Off-Line Parameter Estimation of Induction Motor Based on a Multitime Scale Approach

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Abstract: The paper presents a method for parameters estimation of squirrel cage induction motor. The state-space model of the system can be decomposed in two quasi-decoupled systems because the dynamic of electrical system and the one of the magnetizing system are different. It is used a multitime scale decomposition to obtain reduced operational reactances. In the experiments the induction machine is at standstill and supplied by a pseudorandom binary sequences voltage. This method allows an increase of the parameters accuracy obtained through estimation. The physical parameters of the equivalent system of the phase equivalent circuit of induction machine are determined based on the information from the identification experiments specific to each model.

Key-Words: identification, induction machine, parameter estimation, multime scale system, reduced order model, excitation signal.

1 Introduction
Modern control techniques, like adaptive, optimal, predictive or diagnosis, are based on a priori information from the process. Generally these techniques use minimal information about the process concretised in a model (structured information) [1],[2]. It is desired a minimum order model able to preserve the essence of the dynamic of the modelled system. For the AC machines parameter identification are three groups of methods [3].

The first ones are based on theory of the electromagnetic field. For a specified operating point, the magnetic state of the machine can be computed by a finite-element method and the parameters derived from it. The second category is called optimal adjustment of parameters. These methods use the identification methods to obtain an approximate model. This model matches the frequency or time response of the original system. The most known method from this category is the least squares approximation method. The last category consists in a group of methods, which are based on the identification of the dominant eigenvalues of the original system and their inclusion in the reduced model. This is called the preservation of the dominant modes of the system. The method presented in this paper is from this category.

The induction motor is suited for a decomposition in three subsystems, each of them with a distinct dynamic: the electrical subsystem which is the faster one, the magnetizing subsystem, which is fast, and the mechanical subsystem, which is slow. In order to increase the precision of the electrical and magnetizing parameters it is proposed the decomposition of the electromagnetic system of the induction motor in two quasi-decoupled systems. After that, the parameters of each system are estimated separately in identification experimental condition suited for the dynamic of each of these systems.

The parameter estimation experiments must provide enough excitation of the natural modes in order to ensure the consistency of the estimated parameters. If the spectrum of the system matrix is large, the precision of the estimated parameters is low because is hard to ensure an optimum compromise between the sampling period, the spectrum of the excitation signal, the energy of it to excite not the non-linear dynamic regimes etc.

2 The Multime Scale Technique
Let consider a state-space model of a linear multivariable system with appropriate dimensions of the matrix and vectors:
\[
\begin{aligned}
\frac{dX}{dt} &= AX + BU \\
Y &= CX
\end{aligned}
\] (1)

On the basis of physical significance, or using the constituent matrix, the system can be restructured in two subsystems characterised by states with highly different dynamic [3], [4].

\[
\begin{bmatrix}
\frac{dx_1}{dt} \\
\frac{dx_2}{dt}
\end{bmatrix}
= \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
+ \begin{bmatrix}
B_1 \\
B_2
\end{bmatrix} u
\] (2)

where \(x_1\) represent the “slow” states and \(x_2\) represent the “fast” states. To highlight the different time scales one can use an adimensional small positive parameter \(\varepsilon\), and by abuse of notation, the eq. 2 can be rewritten as:

\[
\begin{bmatrix}
\frac{dx_1}{\varepsilon} \\
\frac{dx_2}{\varepsilon}
\end{bmatrix}
= \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
+ \begin{bmatrix}
B_1 \\
B_2
\end{bmatrix} u
\] (4)

The derivative of the component \(x_2\) is of \(1/\varepsilon\) order, and this component will have a high dynamic. Sometimes the parameter \(\varepsilon\) is considered as describing a parasitic effect and the designer is tempted to ignore the term \(\varepsilon \frac{dx_2}{dt}\) when he wants to obtain a reduced order model. For the slow time scale the process is modelled with the differential equation system for the states \(x_1\) and some algebraic relations for the states \(x_2\). This method is called sometimes the quasi-stationary method because the fast part of the process drives on \(x_2\) at a quasi-stationary value, where \(x_1\) follow the reduced order model output.

Even if the dynamic evolution of the variables is different, their evolution is not completely independent. This is way it is possible a decomposition of them:

\[
\begin{aligned}
x_1 &\approx \bar{x}_1 + \hat{x}_1 \\
&= \bar{u} + \bar{u}
\end{aligned}
\]

\[
\begin{aligned}
x_2 &\approx \bar{x}_2 + x_2 \\
&= \bar{y} + y
\end{aligned}
\] (5)

In (5) we have denoted with \(\bar{\cdot}\) the variables specific to the slow time scale and with \(\hat{\cdot}\) the variables specific to the fast time scale.

The reduced model, specific for the slow time scale, can be obtained considering the parameter \(\varepsilon\) null.

\[
\begin{bmatrix}
\frac{d\bar{x}_1}{dt} \\
\frac{d\bar{x}_2}{dt}
\end{bmatrix}
= \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
\bar{x}_1 \\
\bar{x}_2
\end{bmatrix}
+ \begin{bmatrix}
B_1 \\
B_2
\end{bmatrix} u
\] (6)

which means a system of differential equations and a system of algebraic equations:

\[
\begin{aligned}
\frac{dx_1}{dt} &= \left( A_{11} - A_{12} A_{22}^{-1} A_{21} \right) \bar{x}_1 + \left( A_{12} \right)^{-1} B_1 u \\
\bar{y} &= \left( C_1 - C_2 A_{22} A_{21}^{-1} \right) \bar{x}_1 - C_2 A_{22} \bar{u}
\end{aligned}
\] (7)

If is considered the fast scale time yields:

\[
\begin{aligned}
\frac{d\hat{x}_1}{dt} &= \varepsilon A_{11} \hat{x}_1 + \varepsilon A_{12} \hat{x}_2 + \varepsilon B_1 \hat{u} \\
\frac{d\hat{x}_2}{dt} &= A_{22} \hat{x}_2 + B_2 \hat{u} + A_{21} \hat{x}_1
\end{aligned}
\] (8)

From the slow part equation

\[
\begin{aligned}
\frac{d\bar{x}_1}{dt} &= \varepsilon A_{11} \bar{x}_1 + \varepsilon A_{12} \bar{x}_2 + \varepsilon B_1 \bar{u} \\
\frac{d\bar{x}_2}{dt} &= A_{22} \bar{x}_2 + B_2 \bar{u} + A_{21} \bar{x}_1
\end{aligned}
\] (9)

one can see that the derivative of the component \(x_1\) is of \(1/\varepsilon\) order, which means a slow variation \(\frac{dx_1}{dt} \approx 0\), the state equation for the fast part being:

\[
\begin{aligned}
\frac{d\hat{x}_2}{dt} &= A_{22} \hat{x}_2 + B_2 \hat{u} + A_{21} \hat{x}_1 \\
y &= C_1 \hat{x}_1 + C_2 \hat{x}_2
\end{aligned}
\] (10)

If we look at the term \(\hat{x}_1\) as a disturbing factor almost constant, one can associate to the fast time scale system a differential equation system in the form:

\[
\begin{aligned}
\frac{d\hat{x}_2}{dt} &= A_{22} \hat{x}_2 + B_2 \hat{u} \\
y &= \varepsilon A_{11} \bar{x}_1 + \varepsilon A_{12} \bar{x}_2 + \varepsilon B_1 \bar{u}
\end{aligned}
\] (11)

This way is obtained a “limit layer” model with the propriety that it will influence the system dynamic only in the proximity of the initial moment.

Some authors [2] show that if the system (6) has \(n_1\) eigenvalues \(\hat{\lambda}_i\) and the system (11) has \(n_2\) eigenvalues \(\hat{\mu}_j\), the spectrum of the matrix \(A\) in (4) has the eigenvalues of the form:

\[
\begin{aligned}
\hat{\lambda}_i + \omega_1(\varepsilon) \\
\frac{1}{\varepsilon} \left( \hat{\mu}_j + \omega_2(\varepsilon) \right)
\end{aligned}
\]

where \(\lim_{\varepsilon \to 0} \omega_1(\varepsilon) = \lim_{\varepsilon \to 0} \omega_2(\varepsilon) = 0\).
the matrix \(A_{11} - A_{12}A_{22}^{-1}A_{21}\) and \(n_2\) fast modes associated with of the matrix \(A_{22}\).

3 Decomposition of the Squirrel Cage Induction Motor Electromagnetic Model

Let us consider the voltage feed squirrel cage induction motor. The electromagnetic of this machine can be represented with several equivalent state-space models. If we choose as complex variables in the Park d-q axis the phasors \(i_s\) and \(\Psi_r\) in the stator frame reference \((\omega_s=0)\), the model of the machine is:

\[
\begin{align*}
\dot{X} &= AX + BU \\
Y &= CX
\end{align*}
\]

where

\[
X = \begin{bmatrix} \Psi_r \\ i_s \end{bmatrix}^T; \quad U = u_s; \quad Y = i_s;
\]

\[
A = \begin{bmatrix} -\frac{1}{T_f} + j\omega_s & \frac{L_m}{T_f} \\ \frac{1-\sigma}{\sigma L_m} \left( \frac{1}{T_r} + j\omega_s \right) & -\frac{1}{\sigma T_i} - \frac{1-\sigma}{T_r} \end{bmatrix}
\]

\[
B = \begin{bmatrix} 0 \\ \frac{1}{\sigma L_s} \end{bmatrix}; \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}^T
\]

It is well known that the variations of the stator state variables are much faster than the variations of the rotor flux. Some authors consider that the rotor variables can be obtained using the equations of the stator voltage in steady-state. A better modelling of the fast and slow modes can be achieved considering that the rotor transient regime starts before the stator transient regime is finished [5].

The model from (12) is already structured in two subsystems with different dynamics from physical considerations. The stator current \(i_s\) is the variable with a fast dynamic and rotor flux \(\Psi_r\) is the variable with a slow dynamic.

The comparison between this model and the model (4) leads to the observation that the system has two complex variables and two eigenvalues:

\[
\lambda_{1,2} = -\frac{1}{2} \left[ \frac{1}{2} \left( \frac{1}{T_f} + \frac{1}{T_r} \right) \right] \pm \frac{1}{2} \sqrt{\left( \frac{1}{2} \left( \frac{1}{T_f} + \frac{1}{T_r} \right) \right)^2 - 4 \left( \frac{1}{\sigma T_i} + \frac{1}{T_r} \right) - j\omega_s}
\]

For the particular case of the squirrel cage induction motor, the matrix used in the model (4) are:

\[
A_{11} = -\frac{1}{T_f} + j\omega_s; \quad A_{12} = \frac{L_m}{T_f}; \quad A_{21} = \frac{1}{\sigma T_i} \left( \frac{1}{T_r} + \frac{1-\sigma}{T_r} \right) \quad (14)
\]

\[
A_{22} = 1 - \frac{\sigma}{\sigma L_m} \quad (15)
\]

\[
B_1 = 0; \quad B_2 = \frac{1}{\sigma L_s} \quad (16)
\]

With the new notations

\[
R_s^r = R_s + \left( \frac{L_m}{\sigma L_s} \right)^2
\]

\[
T_s^r = \frac{\sigma L_s}{R_s^r}
\]

the subsystem with a high dynamic becomes

\[
\frac{di_s}{dt} = -\frac{1}{T_s^r} i_s + \frac{1}{\sigma L_s} u_s
\]

and the natural fast mode is given by the eigenvalue

\[
\lambda_1 = -\frac{1}{T_s^r}
\]

The term \(A_{22}\) becomes

\[
\lambda_2 = \frac{1}{T_s^r} R_s^r
\]

The reduced model described by (7) yields in the form

\[
\begin{align*}
\frac{d}{dt} \begin{bmatrix} \Psi_r \\ i_s \end{bmatrix} &= \begin{bmatrix} -\frac{1}{T_f} R_s^r \frac{L_m}{T_f} & \frac{L_m}{T_f} \\ \frac{1-\sigma}{\sigma L_m} \left( \frac{1}{T_r} + j\omega_s \right) & -\frac{1}{\sigma T_i} - \frac{1-\sigma}{T_r} \end{bmatrix} \begin{bmatrix} \Psi_r \\ i_s \end{bmatrix} + \begin{bmatrix} \frac{1}{\sigma L_s} R_s^r \end{bmatrix} u_s \\
\dot{i}_s &= \frac{L_m}{\sigma L_s T_r} \Psi_r + \frac{1}{R_s^r} u_s
\end{align*}
\]

and the dominant mode associated with the slow dynamic subsystem is determined by the next eigenvalue.

\[
\lambda_2 = -\frac{1}{T_s^r} R_s^r
\]

The transfer functions associated with the two models are:

\[
H_r(s) = \frac{i_s(s)}{u_s(s)} = \frac{1}{\sigma L_s} \quad \frac{b_{of}}{s + \frac{1}{T_s^r}} = \frac{b_{of}}{s + a_{of}} \quad (22)
\]

where the coefficients are:
\[ b_{of} = \frac{1}{\sigma L_s} \]  
(23)

\[ a_{of} = \frac{1}{T_s} = \frac{R_s^{\text{ref}}}{\sigma L_s} \]  
(24)

\[ b_{ls} = \frac{1}{R_s} \]  
(25)

\[ b_{os} = \frac{1}{R_s} \frac{T_r}{T_s} \]  
(26)

\[ a_{os} = \frac{1}{T_r} \frac{R_s}{R_s^{\text{ref}}} \]  
(27)

The estimation of the coefficients from (23)-(27) allows a partially calculation of the physical parameters of the squirrel cage induction motor. For a complete estimation is necessary to estimate the stator leakage inductance \( L_{\sigma_s} \) with a specific experiment. It can be seen a redundancy in the evaluation of the reflected resistance \( R_s^{\text{ref}} \). The physical parameters are determined with the equations:

\[ \sigma L_s = \frac{1}{b_{of}} \]  
(28)

\[ R_s^{\text{ref}} = \frac{a_{of}}{b_{of}} = \frac{1}{b_{ls}} \]  
(29)

\[ T_r = \frac{b_{ls}}{b_{os}} \]  
(30)

\[ R_s = \frac{a_{os}}{b_{os}} \]  
(31)

\[ R_s^{\text{ref}} - R_s = \left( \frac{L_m}{L_r} \right)^2 R_r \]  
(32)

\[ R_s^{\text{ref}} T_r = \left( \frac{L_m}{L_r} \right)^2 R_r \frac{L_m}{L_r} \Rightarrow \frac{L_m}{L_r} = \frac{R_s^{\text{ref}}}{R_r} T_r \]  
(33)

\[ \sigma = 1 - \frac{L_m}{L_s L_r} \Rightarrow (1 - \sigma) = \frac{L_m}{L_s L_r} \Leftrightarrow \]  
(34)

\[ L_s = \sigma L_s + R_s^{\text{ref}} T_r = \frac{1}{b_{of}} + \frac{R_s^{\text{ref}}}{b_{of}} T_r \]  
(35)

\[ \sigma = \frac{\sigma L_s}{L_s} = \frac{\frac{1}{b_{of}}}{1 + \frac{R_s^{\text{ref}}}{b_{of}} T_r} \]  
(36)

\[ L_m = L_s - L_{\sigma_s} \]  
(37)

\[ L_r = \frac{L_m}{(1 - \sigma) L_s} \]  
(38)

\[ R_r = \frac{L_m}{T_r} \]  
(39)

### 4 Model Structure of the Experimental Data

The squirrel cage induction motor was configured as in Fig.1.

This configuration is characterised by the following relations between the phase variables and the measured variables:

\[ i_B = i_c = \frac{-i_A}{2} = \frac{-i_{\text{max}}}{2} \]  
(40)

\[ u_A = \frac{2}{3} u_{\text{max}}; u_B = u_C = -\frac{1}{3} u_{\text{max}} \]  
(41)

Using the definition of the Park vectors,

\[ i_s = \frac{2}{3} \left( i_A e^{i\theta_A} + i_B e^{i\frac{2\pi}{3}} + i_C e^{i\frac{4\pi}{3}} \right) \]  
(42)

\[ u_s = \frac{2}{3} \left( u_A e^{i\theta_A} + u_B e^{i\frac{2\pi}{3}} + u_C e^{i\frac{4\pi}{3}} \right) \]  
(43)

the components in the d-q axis have the values:

\[ i_d = \frac{2}{3} \left( i_A + i_B + i_C \right) = \frac{2}{3} \left( i_{\text{max}} + i_{\text{max}} + i_{\text{max}} \right) = \frac{2}{3} i_{\text{max}} \]  
(44)

\[ i_{sd} = \frac{2}{3} \left( 0 i_A + \sqrt{3} \left( i_B - i_C \right) \right) = 0 \]  
(45)

\[ u_d = \frac{2}{3} \left( u_A - u_B + u_C \right) = \frac{2}{3} \left( u_{\text{max}} + u_{\text{max}} + u_{\text{max}} \right) = \frac{2}{3} i_{\text{max}} \]  
(46)

\[ u_{sd} = \frac{2}{3} \left( 0 u_A + \sqrt{3} \left( u_B - u_C \right) \right) = 0 \]  
(47)

The use of the least squares needs to ensure a persistent input signal. To excite all the modes of the system it was used a PseudoBinary Random Sequence (PBRS). In the experiments this signal was generated software, and it was amplified. The design of PBRS must be related to the dynamic characteristics of the identified process [6]. The use of mutitime scale imposes to the identification signal...
to satisfy a condition of “discriminate persistent excitation”. That means to excite the fast or the slow mode of the system without an excitation, if possible, of the others.

The analytical forms of the eigenvalues of the squirrel cage induction motor are already determined in (18) and (21). If we have not other a priori information, an approximate value of the parameters can be obtained through the passport data of the machine [7]. If we consider the stator resistance equal with the reflected resistance $R_{sref}$, the eigenvalues given in (18) and (21) can be determined. The data of the motor used in the experiment are:

Type MA 19F165 -6A  $n_n=930$ rot/min
$f_c=50$ Hz  $\Delta/\gamma$  220/380V
3.05/1.76A  $P=0.55$ kW  $\cos \varphi=0.687$
Iz. Class B  IP 54  $m=14$kg
$p=3$.

Taking into account the fast subsystem, the sampling frequency was chosen 1000Hz. For the reduced model the frequency of the PBRS was chosen 50Hz and the number of cells of the shift register was chosen $N=9$. For the fast subsystem or the “limit layer” system the frequency of PBRS is increased 10 times. Fig. 2 is presents the power spectrum of PBRS of the signal used in the determination of the reduced order system coefficients and in Fig. 3 is presented the power spectrum of BPRS of the signal used in the determination of the “limit layer” model coefficients.

### 5 Experimental Results

The same procedure applied finding the sample period, can be applied also to determine the positions of the eigenvalues in the frequency band for the two subsystems. With (18) and (21), and the typical values of the induction machine yields:

$$\lambda_1 = 1 - \frac{R_s + \left( \frac{L_m}{L_r} \right)^2}{R_r} \cong \frac{2\alpha}{\sigma T_s} \cong 600 \text{Hz}$$

$$\lambda_2 = 1 - \frac{R_s + \left( \frac{L_m}{L_r} \right)^2}{R_r} \cong \frac{1}{2T_r} \cong 15 \text{Hz}$$

The first experiment was dedicated to the reduced order model. It was used a Butterworth filter of $5^{th}$ order to filter the voltages (inputs) and currents (outputs) from the process. This was done because we want not to excite “limit layer” system. The power spectrum of the filtered signal injected in the system is presented in Fig. 4. The power spectrum of the acquired signal from the process is presented in Fig. 5.

The coefficients of the continuous model given in the second relation of (23) are evaluated using the Poisson moment functional technique. The pole of filters chain was set $\lambda=50$. The resistance $R_s$ was computed as the ratio of the continuous components of the two spectrums shown in Fig. 4 and 5. On the basis of (29)-(33), the parameters obtained are shown in Table 1.

Similarly, adopting $L_m=0.0348$H, the other parameters, computed on the basis of relations (36)-(39), are presented in Table 2.
6 Conclusions
The paper presents a new technique of physical parameters estimation for the squirrel cage induction motor based on the decomposition of the electromagnetic system in two subsystems with different dynamic. The estimation accuracy increases because are sequentially processed signals with a different spectrum. The reduced order model describes well the low frequency dynamic, and can be used in scalar control. The vectorial control can be better implemented with the parameters obtained with the described methodology.

References