Dual Quantization \(\Sigma\Delta\) Modulator with Noise Shaping Improvement

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Abstract: This work presents a multi-level sigma-delta modulator (SDM) with dual quantization technique to improve noise shaping. The order of the noise shaping can be obtained by the digital part without complicated analog circuits. Conventionally, a multi-level SDM uses a multi-bit digital to analog converter (DAC) in the internal feedback loop. Due to the non-linearity, it is difficult to implement the multi-bit DAC accurately. In order to prevent the inaccurate multi-bit DAC, we use a one-bit quantizer in the feedback loop and a multi-level quantizer in the feed-forward path. The simulation shows the SNR of the proposed SDM architecture is the same as or even better than the conventional multi-level SDM.

Key-words: Multi-level sigma delta modulator, Dual quantization

1 Introduction
The sigma delta modulator (SDM) is a very popular approach to realize an analog to digital converter especially in the audio domain [1], and is easy to be implemented in switched-capacitor circuit by CMOS process. In aspect of filtering, the transfer functions of the input signal and the error of the quantizer are different in the sigma delta modulator. Usually, we treat the error of the quantizer as an uniform distribution noise, and it is inverse proportional to the bit number of the quantizer (N); the noise power is about \([V_{\text{ref}}/(2^N-1)]^2/12\). For the input signal, it works like an all-pass filter, which let all input signal pass through without any attenuation. For the error of quantizer, it works like a high-pass filter. It can shape the noise to the high frequency domain, and the high frequency noise can filter out by the followed decimator. This feature let the noise in base-band of SDM be lower than that of the Nyquist rate ADC. Therefore, the most attractive immmession of the SDM is that it has high resolution but does not need high accurate analog circuit. The main technique for high resolution in a SDM is oversampling. The basic concept of oversampling is that we employ a sampling signal whose rate (f_s) is much higher than the bandwidth of the input signal (f_0). According to the oversampling theorem [2], the signal to noise ratio (SNR) is proportional to the oversampling ratio (OSR). However, due to the hardware physical limit, the operation frequency of the system is limited by circuit consideration, such as gain, slew rate, unity-gain bandwidth of the opamp. There are several methods to increase the SNR of the SDM, such as higher order approach or multi-level technique. Higher order approaches may cause the SDM to be unstable [1]. For the multi-level technique, it requires a multi-bit DAC in the feedback loop. However an accurate DAC is very difficult to design and makes the SDM to be very complicated [2].

In this paper, we propose a new SDM architecture to avoid the multi-bit DAC but get a good SNR. We use a single bit quantizer to replace the multi-bit DAC in the feedback loop, and arrange a multi-level quantizer in the feed forward path to compensate the noise shaping. The simulation shows this architecture is as good as the conventional multi-level SDM.

2 Multi-level and Multi-Level Sigma Delta Modulator
Generally, the dynamic range (DR) is dominated by the oversampling rate, the quantization level, and the order of the shaping. It is described as

\[
DR = \frac{3}{2} \left(\frac{2L+1}{\pi^{2L}}\right) \left(2^N - 1\right)^2 R^{2L+1}.
\]

(1)

Where \(R\) is the oversampling ratio; \(L\) is the order of the modulator, and \(N\) is the bit number of the quantizer.

From equation (1), we can get the following two results:
1. There are two ways to obtain high resolution with low oversampling ratio; high-order of the modulator for more noise shaping and multi-level of the quantizer for less quantization error.
2. From Fig. 1, the high-level approach has better performance than the high order approach. The trade off is the cost of the implementation.

![Image of Dynamic range vs. Oversampling ratio](image)

**Fig. 1** Dynamic range vs. Oversampling ratio

The increase of the DR is proportional to its order and oversampling ratio. However, both of them have their own problems. If the order of a SDM is more than two, the system may be unstable [2]. In a high level architecture, multi-bit DACs are needed. The multi-level DAC may cause non-linearity problems [1][2]. There are many methods proposed to overcome the DAC problems [2][3][4][5]. Leslie-Singh use cascaded architecture and a one-bit DAC in the feedback loop to prevent the non-linearity problem [2]. Some researches use the techniques such as dynamic element matching (DEM) algorithm or data weighted averaging (DWA) method to increase the accuracy of the DAC to reduce the non-linearity problems [3]. Some researches combine the best aspect of the multi-level quantization and SDM to avoid the strict linearity requirement for traditional multi-level SDM [4]. People also use an interstate feedback to attenuate the total error in the base band [5].

3 **The Proposed Approach**

Because it is easy to control and analyze, the second order single-bit SDM is well developed. Based on the approach proposed by Leslie-Singh [2] and the second order single-bit SDM, we propose a new architecture with dual-quantization technique. The proposed architecture is showed in Fig. 2. In the new architecture, the error \(e_z\) of the multi-level quantizer is multiplied by the function \(F(z)\), and then it sums \(e_z\) and the error \(e_1\) of the one-bit quantizer. The digital parts, \(H_1\) and \(H_2\), are used to cancel the error of the one-bit quantizer error and to maintain the modulator to be multi-level shaping. Due to the function \(F(z)\), we can gain the order of NTF effectively.

This approach has the following advantages: (a) The non-linearity problem of DAC in the feedback loop of the first stage is avoided. (b) The quantization error of the one-bit quantizer can be cancelled in the followed digital filter, and the remained noise is the quantization error of the multi-level quantizer that is much less than that of the one-bit quantizer. (c) Theoretically, we can improve the order of the noise shaping by changing the transfer function \(F(z)\). The brief derivation of the ideal approach is described as follows,
\[ y = STF \cdot (H_1 + H_2) \cdot x \]
\[ + (NTF \cdot H_1 + (1 - NTF) \cdot H_2) \cdot e_1 \]
\[ + (1 + F) \cdot H_2 \cdot e_2 \]
(2)

Let
\[ \begin{cases} 
NTF \cdot e_1 = (NTF \cdot H_1 + (1 - NTF) \cdot H_2) = 0 \\
H_1 + H_2 = 1 
\end{cases} \]
(3)

then
\[ y = STF \cdot x + (1 + F) \cdot NTF \cdot e_2 \]
(4)

As mentioned before, equation (3) has to be satisfied to cancel the error of the one-bit quantizer. Let the transfer function of input signal, \( x \), be held as STF, and we can obtain equation (4). According to equation (2), \( e_2 \) must be shaped as a high-pass function. Intuitively, we can design \((1+F)\) to be the same as NTF:
\[ F = NTF - 1 \]
(5)

Because the NTF of this architecture is the square of the NTF of the first stage, the number of pole and zero is twice of the first stage. It is easy to prove that the proposed architecture has four zeros at \( z=1 \) and four poles at the roots of the denominator of the NTF. The stable condition of this system is to set all poles in the unit circle at \( z \)-domain. If we compare NTFs of the proposed architecture and the second order architecture, we can find that there is a crossed point between these two transfer functions, as shown in Fig. 3. When we choose the bandwidth of a base band to be wider than this critical frequency, the performance of the proposed architecture cannot be better than the performance of the second order multi-level SDM. Therefore, the noise contributions of the quantization error of the second order architecture and the proposed architecture will be dominated at frequency where
\[ \begin{align} 
|NTF| \cdot \rho_{e_1} & \geq \left| (1 + F) \cdot NTF \right| \cdot \rho_{e_2} 
\end{align} \]
(6)
Where \( \rho_{e_1} \) is the power of \( e_1 \) (M bits), and \( \rho_{e_2} \) is the power of \( e_2 \) (N bits). Combining equations (5) and (6), we get
\[ |NTF| \geq \frac{\rho_{e_1}}{\rho_{e_2}} \].
(7)

Assuming that the frequency range of interest is much lower than the Nyquist frequency, we may approximate equation (7) by
\[ OSR \geq \pi \cdot 2^{N-M}. \]
(8)

![Mathematical NTF spectrums of the second order approach and the proposed approach](image)

### 4 Simulation Results

The proposed architecture has been simulated in MATLAB. The main modulator is a second order with one-bit quantizer, and the second stage is a four-bit quantizer in the feed-forward path. We simulate it with a sinusoid signal (-6dB), the sampling rate is 2.56MHz; the oversampling rate is 64. An ideal second order SDM with four-bit quantizer is simulated to compare the proposed architecture. After scaling process, the path parameters are listed in Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Proposed SDM</th>
<th>Ideal second order SDM</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>A2</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>B1</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>B2</td>
<td>0.5</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 1 Path parameters

Fig. 4 shows the output power spectrums of the proposed SDM and second order SDM. It is just like the mathematical NTF spectrums of Fig. 3. Fig. 5 shows a comparison of the ideal proposed SDM and the proposed SDM with \( e_2' \) unequalling \( e_2 \). From Table 2, we can find that the proposed SDM decreases about 5dB when \( e_2' \) is considered, but the second-order SDM decreases about 20dB when the DAC mismatch is considered.
5 Conclusion

A multi-level sigma delta modulator with dual quantization is proposed in this paper. The proposed SDM can get order of noise transfer function as we want by F(z). By this approach, the remained noise is the quantization error of the multi-level quantizer, which is much less than that of the one-bit quantizer. In the new SDM, we do not need to consider the non-linearity of the DAC in the feedback path.

Reference: