Semi-active Suspension System with Electro-Rheological Damper

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Abstract: The design of suspension is very important to improve comfortableness and security of automobiles and structures. This paper treats the semi-active suspension system for automobile. As an actuator, a variable damper is constructed with ER (Electro- Rheological) fluid. A bilinear optimal control is applied to regulate the viscosity of the fluid. The performance of the system is confirmed with experiments.

Key-Words: ER Damper, Semi-active suspension, Bilinear optimal control

1 Introduction
In recent years, active suspension or semi-active suspension have been adopted to vehicles and buildings to improve ride-comfort and stability [1,2,3]. Although active suspension can reduce vibration more effectively, the equipment’s cost and energy consumption are not desirable, and when a control unit breaks, the safety of the system can't be guaranteed. On the other hand, the semi-active suspension in which the coefficient of viscosity is controlled can avoid those problems.

In this study, we treated such kinds of semi-active suspension system. Our research objective are to investigate the possibility of ER fluid [4], provide an effective control method, and establish a practical useful control system for vertical and horizontal vibration of vehicles.

This paper is the first step of our study. We introduce a new mechanism which uses ER fluid in the damper, an effective control method based on bilinear optimal control, and the performance tests with experiment.

2 System Construction
Figure 1 shows the image of experimental system where a balance structure is used to make experiment easier.

The principle of ER damper is to utilize the characteristic that the viscosity of fluid changes with voltage charged on. The ER damper cooperating with spring composes the single semi-active suspension in the set.

It is clear that the kinematics equation of the system can be treated as follow

\[ m \ddot{x} + C(x - x_o) + k(x - x_o) = 0 \]  \hspace{1cm} (1)

Where \( m \) is mass of \( M \); \( k \) is spring constant, \( x \) and \( x_o \) are displacements.

The structure of ER damper is shown in fig.2. ER fluid is filled in the space between cylinder and piston rod, and high voltage is added on the piston. Therefore,
the viscosity coefficient $C$ consists of uncharged part $c_0$ and charged part $c(v)$:

$$ C = c_0 + c(v) \quad (2) $$

Figure 2: Structure of ER Damper

3 Control Design

For system (1), the bilinear optimal theory [5] is applied to design a control rule. As usual doing, equation (1) can be represented to the following state form:

$$ \dot{X}(t) = AX(t) + BV(t)u(t) + \omega(t) \quad (4) $$

Where

$$ \dot{X}(t) = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}, \quad V(t) = \dot{x} \quad \omega(t) = \ddot{x}_w $$

$$ A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c_0}{m} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -\frac{\dot{x}}{m} \end{bmatrix} $$

$$ u(t) = c(v(t)) $$

$\omega(t)$ is disturbance from road condition, which is supposed to be a white noise.

Now, we can state out control problem as:

[Problem:]
Find a control $u(t)$ for system (4) such that
1. Closed loop system is stable.
2. The value of the following criterion function is minimum.

$$ J = \int [x^T(t)Qx(t) + u^T(t)Ru(t)]dt $$

$$ J = \int_0^\infty F(x,u,t)dt \quad (6) $$

here $Q$ and $R$ are weightings matrices with suitable size.

In the following, we simply explain a deriving process to the problem.
At first, the minimum of (6) is denoted as $W(x,t)$, that is

$$ W(x,t) = \min \int F(x,u,t)dt \quad (7) $$

It can be shown that the Hamilton-Jacobi-Bellman equation of the optimal condition (6) is as follows.

$$ -\frac{\partial W}{\partial t} = \min \left\{ F(x,u,t) + f(x,u,t)\frac{\partial W}{\partial x} \right\} $$

$$ = \min \{ x^T(t)Qx(t) + u^T(t)Ru(t) \} $$

$$ + [Ax(t) + BV(t)u(t)]^T \frac{\partial W}{\partial x} $$

$$ + \frac{1}{2} \frac{\partial}{\partial x} \omega(t) \left[ \frac{\partial}{\partial x} W(x,t) \right] $$

$$ = \min \{ x^T(t)Qx(t) + u^T(t)A^T \frac{\partial W(x,t)}{\partial x} \} $$

$$ + \frac{1}{2} \frac{\partial}{\partial x} \omega(t) \left[ \frac{\partial}{\partial x} W(t) \right] $$

$$ + \left[ u(t) + \frac{1}{2} V^T R^{-1} B^T \frac{\partial W(x,t)}{\partial x} \right] $$

$$ R \left[ u(t) + \frac{1}{2} V^T R^{-1} B^T \frac{\partial W(x,t)}{\partial x} \right] $$

$$ - \frac{1}{4} \left[ V^T R^{-1} B^T \frac{\partial W(x,t)}{\partial x} \right] R^{-1} \left[ V^T R^{-1} \frac{\partial W(x,t)}{\partial x} \right] $$

$$ \frac{\partial}{\partial x} \frac{\partial W(x,t)}{\partial x} $$

(8)

Hence, the control $u^*(t)$ which minimizes the
right-hand side of (8) is given by

\[ u^*(t) = -\frac{1}{2} V^T [R]^{-1} B^T \frac{\partial W(x,t)}{\partial x} \]  

(9)

then equation (8) becomes

\[
\begin{align*}
\frac{\partial W}{\partial t} &= x^T(t)Q(t) + x^T(t)A^T \frac{\partial W(x,t)}{\partial x} \\
+ &\frac{1}{2} \left[ \frac{\partial}{\partial x} \right]^T \left[ \frac{\partial}{\partial x} \right] W(x,t) \\
- &\frac{1}{4} \left[ V^T B^T \frac{\partial W(x,t)}{\partial x} \right]^T R^{-1} \left[ V^T B^T \frac{\partial W(x,t)}{\partial x} \right]
\end{align*}
\]

(10)

On the other hand, the minimum \( W(x,t) \) is assumed to be the following form

\[ W(x,t) = x^T(t)P(t)x(t) \]  

(11)

Substituting (11) to (10) yields

\[
\begin{align*}
-x^T(t) \dot{P}(t)x(t) &= x^T(t)Q(t) + x^T(t)P(t)Ax(t) \\
- &\frac{1}{4} \left[ 2x^T(t)P(t)V^T B^T \right]^T R^{-1} \left[ 2x^T(t)P(t)V^T B^T \right] \\
 &= x^T(t)Q(t) + x^T(t)P(t)Ax(t) \\
+ &\frac{1}{4} \left[ 2x^T(t)P(t)V^T B^T \right] x(t) \\
- &x^T(t)P(t)VBR^1B^T V^T P(t)x(t)
\end{align*}
\]

(12)

Where \( P(t) \geq 0 \) satisfies the following equation

\[
\dot{P}(t) = P(t)A + A^T P(t) - P(t)VBR^1 B^T V^T P(t) + Q \]  

(13)

Then the optimal solution is

\[
\begin{align*}
u^*(t) &= -\frac{1}{2} V^T [R]^{-1} B^T \frac{\partial W(x,t)}{\partial x} \\
&= -\frac{1}{2} V^T [R]^{-1} B^T \left[ 2x^T(t)P(t) \right] \\
&= V^T R^{-1} B^T \left[ P(t) \right]^T x(t)
\end{align*}
\]

(14)

Notice that the Riccati equation should be solved step by step. In some other papers, the performance about input is \((Vu)^T R(Vu)\) in the criterion function, which results undesirable high gain feedback. In this paper, considering the components of the system, we chose \( u^TRu \) to be restricted.

4 Experimental Results

This section shows some results about basic experiment of ER fluid and performance tests of the system.

4.1 ER damper

(1) The constant \( c_0 \) is obtained by damping test without charge. The result is

\[ c_0 = 0.5059 \ [Ns/m] \]

(2) Figure 3 is a measured relation of add voltage to damping force. From this graph, \( c(v) \) can be approximated as:

\[ c(v) = 1.1 \times 10^{-3} V \ [Ns/m] \]  

(15)

![Figure 3: Characteristic of Viscosity Coefficient](image-url)
4.2 Control performance

Figure 4 and 5 is the control results. Here, time responses and frequency response are presented, respectively. In those experiments, the weighting parameters are selected as

\[ Q = \begin{bmatrix} 30 & 0 \\ 0 & 1 \end{bmatrix}, \quad R = 1 \]

The ER damper has good sensitivity, and semi-active damping system with ER damper and bilinear controller can reduce vibration more effectively.

5 Conclusion

In this study, we constructed a semi-active suspension system with ER damper and bilinear optimal controller. The results suggest that such kind of system is expectable. We intend to develop a practical suspension system for vehicle depends on those results.

References:


