Performance Assessment of Wavelength Routing Optical Networks with Regular Degree-Three Topologies of Minimum Diameter

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Abstract: - In this paper, we present an assessment of the blocking performance in wavelength routing optical networks with degree-three topologies of minimum diameter. It is analysed a general family of degree-three topologies, of which the chordal ring family is a particular case. Performance results show that all topologies of absolute minimum diameter have exactly the same path blocking probabilities.

Key-Words: - Wavelength routing optical networks, traffic performance, regular degree-three topologies, wide area networks, optical networking, performance evaluation.

1 Introduction

IP-over-WDM (IP: Internet Protocol, WDM: Wavelength Division Multiplexing) networks are expected to offer an infrastructure for the next generation Internet [1]-[6]. Actually, the world-wide deployment of WDM optical links, to satisfy the bandwidth requirements imposed by the traffic growth, is seen as the first phase of optical networking and recent technology developments, such as the advent of Optical Add/Drop Multiplexers (OADMs) and Optical Cross-Connects (OXCs), are enabling the evolution from those point-to-point WDM links to wavelength routing networks. Optical networks with wavelength routing in a mesh topology are now being object of intense research. In [7], it is presented a study of the influence of nodal degree on the fibre length, capacity utilisation, and average and maximum path lengths of wavelength routed mesh networks. It is shown that average nodal degrees varying between 3 and 4.5 are of particular interest.

In this paper, we consider degree-three topologies, of which the chordal ring family is a particular case. Chordal rings are a well-known family of regular topologies with nodal degree of 3 and were proposed by Arden and Lee [8], in the early eighties, for interconnection of multi-computer systems. Since then, some studies have been published concerning properties of chordal rings [9]-[13].

Recently, Freire and da Silva [14] have investigated the influence of the chord length on the traffic performance of wavelength routing chordal ring networks. They have shown that the best network performance is obtained for the chord length that leads to the smallest network diameter. In [15], the same authors have shown that the performance of a chordal ring network (which has a nodal degree of 3), with a chord length that leads to the smallest diameter, is similar to the performance of a mesh-torus network (which has a nodal degree of 4). Since a chordal ring network with N nodes has $3N$ links and a mesh-torus network with N nodes has $4N$ links, the choice of a chordal ring with minimum diameter, instead of mesh-torus, reduces network links by 25%. Moreover, since chordal rings have lower nodal degree, they require in each switch, a smaller number of node-to-node interfacing (NNI) ports.

However, there are some restrictions that limit the practical implementation of chordal rings with the smallest network diameter (as well as mesh-torus), when compared with other chord lengths. In fact, the smallest network diameter in a chordal ring was obtained with the chord length of $\sqrt{N} + 3$, for an N -node chordal ring, where N is a square ($N=m^2$) and $N \geq 64$. These restrictions are not imposed to other chord lengths such as $N/4$ or 3. In this paper, we investigate the existence of other families of degree-three topologies with minimum diameter and we present a performance comparison of their

performance with the performance of chordal rings of minimum diameter.

The remainder of this paper is organised as follows. The model used to compute the path blocking probability in wavelength routed networks, with degree-three topologies, is briefly described in section 2. A performance assessment of the blocking performance in these kind of networks with minimum diameter is presented in section 3 and main conclusions are presented in section 4.

2 Evaluation of Path Blocking Performance

One of the key performance metrics for optical networks with wavelength routing is the path blocking probability, i. e., the probability of a connection request be denied due to unavailable optical paths.

To compute the path blocking probability in optical networks with wavelength interchange, we have used the model presented in [16], since it applies to topologies with low connectivity, has a moderate computational complexity, and takes into account dynamic traffic and the correlation between the wavelengths used on successive links of a multi-link path.

The following assumptions are used in the model [16]: 1) Session requests arrive at each node according to a Poisson process, with each session equally likely to be destined to any of the remaining nodes. 2) Session holding time is exponentially distributed. 3) The path used by a session is chosen according to a pre-specified criterion (e.g. random selection of a shortest path), and does not depend on the state of the links that make up a path; a session is blocked if the chosen path can not accommodate it; alternate path routing is not allowed. 4) The number of wavelengths per link, F , is the same on all links; each node is capable of transmitting and receiving on any of the F wavelengths; each session requires a full wavelength on each link it traverses. 5) Wavelengths are assigned to a session randomly from the set of free wavelengths on the associated path.

In addition to the above assumptions, it is assumed in [16] that, given the loads on links 1, 2, ..., $i-1$, the load on link i of a path depends only on the load on link $i-1$ (Markovian correlation model). The analysis presented in [16] also assumes that the hop-length distribution is known, as well as the arrival rates of sessions at a link that continue, and those that do not, to the next link of a path. The session arrival rates at links have been estimated from the arrival rates of sessions to nodes, as in [16]. The hop-length distribution is a function of the

network topology and the routing algorithm, and is easily determined for most regular topologies with the shortest-path algorithm.

Concerning hop-length distribution for degree-three topologies, in some particular cases, we have found general expression for the hop-length distribution, but even for the chordal ring family, we were unable to find a general expression for the hop-length distribution, as a function of number of nodes and chord lengths. In this work, instead of defining the hop-length distribution through an analytical equation used by the analytical framework to compute the path blocking probability, we developed an algorithm that provides the hop-length distribution for a given N -node topology of the DTT(w_1, w_2, w_3) type. An explanation of this kind of topologies follows.

A chordal ring is basically a ring topology, in which each node has an additional link, called a chord. The number of nodes in a chordal ring is assumed to be even, and nodes are indexed as 0, 1, 2, ..., $N-1$ around the N -node ring. It is also assumed that each odd-numbered node i ($i=1, 3, \dots, N-1$) is connected to a node $(i+w) \bmod N$, where w is the chord length, which is assumed to be positive odd [8]. For a given number of nodes there is an optimal chord length that leads to the smallest network diameter. The network diameter is the largest among all of the shortest path lengths between all pairs of nodes, being the length of a path determined by the number of hops.

In each node of a chordal ring, we have a link to the previous node, a link to the next node and a chord. Suppose now that the links to the previous and to the next nodes are replaced by chords. Thus, each node has three chords, instead one. Let w_1, w_2 , and w_3 be the corresponding chord lengths, and N the number of nodes. We represent a general degree-three topology by DTT(w_1, w_2, w_3), assuming that each odd-numbered node i ($i=1, 3, \dots, N-1$) is connected to the nodes $(i+w_1) \bmod N$, $(i+w_2) \bmod N$, and $(i+w_3) \bmod N$, where chord lengths, w_1, w_2 , and w_3 are assumed to be positive odd, with $w_1 \leq N-1$, $w_2 \leq N-1$, and $w_3 \leq N-1$, and $w_i \neq w_j, \forall i \neq j, 1 \leq i, j \leq 3$. In this notation, a chordal ring with chord length w_3 is simply represented by DTT(1, $N-1, w_3$). In this paper we concentrate on the DTT(1, w_2, w_3) family, of which the chordal ring family is a special case. As an example, figure 1 shows two degree-three topologies: DTT(1, $N-1, 9$) (chordal ring with a chord length of 9) and DTT(1, 3, 9), both for $N=20$ nodes.

3 Assessment of Blocking Performance in Wavelength Routing Networks with Regular Degree-three Topologies

In this section, we present an assessment of the blocking performance in wavelength routing networks with a topology of the type $DTT(1, w_2, w_3)$. The performance analysis is focused in networks with 100 nodes.

Fig. 1. Schematic representation of two regular degree-three topologies for networks with 20 nodes:
a) $DTT(1, N-1, 9)$ (Chordal ring network with a chord length of 9);
b) $DTT(1, 3, 9)$.

Figure 2 shows the maximum (absolute) and the minimum (absolute) diameter in each of the following families $DTT(1, 3, w_3)$, $DTT(1, 5, w_3)$, ..., $DTT(1, N-1, w_3)$. Note that, in this figure, the diameter is displayed as a function of w_2 . As may be seen, there is a symmetry in this figure. Moreover, in some ranges there is also a periodic variation of the network diameter. One important observation that can be made from this figure is that the minimum (absolute) diameter of the general family $DTT(1, w_2, w_3)$ is 9, i. e., the same minimum diameter observed for the chordal ring family.

Figure 3 shows the network diameter for three interesting families of figure 2: $DTT(1, 11, w_3)$, in which the diameter ranges between 10 and 12, $DTT(1, 21, w_3)$, in which the diameter is always 10, and $DTT(1, 51, w_3)$, in which the diameter is always 25. For the case of $DTT(1, 11, w_3)$, the network is not connected for $w_3=21$, $w_3=31$, $w_3=41$, $w_3=51$, $w_3=61$, $w_3=71$, $w_3=81$, and $w_3=91$. The case of $w_3=11$ is not allowed, since, with $w_2=11$, it reduces a degree topology to a degree two topology. This family of topologies is very interesting because it always has a diameter slightly higher than the minimum. The $DTT(1, 21, w_3)$ family, although it has always a diameter of 10, it is not so interesting because, in this family, the network is connected only for a small number of values of w_3 : $w_3=3$, $w_3=7$, $w_3=15$, $w_3=19$, $w_3=23$, $w_3=27$, $w_3=35$, and $w_3=39$, $w_3=43$, $w_3=47$, $w_3=55$, $w_3=59$, $w_3=63$, $w_3=67$, $w_3=75$, $w_3=79$, $w_3=83$, $w_3=87$, $w_3=95$, and $w_3=99$. Our performance results show that the blocking probability changes in the same way as diameter. The minimum diameter leads to the best performance, whereas the maximum diameter leads to the worst performance. The $DTT(1, 51, w_3)$ family is not interesting, because it leads to very high blocking probabilities, since their diameter is far from the minimum diameter.

Figure 4 shows the number of absolute minimum diameters in each of the following families: $DTT(1, 3, w_3)$, $DTT(1, 5, w_3)$, ..., $DTT(1, N-1, w_3)$. As can be seen, the number of minimum diameters, as a function of w_3 , changes periodically. Moreover, the maximum number of minimum diameters is 4.

One interesting result that we also found is concerned with the diameters of $DTT(1, 3, w_3)$ and $DTT(1, N-1, w_3)$ families. The last one is the chordal ring family with a chord length of w_3 . Figure 5 shows the diameters for these two families.

Fig. 2. Minimum and maximum network diameters for DTT (1, w_2 , w_3), with $3 \leq w_2 \leq 99$, and $3 \leq w_3 \leq 99$.

Fig. 3. Minimum network diameters for DTT (1, 11, w_3), DTT (1, 21, w_3), and DTT(1,51, w_3), with $3 \leq w_3 \leq 99$.

Fig. 4. Number of minimum diameters in each DTT (1, w_2 , w_3), with $3 \leq w_2 \leq 99$, and $3 \leq w_3 \leq 99$.

Fig. 5. Network diameters for DTT (1, $N-1$, w_3), DTT (1, 3, w_3), with $3 \leq w_3 \leq 99$.

Fig. 6. Path blocking probability versus load per node for networks with DTT(1,99,13) (chordal ring with minimum diameter) and DTT(1, 3, 15) topologies, without wavelength interchange. $N=100$; F : number of wavelengths per link.

Fig. 7. Path blocking probability versus converter density for networks with DTT(1,99,13) (chordal ring with minimum diameter) and DTT(1, 3, 15) topologies. $N=100$; Load per node: 0.5 Erlang; F : number of wavelengths per link.

Fig. 8. Path blocking probability versus load per node for networks with DTT(1,19,27) and DTT(1,43,37) topologies, without wavelength interchange. $N=100$; F : number of wavelengths per link.

As can be seen, the diameter of the DTT(1, 3, w_3) is a shifted version of the diameter of the chordal ring family. For instance, the diameters of DTT(1, $N-1$, 3) and DTT(1, 3, 5) is 26, the diameters of DTT(1, $N-1$, 5) and DTT(1, 3, 7) is 18, the diameters of (1, $N-1$, 7) and DTT(1, 3, 9) is 14, and so on (see figure 5).

Figure 6 shows the path blocking probability of wavelength routing networks with topologies of DTT(1,99,13) and DTT(1, 3, 15). As can be seen, for each number of wavelengths per link, these networks have exactly the same path blocking probability. This observation is confirmed in figure 7, which shows the path blocking probability as a function of the wavelength converter density. Although not shown in both figures, DTT(1, 3, 15), DTT(1, 3, 17), DTT(1, 3, 87), DTT(1, 3, 89), and DTT(1, 99, 13), DTT(1, 99, 15), DTT(1, 99, 85), and DTT(1, 99, 87) topologies, which have a minimum diameter of 9, have exactly the same path blocking probability.

After these results, we analysed the blocking performance of wavelength routing networks with all degree-three topologies of minimum diameter and we found that all topologies of minimum diameter, belonging to the DTT(1, w_2 , w_3) family, have exactly the same path blocking probability of chordal ring with minimum diameter. As an example, figure 8 shows path blocking probabilities for DTT(1,19,27) and DTT(1, 43, 37) topologies, which have a diameter of 9. These curves are exactly the same as the ones of figure 6. We looked for an explanation for these observations and we found that all degree-three topologies of the type DTT(1, w_2 ,

w_3), with a minimum diameter of 9, have exactly the same hop length distribution.

4. Conclusions

We presented an assessment of the blocking performance in wavelength routing optical networks with degree-three topologies of minimum diameter. The network diameter of 100-node topologies of the DTT(1, w_2 , w_3) family have been analysed. Since minimum diameters lead to the best network performance, we focused the analysis in topologies with minimum diameter. Obtained results show that all 100-node topologies of the type DTT(1, w_2 , w_3), with a minimum diameter of 9, have exactly the same path blocking probabilities. This observation may be explained by the fact that all degree-three topologies of the type DTT(1, w_2 , w_3), with a minimum diameter of 9, have exactly the same hop length distribution.

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