR.

1 Introduction

Spread Spectrum has been under development for the last 50 years. The interest in spread spectrum for civilian applications is growing, particularly in the area of mobile communications. One of many reasons for this growing interest is the huge demand of spectrum that has become more and more requisite for modern communication systems. Antijamming, antiinterference, privacy and low power spectral density are some of the advantages that the SS technique encompasses and strengthen their use to overlay schemes. Both field tests and analyses have provided a perspective as to what the capabilities of such a system are. The mutual interference, which in the conditions of high density of these systems becomes unavoidable, through the estimation of the interference noise, serves as a measure for accepting their coexistence. This overlay concept has been demonstrated in both PCS band [1] and cellular band [2].

One standard method for broadcasting audio is analog frequency modulation (FM) in the 87.5 – 108 MHz band. Standard efforts are underway to define and evaluate transmission methods for digital audio broadcasting [3]. The result of CD quality is expected for the received audio signal. Recently, due to the spectrum vacancy, there has been a lot of interest in utilizing the analog FM band. Of course there is a constraint corresponding the performance of the existing analog FM. Some experimental results concerning frequency hopping (FH) SS interference on narrowband FM signals can be found in [4].

This spectrum overlay can increase communications capacity and spectral efficiency, but may cause the following types of interference: i) interference from the narrowband FM stations to the SS system and ii) interference from the overlaid wideband SS system on the FM receivers. The latter is the scope of this paper.

2 Description of Analog System

In this section we briefly describe the analog FM standard with some of the associated system parameters that were used in this paper. Commercial FM radio broadcasting utilizes the frequency band
87.5-108 MHz for transmission of voice and music signals. The channel spacing is 200 kHz and the peak-frequency deviation is fixed at 75 kHz. Pre-emphasis is generally used to improve the demodulator performance at the receiver. The pre-emphasis filter has the following response:

\[ H_{p}(f) = 1 + j \frac{f}{f_{o}} \]  

where \( f_{o} = 3.1 \text{ kHz} \) is the 3-dB frequency of the RC filter having a time constant 50 µs. The stereo message signal is a composite signal produced by the sum of left and right channels and the difference of them is used to AM modulate (DSB-SC) a 38 kHz carrier that is generated from a 19 kHz oscillator. A pilot tone at the frequency of 19 kHz is added to the signal for the purpose of demodulating the DSB-SC AM signal. The stereo signal \( m(t) \), whose spectrum is depicted in Fig. 1, is then transmitted by analog FM modulation, i.e.,

\[ x_{FM}(t) = \cos \left( 2\pi f_{c} t + 2\pi f_{c} \int_{-\infty}^{\tau} m(\tau) d\tau \right) \]  

Since the signal is embedded in the frequency of the carrier, any amplitude variations in the received signal are result of additive noise and interference. The hard limiter removes any amplitude variations in the received signal at the output of the IF amplifier by band-limiting the signal. A bandpass filter centered at \( f_{IF} = 10.7 \text{ MHz} \) with a bandwidth of 200 kHz is included in the limiter to remove higher order frequency components introduced by the nonlinearity inherent in the limiter. A balanced discriminator is used for frequency demodulation. The resulting message signal is then passed to the audio amplifier, which performs the functions of de-emphasis, with de-emphasis filter response \( H_{d}(f) = \frac{j}{H_{p}(f)} \), and amplification. A low pass filter, used to remove out-of-band noise further, filters the output of the audio amplifier and its output is used to drive a loudspeaker. Note that FM sub-carrier modulation for data (such as SCA and RDS) has not been included in this study.

### 3 Description Of Spread Spectrum System

The spread spectrum signal is given by [5] as

\[ C(t) = \sqrt{2P_{i}}d(t-\tau)p(t-\tau)\cos(\omega_{o}t+\theta) \]  

where \( P_{i} \) is the received power of the spread spectrum, \( \tau \) is the time delay of the signal uniformly distributed into \([0,T]\), \( \theta \) is the phase angle uniformly distributed on \([0,2\pi]\), \( d(t) \) is the modulating digital signal, \( P[d(t)=1] = P[d(t)=-1] = 0.5 \), with \( T \) seconds duration, and \( \omega_{o} \) is the carrier frequency of the signal. \( p(t) \) is the spreading code given by

\[ p(t) = \sum_{i=-\infty}^{\infty} p_{i}\xi(t-iT_{s}) \]  

where \( p_{i} \) is one chip of random binary sequence \( \{p_{i}\} \), which consists of independent symbols with equal probability, \( T_{s} \) is the spreading code chip duration and \( \xi \) is the chip waveform, assumed rectangular.

The power spectral density of the spreading sequence is a line spectrum [5]:

\[ S_{PN}(f) = \sum_{k=-\infty}^{\infty} \sum_{k=1}^{N} \frac{1}{N^{2}} \left( \frac{f-kf_{s}}{N} \right)^{2} \]  

\[ + \frac{1}{N^{2}} \delta(f) \]  

where \( f_{s} \) is the spreading code rate and \( N = 2^{r} - 1 \) is its length. It is assumed that \( r \) shift registers generate the spreading sequence.

In this paper we assumed unmodulated PN. Therefore, the spectral density of the total interference, \( S_{i}(f) \), will be a line spectrum that resembles \( S_{PN}(f) \) frequency translated by \( \pm f_{o} \), given by [6]:

![Fig. 1. Spectrum composition of the stereo FM signal](image-url)
4 Interference Analysis

Consider a frequency modulation system with carrier frequency $f_c$, disturbed by a single spread spectrum interferer component of frequency $f_c + f_i$ at the receiver input. The FM signal represented by the expression:

$$u_{FM}(t) = \sqrt{2P_o} \cos \left[ \omega_c t + \varphi(t) \right]$$

where $P_o$ is the mean power and the instantaneous phase deviation is given by

$$\varphi(t) = 2\pi \Delta f \int x_{FM}(\tau) h_p(\tau) d\tau$$

where $x_{FM}(t)$ is the modulating FM signal (audio), $\Delta f$ is the frequency deviation and $h_p(t)$ is the pre-emphasis transfer function. Symbol $\otimes$ stands for convolution. The system model is depicted in Fig. 2.

It is assumed that all the signals at the receiver input are mutually statistically independent. The spread spectrum interference is represented by

$$u_i(t) = B(t) \cos \left[ \omega_i t + \theta_i \right]$$

where $B(t) = \sqrt{2P_i} c(t)$ and $\omega_i$ is the frequency difference.

The signal at the receiver input is

$$x(t) = u_i(t) + u_{FM}(t) + n(t) =$$

$$\cos \left[ \omega_c t + \varphi(t) \right] + \frac{B(t)}{\sqrt{2P_o}} \cos \left[ \left( \omega_c + \omega_i \right) t + \theta_i \right] +$$

$$\frac{r(t)}{\sqrt{2P_o}} \cos \left[ \omega_c t + \varphi_n(t) \right]$$

where $n(t)$ is the AWGN, $r(t)$ is the Rayleigh distributed noise envelope and $\varphi_n(t)$ is the uniformly distributed noise phase. Following the noise analysis for angle modulation, the input at the limiter-discriminator can be expressed as:

$$u_M(t) = R(t) \cos \left[ \omega_c t + \varphi(t) + \varepsilon(t) \right]$$

where

$$R(t) = \left[ 1 + \frac{r(t) \sin \left[ \varphi_n(t) - \varphi(t) \right]}{\sqrt{2P_o} + \frac{r(t) \cos \left[ \varphi_n(t) - \varphi(t) \right]}{B(t) \cos \left[ \omega_c t - \Theta(t) \right]} \right]^2 +$$

$$\frac{r(t) \sin \left[ \varphi_n(t) - \varphi(t) \right]}{\sqrt{2P_o} + \frac{r(t) \cos \left[ \varphi_n(t) - \varphi(t) \right]}{B(t) \cos \left[ \omega_c t - \Theta(t) \right]} \right]^2 \right]^2$$

and

$$\varepsilon(t) = \tan^{-1} \frac{r(t) \sin \left[ \varphi_n(t) - \varphi(t) \right] + B(t) \sin \left[ \varphi_n(t) - \varphi(t) \right]}{\sqrt{2P_o} + \frac{r(t) \cos \left[ \varphi_n(t) - \varphi(t) \right]}{B(t) \cos \left[ \omega_c t - \Theta(t) \right]} \right]$$

where we have made the substitutions: $\omega_i = \omega_o - \omega_c$ and $\Theta(t) = \theta_i - \varphi(t)$.

The phasor diagram of the discriminator input is depicted in Fig. 3, for the case $\sqrt{2P_o} > r(t) + B(t)$.

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Fig. 2. System block diagram

Fig. 3. Phasor diagram of the FM signal, SS interference and noise at the same frequency and phase shift
The discriminator output will be:
\[ u_t(t) = \frac{1}{2\pi} \frac{d}{dt} \left[ \varphi(t) + \varepsilon(t) \right] \] (13)

If the SNR at the IF filter is large most of the time, \( \sqrt{2P_o} \gg r(t) + B(t), \varepsilon(t) \) can be written
\[ \varepsilon(t) = \frac{r(t)}{\sqrt{2P_o}} \sin[\varphi_o(t) - \varphi(t)] + \frac{B(t)}{\sqrt{2P_o}} \sin[\omega_ot - \Theta(t)] \] (14)

and the discriminator output will be:
\[ u_t(t) = \Delta fm(t) + n_D(t) + G(t) \]

where \( m(t) \) is the FM message signal before the de-emphasis, \( n_D(t) \) is the interference caused by AWGN given by:
\[ n_D(t) = \frac{1}{2\pi} \frac{d}{dt} \left[ \frac{r(t)}{\sqrt{2P_o}} \sin[\varphi_o(t) - \varphi(t)] \right] \] (15)

and \( G(t) \) is the interference at the output of the discriminator caused by spread spectrum, and given by:
\[ G(t) = \frac{1}{2\pi} \frac{d}{dt} \left[ \frac{B(t)}{\sqrt{2P_o}} \sin[\omega_ot - \Theta(t)] \right] = \frac{1}{2\pi \sqrt{2P_o}} \frac{d}{dt} E(t) \] (16)

where \( E(t) = B(t) \sin[\omega_ot - \Theta(t)] \). For the derivation of the power spectral density of the SS interference at the discriminator output, \( S_G(f) \), the power spectrum of \( E(t) \) must be first calculated. Considering unmodulated case for the FM signal, \( E(t) \) will be a zero-mean, uncorrelated, WSS random process. Therefore, its spectral density, \( S_E(f) \), will be identical to \( S_G(f) \), given in eq.5, with center frequency \( f_s = \left| f_o - f_c \right| \). Hence, the PSD of \( G(t) \) will be:
\[ S_G(f) = \frac{1}{2P_o} f^2 S_E(f) \] (17)

and taking into consideration only the positive carrier frequency side of the spectrum from eq.5 and for the case of \( \frac{1}{T} > f_s \), can be approximated by:
\[ S_G(f) = \frac{P_i}{2P_o} \frac{1}{f_s} \left[ \frac{\sin \pi(f - f_s) / f_s}{\pi(f - f_s) / f_s} \right]^2 \] (18)

The effect of modulation is to produce frequency components in the output for \( f > W \), the low pass filter bandwidth, which are removed by the baseband low pass filter [7].

The PSD of the AWGN at the discriminator output will be given by:
\[ S_n(f) = \frac{1}{(2\pi)^2} \frac{P_i}{2P_o} \frac{1}{f_s} \left[ \frac{\sin \pi(f - f_s) / f_s}{\pi(f - f_s) / f_s} \right]^2 \] (19)

where \( N_o \) is the amplitude of the one side power spectral density of the AWGN.

The white noise interference power at the receiver output will be:
\[ n_o = \int_{-W}^{W} S_n(f) df = \frac{N_o W^3}{3P_o} \] (20)

The spread spectrum interference at the receiver output will be
\[ I(f) = \int_{-W}^{W} S_G(f) df = \frac{P_i}{2P_o} \frac{1}{f_s} \int_{-W}^{W} \left[ \frac{\sin \pi(f - f_s) / f_s}{\pi(f - f_s) / f_s} \right]^2 df \] (21)

For our case of sinusoidal message signal, the power of the desired signal will be given by [8] \( u_o = \left( \Delta f \right)^2 m^2(t) \), where bar denotes time average and we assumed \( m^2(t) = 1 \). Thus:
\[ SNR_o = \frac{\left( \Delta f \right)^2}{\frac{P_i}{2P_o} \frac{1}{f_s} \int_{-W}^{W} \left[ \frac{\sin \pi(f - f_s) / f_s}{\pi(f - f_s) / f_s} \right]^2 df + \frac{N_o W^3}{3P_o}} \] (22)

## 5 Numerical Results

An ambient noise temperature of 10000 K is representative of the FM frequency band. Taking into account the requirements for Analog FM system of section 2, we express the AF SNR out of eq.22 for various spreading scenarios and values of frequency separation. The results are depicted in Fig. 4 and Fig. 5.
6 Conclusion

The impact on performance of the host FM signal in the presence of various SS configurations has been analyzed. An analytical expression relating RF Carrier-to-Interference Ratio (CIR) to baseband signal-to-noise ratio (SNR) output for the Analog FM system, under SS interference is derived. Careful spectral placement and power level setting assure minimal impact to the coexisting FM signal. We conclude to the following inferences: i) the worst SNR performance occurs when the carrier frequencies are identical, ii) the carrier placement of the SS system has minimal impact to AF SNR output and iii) the null-to-null bandwidth of the SS signal is inversely proportional to SNR degradation.

References:


