Abstract: - Two simple circuits containing one operational amplifier (op-amp), i.e. current controlled-voltage source (CCVS) and inverting voltage amplifier, are analysed. In the first step the op-amp is considered ideal, while in the second step the simple one-pole model of the op-amp is used. The corresponding ideal and RL equivalent circuits and typical network functions are calculated for both ideal and one-pole model case.

Key-Words: - Operational Amplifier, Ideal Model, One-Pole Model

1 Introduction

The analysis of linear resistive circuits containing operational amplifiers (op-amp) is usually realized in two steps. In the first step the op-amp is considered ideal (Fig 1a), all the network functions obtained do not depend on frequency and represent “DC” solution without any information about op-amp input polarity, stability conditions, etc. [1].

In the second step the so-called one-pole model [2] is usually used where op-amp is considered ideal voltage amplifier (Fig. 1b) with simple frequency depended voltage gain having high DC gain and one simple dominant pole, i.e.

\[ A(s) = \pm \frac{A_0}{1 + s \tau_0}, \quad A_0 >> 1, \quad \tau_0 = \frac{1}{\omega_0} \quad (1) \]

where \( \omega_0 \) is the corresponding (-3 dB) frequency. It can be rewritten into the form

\[ A(s) = \pm \frac{1}{1/A_0 + s/\omega_0} = \frac{1}{1/A_0 + s/\omega_t} = \frac{1}{1/A_0 + s\tau}, \quad (1a) \]

where the so-called transition frequency \( \omega_t = \frac{1}{\tau} = A_0 \omega_0 \) represents the product of DC gain \( A_0 \) and bandwidth \( \omega_0 \) (GBP). All the commercially available voltage-mode op-amps have usually the internal compensation of their open-loop gain frequency characteristic and then can be described with sufficient accuracy by this simple model [2].

For the analysis the one-pole model can further be simplified and expressed separately for two typical frequency ranges, i.e. \( \omega << \omega_0 \) and \( \omega >> \omega_0 \).

a) When \( \omega << \omega_0 \) then \( \omega \tau << 1/A_0 \), the voltage gain function (1) does not depend on frequency and can be expressed by the simplified form

\[ A(s) = \pm A_0 \quad (2a) \]

Because \( A_0 >> 1 \), the inverse value \( A_0^{-1} \to 0 \) and such a situation is described with sufficient accuracy by ideal op-amp model without input polarity (Fig 1a).
b) When $\omega >> \omega_0$ then $\omega \tau >> 1/A_0$ and the voltage gain function (1) can be simplified as

$$A(s) \approx \pm \frac{1}{s\tau}, \quad (2b)$$

which corresponds to ideal integrator that includes the op-amp input polarity and its basic frequency properties. It enables us to analyze the corresponding frequency characteristic of the complete circuit and determine the correct op-amp input polarity from the stability viewpoint. From this reason this simple modification of one-pole model is currently used in theory of R filters [3], compound amplifiers [4], and phase-compensated amplifiers [5].

The next two examples illustrate its utilization in two-step analysis of simple op-amp structures, i.e. one of controlled sources and non-inverting voltage amplifier, where also all related transfer functions and some non-conventional simple equivalent RL circuit are shown.

2 Current controlled-voltage source (CCVS)

2.1 Ideal model

The well known ideal op-amp structure with single feedback resistor (Fig. 2a) has also very simple two-port impedance matrix (Fig. 2b) with zero input- and output- , and nonzero transfer impedances

$$z_{11} = z_{22} = 0, \quad z_{21} = -R. \quad (3a)$$

It evidently represents ideal CCVS (Fig. 2c) having its "DC" transfer parameter $W = -R$. It is in accordance with ideal op-amp infinity gain $K_{12} \rightarrow \infty$.

2.2 One-pole model

To obtain the parasitic pole of the basic structure (Fig. 3a) in the left half of the s-plane the inverting op-amp input is used. By simple analysis its two-port impedance matrix is determined (Fig. 3b) that shows how the simplified frequency dependent op-amp gain (2b) modifies its impedance parameters. In comparison with ideal case (3a) it includes nonzero and frequency dependent input- and transfer- , and zero output impedances

$$z_{11}(s) = \frac{stR}{1 + st} = stR\epsilon(s), \quad z_{22} = 0,$$

$$z_{21}(s) = -R \frac{1}{1 + st} = -R\epsilon(s), \quad (3b)$$

where $\epsilon(s) = \frac{1}{1 + st}$ is the so-called error function [2] and represents the dominant parasitic pole of the analyzed structure. Then the equivalent circuit can be expressed as the cascade connection of parasitic RL two terminal element and non-ideal CCVS (Fig. 3c). The parasitic voltage gain of this structure is the same as the simplified op-amp voltage gain function (2b)

$$K_{12}(s) = \frac{z_{21}(s)}{z_{11}(s)} = -\frac{1}{st}. \quad (3c)$$
3 Inverting voltage amplifier

3.1 Ideal model

Connecting the additional input resistor $R_0$ to the previous CCVS equivalent structure the well known inverting voltage amplifier is obtained. Its ideal op-amp circuit model (Fig. 4a) has also simple two-port impedance matrix (Fig. 2b) with nonzero input- and transfer-, and zero output impedances

$$z_{11} = R_0, \quad z_{21} = -R, \quad z_{22} = 0. \quad (4a)$$

It entails “DC” voltage gain $A = K_{12} = \frac{z_{21}}{z_{11}} = -\frac{R}{R_0}$, so that its ideal equivalent circuit model can be expressed either as the cascade connection of the input resistor $R_0$ and ideal CCVS or the inverting voltage amplifier with finite input impedance (Fig. 4c).

$$A(s) = K_{12}(s) = \frac{z_{21}(s)}{z_{11}(s)} = -\frac{R}{R_0} e_{LP}(s) \quad (4c)$$

and parasitic frequency dependent input impedance (Fig. 5c). The error function $e_{LP} = \frac{1}{1 + s\tau_A}$ is related to this voltage gain only and its time constant $\tau_A = \tau(1 + \frac{R}{R_0}) > \tau$. It entails lower bandwidth of the inverting low-pass R filter with cut-off frequency $\omega_A = \frac{1}{\tau_A}$ which this amplifier represents as R filter (Fig. 6a). Expressing another possible voltage gain between ports 1 and 3

$$K_{13}(s) = \frac{R}{R_0} \frac{s\tau}{1 + s\tau} = \frac{R}{R_0} e_{HP}(s) \quad (4d)$$

we obtain the non-inverting high-pass R filter with the same $\omega_A$ and absolute value of “DC” voltage gain, however, with nonzero output impedance (Fig. 6b).

3.2 One-pole model

Consider again the inverting op-amp input and utilizing the same analysis procedure of the basic structure (Fig. 5a) its two-port impedance matrix is obtained (Fig. 5b). It includes nonzero and frequency dependent input- and transfer-, and zero output impedances

$$z_{11}(s) = R_0 + \frac{stR}{1 + st} = R_0 + s\tau R \epsilon(s), \quad z_{22} = 0,$$

$$z_{21}(s) = -\frac{1}{1 + s\tau} = -R \epsilon(s), \quad (4b)$$

where $\epsilon(s)$ is the same basic error function as in CCVS model but here related to impedance parameters only. The corresponding equivalent circuit model can represent either cascade connection of parasitic $R_0$ and RL two terminal elements and non-ideal CCVS or the inverting voltage amplifier with frequency dependent voltage gain

$$A(s) = K_{12}(s) = \frac{z_{21}(s)}{z_{11}(s)} =$$

Choosing the floating output port between nodes 2 and 3, the voltage gain can be expressed as the difference

$$K_{1,23}(s) = K_{12}(s) - K_{13}(s) =$$
\[ \frac{R}{R_0} \frac{1 + s\tau}{1 + s\tau (1 + \frac{R}{R_0})} = \frac{R}{R_0} \frac{1 + s\tau}{1 + s\tau A} \quad (4e) \]

and the corresponding combined frequency characteristic is shown in Fig. 6c.

Fig. 6. Frequency characteristics of the individual R filters in the inverting amplifier. a) Low-pass, b) high-pass, c) combined type.

4 Conclusion

Two step analysis of simple resistive op-amp structures and their corresponding non-conventional RL equivalents circuits are presented. By the similar way also some new relations in R filters and phase-compensated amplifiers can systematically be investigated and found.

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