AUTOMATED RECONSTRUCTION OF FRAGMENTED, 1600 B.C. WALL PAINTINGS

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Abstract: - In this paper a novel general methodology is introduced for the computer-aided reconstruction of the magnificent wall-paintings of the Greek island Thera, painted in the middle of the second millennium BC. These wall-paintings are excavated in fragments and, as a result, their reconstruction is a painstaking and a time-consuming process. Therefore, in order to facilitate and speed up this process a proper system has been developed based on the introduced methodology. According to this methodology each fragment is photographed, its picture is introduced to the computer, its contour is obtained and subsequently all fragments contours are compared in a manner proposed herein. Both the system and the methodology presented here, extract the maximum possible information from the contour shape of fragments of an arbitrary initially unbroken plane object, to point out possible fragments matching. This methodology has been applied to reconstruct, for the first time, unpublished wall-paintings parts from a set of 936 fragments.

Key - Words: Contour-shape image reconstruction, image processing, jigsaw puzzle automatic solving, computers in archaeology, reconstruction of fragmented wall-paintings

1 Introduction

The discovery of the wall-paintings at Akrotiri of the Greek island Thera, is of outstanding importance for human knowledge of the early Aegean world and not only. As with the treasures of Pompeii and Herculaneum, the wall-paintings of Thera were preserved due to the seal of the pumice from the great eruption of a volcano [1]. As a rule, the walls decorated with paintings no longer survive. They collapsed together with their painted coat before the volcanic eruption, due to particularly strong earthquakes. Thus, a single painting is usually scattered into many fragments mixed with the fragments of other wall-paintings, too. The restoration of the wall-paintings from the fragments is a very painstaking and time consuming process frequently demanding many months or even years of dedicated, experienced personnel work for a single wall-painting restoration. Therefore, the development of a system that will contribute to the automatic restoration of these wall-paintings is of fundamental importance for this archaeological research, but for many others too, which face the problem of an image reconstruction from excavated fragments.

Each excavated wall-painting fragment after being cleaned, is being photographed with a very strict protocol. Subsequently, the obtained image is processed and eventually each photographed fragment is embedded into a white background frame, which we call the absolute frame of reference of the specific fragment (see Fig. 1).

The problem tackled in this paper may be considered to be associated with automatic jigsaw puzzle solving. However, as we will point out below, the two
problems solutions manifest drastic and essential differences. In fact, a number of papers deal with automatic puzzle solving using methods and techniques such as: a set of critical isthmus points ([12], [3]), method for solving the "traveling salesman problem" ([4]), various features (side curvature, convexity/concavity, Euclidean distance between adjacent corners) ([5], [6], [7], [8], [9]), etc. The problem of automatic reconstruction of the prehistoric city of Thera wall-paintings is in essence drastically different than the one of automatic jigsaw puzzle solving. In fact, 1. No a priori knowledge concerning the shape of each piece (fragment) is given and therefore one cannot presuppose the existence of breakpoints on the boundary curve of it. 2. No frame pieces exist, i.e. pieces whose at least one side is a straight line segment. 3. The fragments size and shape varies dramatically in contrast to what happens in jigsaw puzzles. 4. Features such as convexity/concavity, Euclidean distance between adjacent corners etc., cannot contribute to our problem solution since we expect that gaps exist between adjacent fragments of each wall-painting, due to wear. 5. No a priori knowledge about the picture content exists. 6. No unique solution concerning the matching of two fragments exists. 7. Finally, very frequently, many different wall-paintings are excavated mixed altogether, due to the collapse of the two or three floor building whose walls were initially decorated by those paintings. Hence, the problem of the automatic jigsaw puzzle solution is a “subcase” of the problem tackled in this paper. Therefore, the method introduced here and the related system can be very well employed to reconstruct any broken into fragments object employing contour shape information only. Although the method and system presented here have proven very powerful in reconstructing wall paintings for the first time, we must stress that, if one wishes to develop a complete system of automatic reconstruction of an image from its constituent fragments, one may take into account many other parameters, too, such as a) matching between internal contours of the fragments b) colour continuation between actually adjacent fragments, c) continuation of the thematic content d) crack continuation e) geological texture of the fragment, etc.

2 Preliminary fragments processing

2.1 Obtaining the fragment image contour
Fragments are embedded very carefully into thin sand along with a colour palette and a scale so that proper processing can be applied later to compensate possible colour and size discrepancies due to different shooting conditions. The fragments digital images are stored in a database and processed for quality improvement. Subsequently, “fragment extraction” takes place, in the sense that specific developed image segmentation algorithms are applied to the obtained image in order to separate each fragment from the background. Thus, finally, one obtains each fragment embedded into a white background, at a random position. This fragment positioning together with axes is considered to be “the absolute reference system or frame” for each fragment, in all subsequent analysis. Next, each fragment contour is obtained. Each contour must have the following properties: A) each pixel must have exactly two neighbouring pixels, B) no isolated pixels or groups of pixels are allowed, C) three pixels must not form a compact right (90°) angle.

2.2 Obtaining rotated contours and dividing into blocks
Consider two actually adjacent fragments. Since their orientation in their absolute frame of reference is completely random, it follows that in order to make them match one must perform a random rotation to at least one of them. In order to account for this random rotation, the contours of each fragment are built corresponding to all fragment orientations obtained after repeated rotation of STEP degrees. Rotation around the origin by an angle \( \phi \) moves point \((x, y)\) into \((X, Y)\) via

\[
\begin{align*}
X &= \cos(\phi) \times y - \sin(\phi) \times x \\
Y &= \sin(\phi) \times y + \cos(\phi) \times x
\end{align*}
\]

(1)

Subsequently, the pixels of each contour are enumerated; the obtained contours are divided into their rectilinear blocks using a 3x3 pixels mask and finally the absolute angle of each block (the angle each block makes with the \((x, y)\) axes of the fragment absolute frame of reference) is defined.

3. The actual method of spotting matching fragments

3.1 The notions of “fixed” and “rotating” chains
One of the two fragments contours is arbitrarily considered as being “fixed” and by convention, the optimum matching figure to this contour is defined (see Fig. 2). In fact, to each pixel of the fixed fragment, its perfectly matching pixel corresponds as shown in Fig. 2. Suppose that two fragments are given and that one wants to decide if their contour shapes match and if yes, where they match. In order to achieve this, one first proceeds as follows:

One considers a length of comparison measured in pixels, say COMP_LEN=250 pixels. At first, one considers a group of COMP_LEN consecutive pixels...
starting from pixel #1 of the fixed fragment. This group of contour pixels is called “fixed chain”.

Subsequently, the other to-be-compared fragment (called the “rotating” one) is considered at a specific orientation. A part of the contour of this rotating fragment is constructed around the fixed one as follows:

- The last pixel #M of the rotating fragment is placed in the perfectly matching position of pixel #1 of the fixed fragment (position PMP1).
- A number of k-1 subsequent pixels of the rotating fragment are placed in the frame of reference of the fixed fragment, by parallel translation.
- The parallel translation of the rotating fragment, say B, in the absolute frame of reference of the fixed fragment, say A, ends when the last #k pixel of B contour, satisfies one of the following:
  a) If one considers the direction along the average absolute angle of the last L pixels of the fixed chain and computes the line at right angles to this direction, which will be called “barrier” line, one stops the building of the “rotating chain” when it intersects the barrier line (see Fig. 2). A consistent choice is to set L equal to COMP_LEN.
  b) If k is greater than a number of pixels called EXC_LEN and condition a) has not occurred; in this case one considers that, de facto, the two contours do not match for this position and orientation of fragments A and B. A good choice seems to be: EXC_LEN = 2*COMP_LEN. We would like to stress that, although this demand may at a first glance seem as a matching criterion, however this is not the case. This demand is in fact set just to speed up the whole process. We have simply made this choice just because extended experiments confirm that, when k becomes greater than 1.2*COMP_LEN then the considered fragments have always violated the matching criteria described below.

3.2 The first area matching criterion

Following a rather typical mathematical criterion, we consider that a measure of shape matching between the fixed chain of length COMP_LEN and the rotating chain of varying length, is the number of pixels enclosed by those two chains and the chain of pixels bridging the last pixels of the fixed and rotating chains (see Fig. 2,3). Notice, that at this stage one counts both the pixels that belong to the gap between the two fragments and those belonging to the two fragments. One does not count, however, neither the pixels of the fixed chain, nor the pixels of the rotating chain that are found to be at a Perfect Matching Position. Therefore, we consider that the contour of two fragments, say A and B, match at pixels #P_A and #P_B respectively, if the number of enclosed pixels between the corresponding chains defined above, is less than a chosen number, say MAX_AREA. The proper choice of MAX_AREA depends on the expected degree of decay the fragments have suffered, as well as the chosen COMP_LEN and the used resolution for the scanning of the fragments photos. In the extreme case, where no decay is expected, one may choose a very small value of MAX_AREA.

Notice, that even for the correct orientation of the rotating fragment, as the value of MAX_AREA grows, there is a non-zero probability that accidental erroneous matching between two chains occurs. An estimation of the relation between MAX_AREA and COMP_LEN that has proven to be very efficient in practice results after statistical experiments. Summarizing, one may state that for a given set of fragments and a given COMP_LEN it is possible to choose MAX_AREA values that essentially minimize random erroneous matching occurrences. In order to further, drastically reduce the number of the erroneous matching occurrences between the fixed and rotating fragment, two other criteria have been used, based on contour information, that are subsequently defined.

3.3 The second area matching criterion

Suppose that the first matching criterion is satisfied for fragments A and B at pixels #P_A and #P_B.
respectively, for a specific rotating chain. If this were an actual matching position, no overlapping between the two adjacent fragments occurs, in the sense that all pixels enclosed by the fixed and the rotating chain should lay in the gap between the two fragments. Hence, the second matching criterion could be the demand that this condition actually occurs. However, in practice, small deviations from the fragments perfect depiction occur, due to imperfections in the shooting and image processing procedures. Then, as a second matching criterion, one demands that the number of pixels enclosed by the fixed and the rotating chain and lay in the gap between the two fragments, is a considerable percentage, say \( GP \) of the total number of enclosed pixels (see Fig. 3).

3.4 The third area matching criterion

Suppose that the first and second matching criteria are satisfied for two fragments A and B at pixels \( \#P_A \) and \( \#P_B \) respectively, and for a certain relative orientation of A and B. At these specific positions and orientation one continues building the fixed and rotating chains until the entire contour of fragments A and B is formed (see Fig. 2,3). As a third criterion one demands that the number of overlapping pixels between the interior of the two fragments contour is smaller than a lower bound, say \( LB \), for the same reasons explained in 3.3 above.

3.5 The “sum of angles difference” matching criterion

We have proven that, if one defines the “sum of angles” \( SA \) for any curve as

\[
SA = \sum_{\text{chain pixels}} \text{block angle}
\]  

then, if the maximum area enclosed by a fixed and rotating chain is \( E \), there is a specific maximum, say \( \mu_{\text{max}} \), of the difference of the “sum of angles” of these two chains. To be precise, if \( d \) is the Euclidean distance between the beginning and end of the fixed chain and if, in order to write a more concise formula, we let \( L = \text{EXC}_\text{LEN} \), then,

\[
\mu_{\text{max}} = \sqrt{d^2 + \frac{\left(2 + E \right)^2}{d^2}} \arctan \left( \frac{2 + E}{d} \right) \left( L - \sqrt{d^2 + \frac{\left(2 + E \right)^2}{d^2}} \right) \pi \frac{2}{2}
\]  

(3)

Therefore, in order to decide if the previously mentioned area criteria will be applied for a couple of chains starting at pixels \( \#P_A \) and \( \#P_B \), the “sum of angles difference” matching criterion is applied first, stating that, if the sum of angles of the fixed and rotating chain differ more than \( \mu_{\text{max}} \), then, \( \text{de facto} \), the two contours do not match for this position and orientation of fragments A and B. We would like to emphasize that the demand that the difference of the sum of angles of the fixed and rotating chain is small enough is not a sufficient condition for matching.

In practice we have reduced the total time of comparison of two fragments by a factor of about twenty (20) or equivalently to five (5) percent of the overall time, by applying this fourth criterion.

4. Application of the methodology

4.1 Implementation of the methodology

A system has been developed implementing the aforementioned matching criteria as follows:

The total number of contour pixels is computed for all fragments (pieces). The application of the method starts with the fragment of greater number of contour pixels called “reference” fragment, tested for matching with all other fragments contour of the set, sequentially. In other words, the reference fragment is considered to be the fixed one, while all other are sequentially considered to be rotating fragments. For each couple of fixed and rotating fragments the following procedure, consisting of four steps at most, is applied.

Step 1: For a specific rotating fragment, the procedure described in 3.1 is initially applied for the orientation of the contour of this fragment in its absolute frame of reference (see Fig. 2).

Step 2: For the orientation of the contour of the rotating fragment in its absolute frame of reference, and for every couple of chains starting at \( \ell, j \) respectively, the aforementioned “sum of angles difference” criterion is applied (see 3.5). The “sum of angles” of both the fixed and rotating chain is computed, say \( SA_f \) and \( SA_r \), and if \( \text{abs} (SA_f - SA_r) > \mu_{\text{max}} \), then we consider, \( \text{de facto} \), that the two contours do not match; if not, we proceed to Step 3 described below.

Step 3: The number of enclosed pixels \( E_{\ell,j} \) between the fixed and rotating chains as well as the number \( G_{\ell,j} \) of pixels lying in the gap between these two fragments are computed. If \( E_{\ell,j} \) is smaller than a specific threshold \( MAX\_AREA \) and \( G_{\ell,j} \) satisfies the second matching criterion, then the system considers that the two fragments in hand may match at the couple of pixels \( (\ell, j) \).

Then one proceeds to checking if the third matching criterion holds. If this criterion is satisfied, too, the system decides conclusively that the two fragments in hand may match at these two pixels. Otherwise, if one of the aforementioned area criteria is not satisfied, the system decides that no matching is possible between the reference fragment and the specific orientation of the rotating fragment at \( (\ell, j) \).
**Step 4:** Finally, all three steps above are repeated for all possible orientations of the rotating fragment obtained via successive rotations of this fragment with rotation step \( \text{STEP} = 1^\circ \).

Each fragment the system suggests that it might match with the reference fragment, and the reference fragment itself, are concatenated at the exact positions \((\ell, j)\), by means of a proper C code. The obtained greater fragment consisting of the concatenated matching ones is used as input to an image-processing tool. In this way the user is able to visualise the system proposition and is able to decide whether this is correct or not. Next, the actually matching fragments are concatenated to form a new single "artificial" fragment and the whole aforementioned procedure is repeated. The fixed reference fragment is considered now to be the big "artificial" piece constructed above. This procedure is repeated until no further matching occurs. In this way, an “island” of matching fragments is formed. Should fragments of the set in hand remain, the whole aforementioned procedure is repeated with reference fragment the one of greater contour length not belonging to the previously constructed island(s), until all fragments are exhausted. Notice, that a histogram of the contour lengths of all initial fragments is formed, and if a considerable variance in the contour lengths is observed, then the whole aforementioned procedure is repeated and for other comparison lengths \(\text{COMP}_\text{LEN}\), too.

### 4.2 Experimental results

In order to test the introduced methodology as well as the developed system we have applied it to a set of fragments. This set comprises nine hundred thirty six (936) fragments belonging to several wall-paintings that have not been reconstructed yet. It must be pointed out that, although the specialised personnel has made a serious effort toward this direction, no considerable matching between fragments of this set has been found due to the large number of pieces, their size, the thematic content of the wall-paintings that made many fragments look alike and the fact that a serious number of fragments was missing.

The authors under strict photographic conditions have for the first time, photographed the fragments. The area of the fragments varied from approximately 1 cm\(^2\) to approximately 3000 cm\(^2\), the more frequently encountered one being about 100 cm\(^2\).

After the initial processing to improve the quality of the obtained images and isolate the fragments as described in 2.1, the main reconstruction procedure has been applied.

Concerning this set of fragments we have used the following parameter values:

- **a)** Two different values of \(\text{COMP}_\text{LEN}\): First 300, subsequently 250,
- **b)** \(\text{EXC}_\text{LEN} = 2 \times \text{COMP}_\text{LEN}\)
- **c)** \(\text{MAX}_\text{AREA} = 1400\) pixels and 1200 respectively

In connection with the above parameter values choice we must point out the following:

We have chosen relatively large values of \(\text{MAX}_\text{AREA}\) to account for expected considerable decay and damage of fragments. We have chosen relatively large values of \(\text{COMP}_\text{LEN}\) for two reasons: First, because the average perimeter length of the fragments was high enough and second due to the large number of fragments in this set.

Starting with the greater contour fragment, we repeated the aforementioned procedures forming islands of matching concatenated fragments. We show two of these islands, not previously reconstructed by scholars, in Figures 4 and 5.

![Fig. 4](image)

We emphasize that we have tried to be very careful in choosing the value of \(\text{MAX}_\text{AREA}\), to reduce the number of accidental erroneous matching between two fragments (see 3.2). In this way, in about 60% of the fragments reported to match, no erroneous matching took place due to accidental contour shape resemblance. In the rest 40% of fragments one to five (1-5) accidental matchings per fragment have been reported. Reducing the \(\text{MAX}_\text{AREA}\) would practically almost exponentially reduce the number of occurrences of accidental contour shape matching, but
in this case we would not be able to account for the relatively big gaps existing between adjacent fragments caused by serious decay and/or by the violence of the breaking procedure. The rejection of the erroneous accidental matching has been made by inspection of the fragments. To obtain an as much as possible automated reconstruction of images from its constituent fragments, one must take into consideration other parameters, too, such as: 1) Colour continuation between actually adjacent fragments, 2) Depicted objects contour continuation, 3) Thematic content continuation, 4) Width of the fragment vertically to its depiction surface, 5) Geological texture of the side opposite to the painted one, etc. The aforementioned are the object of extended study by our team and the related results will be published shortly.

5 Conclusion

In this paper a new methodology is introduced for the computer-aided reconstruction of the Thera wall-paintings, painted in the middle of the second millennium BC. These wall-paintings are excavated in fragments and in order to facilitate and speed up their reconstruction process, a system has been developed based on the proposed methodology. Both the system and the methodology presented here, extract the maximum possible information from fragments contour shape to point out possible fragments matching. The methodology and the system have been used to reconstruct, for the first time, unpublished wall-paintings parts from a set of 936 fragments.

References: