CLUSTERING-BASED BLIND EQUALIZATION FOR TIME VARYING CHANNELS

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Abstract: In this paper a Blind Clustering Based Sequence Equalizer is proposed for the equalization of time varying channels. The clustering based estimation of the channel consists of two steps: a) clusters positions estimation by means of an unsupervised learning technique and b) clusters labeling. In order to track the time varying channel, the received signal sequence is divided in subsequences of length $K$ and the equalizer estimates in every subsequence the clusters positions. Clusters labeling is performed, via a Hidden Markov Model, at the first subsequence. Then cluster labels are updated only if a change in labels is detected. The good tracking characteristics of the equalizer are verified by simulation results. The performance achieved by the proposed blind equalizer is close to the performance of a supervised CBSE, which repeats the training every $K$ data. When the subsequence length $K$ is set to a small value, a significant gain in BER is obtained over a static CBSE.

Key-words: Blind equalization, Clustering techniques, Time varying channels.

1. INTRODUCTION

Signals transmitted through modern communication channels are affected by multipath propagation which results in ISI and fading. To counteract these impairments the receiver should be implemented with a robust equalizer providing tracking of the channel time variations.

In most of the cases, the communication channel is unknown, and the design of the equalizer is performed on the basis of a known training sequence. In many applications, however, it is desirable that the training of the equalizer be performed without the use of a known sequence. Such is the case of time varying channels where frequent repetitions of the training procedure are required. In these systems may be utilized a blind equalizer which does not need the use of a training sequence.

Blind channel equalization is a challenging task and has been the focus of intense research effort. Recently, an interest has risen on approaches based on data clustering techniques [2], [3], [5]. A cluster-based blind channel estimation algorithm consists of two steps: a) data clusters are first estimated via an unsupervised learning technique and b) labeling of the estimated clusters is achieved, by unraveling the information hidden in the sequence of received data [2], [3]. When the channel estimation task is completed, a Cluster Based Sequence Equalizer (CBSE) [4] can be employed to provide signal detection. In [5] a Blind CBSE is proposed which uses a Hidden Markov Model (HMM) for the labeling of the clusters. This approach offers advantages over the other Cluster based
equalizers [2], [3] since it gives extra degrees of freedom to configure the clusters labeling [6].

In this paper the implementation of the Blind CBSE using HMM in time varying channels is proposed. The received sequence is segmented in several subsequences and the proposed equalizer reestimates the clusters positions for each of them. Thus, the time variation of the channel is tracked. Cluster labels are determined at the first subsequence and are updated only when a labeling change occurs. Thus, for the most part of the received data, the estimation of the time varying channel is performed by means of a simple unsupervised learning procedure. The good tracking abilities of the equalizer are verified by simulation results. The MSE and BER performance of the equalizer are investigated for different sizes of the subsequences length.

2. SYSTEM DESCRIPTION

The nonstationary channel environment is modeled by the following discrete time equation

\[ g(t) = \sum_{i=0}^{L} h_i(t) I(t - i) + w(t) = c(t) + w(t), \]

where \( I(t) \) is an equiprobable sequence of transmitted data taken from an \( M \)-ary alphabet, \( w(t) \) is an AWGN sequence and \( h_i(t) \) is the time varying channel impulse response which includes the effects of the transmitter filter, the transmission medium and the receiver filter. In the above relation, \( c(t) \) is the noiseless channel output sequence.

3. CLUSTERING BASED SEQUENCE EQUALIZATION

The equalizer, placed in the receiver part, aims to recover the transmitted sequence of information bits \( I(t) \), based on the corrupted received sequence \( g(t) \).

A CBSE has been proposed in [7] which treats equalization as a classification task. This method focuses on the clusters, which the received data form. The received data samples are clustered around specific points whose number and constellation shape is determined by the spread of the channel and the impairments characteristics.

Consider the \( D \times 1 \) vector of successively received samples: 

\[ \mathbf{g}(t) = [g(t), g(t - 1), ..., g(t - D + 1)]^T. \]

According to eq. (1), in the absence of noise, \( \mathbf{g}(t) \) is associated with \( Q = M^{L+D} \) points in the \( D \)-dimensional space. Each point corresponds to one of the \( M^{L+D} \) possible realizations of the sequence of transmitted bits: 

\( (I(t), ..., I(t - L - D + 1)) \). If the received data are corrupted by AWGN, then the randomness of noise leads to the formation of a cluster around each point. Each cluster is represented by a suitably chosen representative, which corresponds to the noiseless channel response vector in the \( D \)-dimensional space, i.e., 

\( \mathbf{c}(t) = [c(t), ..., c(t - D + 1)]^T, \) with \( c(t) \in \{c(i), i = 1, ..., Q\}, \)

each \( c(i) \) corresponding to one of the possible values of the sequence: 

\( (I(t), ..., I(t - L - D + 1)) \).

Due to the interdependence that ISI imposes on successive received data, only specific transitions among the different clusters are possible. Thus, CBSE employs a Viterbi type procedure dictated by the specific transitions among clusters. Assuming that data are treated in groups of \( D \), then, a \( M^{L+D-1} \) state trellis can be considered, where the state \( S(t) \), at time \( t \), uniquely identifies the channel memory: 

\[ S(t) = (I(t - 1)I(t - 2)...I(t - L - D + 1)). \]

In the Viterbi trellis diagram, the transition from one state, \( S(t - 1) \), to another, \( S(t) \), corresponds to the emission of a specific cluster representative, indicated by the sequence of bits formed by the current state and the new information bit transmitted. We call this sequence of bits label and we denoted it by: 

\[ X(t) = (I(t)I(t - 1)...I(t - L - D + 1)). \]
For the implementation of the Viterbi algorithm an appropriate distance metric is adopted in order to measure the distance between the received data vector and the representatives of the various clusters.

During the training period of the supervised CBSE, each cluster representative is computed by a simple averaging of all the data vectors, \( g(t) \), belonging to the respective cluster [7].

3.1. Supervised CBSE for time varying signals

In the time varying channel case the number of clusters, \( Q \), is fixed for all time instants, however, the positions of the clusters continually change with the time. Consequently, the clusters representatives are time varying signals, i.e., \( c(t) \in \{c_i(t), i = 1, ..., Q\} \). In Figure 1a is illustrated the time variation of the (1-dimensional) clusters formed by a simple time varying channel with transfer function: \( H(z) = 1.0 + (0.2 + 0.0015t)z^{-1} \) [1]. In contrary, the stationary channel with \( H(z) = 1 + 0.2z^{-1} \) forms static clusters which are depicted in Figure 1b.

When the channel is time varying the training procedure should be repeated regularly in order to be able to cope with the changing nature of the channel. The clustering based sequence equalization is accomplished by feeding the Viterbi Algorithm with the time varying cluster estimates.

4. BLIND CLUSTERING BASED ESTIMATION OF TIME VARYING CHANNELS

In the blind CBSE, the unsupervised estimation of the clusters is performed in two steps a) clusters representatives identification and b) labeling of them. As already noted, in time varying channels, clusters positions may vary with time. Moreover, in a time varying environment the correspondence between labels and cluster-
is the fact that the data that belong to a specific cluster are very close to the data belonging to a neighbor cluster. In this case labels change is assumed and the clusters labeling procedure should be repeated.

4.2. HMM and clusters labeling

As already mentioned the clusters labels should be determined a) initially and b) whenever a clusters crossing is observed. For this purpose, a discrete observations HMM is constructed utilizing the already identified values of clusters representatives. Hence, the discrete observations HMM formulated is characterized by the following elements [6]:

1) The states of the model: \( S(t) = (I(t - 1) ... I(t - L)) \). For an M-ary alphabet, the number of the states is \( N = M^L \), that is \( S(t) \in \{1, ..., N\} \).

2) The state transition probabilities, \( a_{ij} \), where: \( a_{ij} = P[S(t + 1) = j | S(t) = i] \), \( 1 \leq i, j \leq N \). In the blind equalization case, \( a_{ij} \) are known and are equal to \( 1/M \), for an allowable transition, or equal to zero, for a not allowable transition. For every allowable transition \( (a_{ij} = 1/M) \) a specific noiseless channel output occurs. In other words, each state transition specifies a cluster label. The cluster labels are specified by: \( X(t) = (I(t) ... I(t - L)) \) with, \( X(t) \in \{1, ..., Q\} \), \( Q = M^{L+1} \).

3) The distinct observation symbols per transition \( V = \{v_k\} \), \( k = 1, ..., Q \). These are assumed to be equal to the clusters representatives.

4) The observation symbol probability distribution in states transition \( i \) to \( j \). This parameter, in our case, corresponds to the probability of a specific cluster representative (symbol) to correspond to specific label (states transition). For simplicity, we use indices of labels and not of states transitions, since there is a unique correspondence between labels and states transitions, as stated earlier. Thus, this element is defined as:

\[ b_h(v_k) = P[v_k \text{ observed} | S(t) = i, S(t + 1) = j] = P[v_k \text{ observed} | X(t) = n], 1 \leq n, k \leq Q, 1 \leq i, j \leq N, \]

where \( n \) corresponds to the label that uniquely specifies a state transition: from \( i \) to \( j \).

5) The initial state distribution:

\( \pi_i = P[S(1) = i] \) for \( 1 \leq i \leq N \).

The discrete observations of the HMM are derived by the sequence of received data. However, due to the presence of noise, the received data, \( g(t) \), in the channel output do not take discrete values (see eq.(1)). This sequence of continuous values can not be fed into the discrete observations HMM, described above. Thus, the received data, \( g(t) \), are quantized to the closer cluster representative and the resulting sequence of discrete data is denoted by the symbol \( y(t) \). This is the discrete observations sequence feeding the HMM.

In the blind channel estimation algorithm, clusters labeling is treated as an HMM learning problem; the unknown probability of a specific cluster to correspond to a specific label is modeled as an unknown parameter of the HMM. Then, the EM (Expectation - Maximization) algorithm is implemented to obtain the Maximum Likelihood (ML) estimates of the formed HMM, [8]. The resulting ML estimate is given by: \( \theta = \arg\max_{\theta} P(Y|\theta) \) where, \( P(Y|\theta) \) is the probability of the observation sequence of length \( K \), \( Y = (y(1), ..., y(K)) \), given the model parameters, \( \theta \). In our case, we define: \( \theta = [b_n(c_k)], n, k = 1, ..., Q \). That is, \( \theta \) is the probability matrix that maps labels to clusters and it is expected to converge to a matrix whose elements are equal to: a) 1, if a specific symbol \( (c_k) \) corresponds to a specific label \( (n) \) and b) zero otherwise.

5. SIMULATION RESULTS

The nonstationary channel environment is simulated by a two ray fading model with: \( H(z) = \)
\(h_t(1) + h_t(2)z^{-1}\), where \(h_t(i), i = 1, 2\) are the time varying channel coefficients. Each channel coefficient, \(h_t(i)\), is generated by low-pass filtering pseudo random white Gaussian noise. The time variation of the coefficients appears in Figure 2. Data are assumed bipolar and the noise variance is set equal to 0.035².

The received sequence is divided in smaller subsequences of length \(K\) each. In each subsequence the unsupervised clustering procedure is implemented by the Isodata algorithm. It should be noted that since the transmitted data are bipolar only half (e.g., positive) of the data should be estimated. During the clustering procedure the case of clusters crossing is also examined. If clusters crossing is detected then the labeling procedure is repeated, otherwise the labels are assumed to be the same to the labels specified in the previous subsequence. It is important to note that in this case the procedure of the time varying channel estimation and tracking is reduced to a simple unsupervised clustering procedure, leading to significant computational savings.

Simulation results show that the proposed blind equalizer exhibits good tracking performance. The tracking ability of the algorithm is improved as \(K\) becomes smaller. The representatives trajectories obtained using the proposed algorithm for different values of \(K\) are plotted in Figure 3. The corresponding MSE for \(K = 40, 100, 1000\) is plotted in Fig. 4. From these figures is seen that tracking is better for \(K = 40\). When \(K\) becomes large, for example \(K = 1000\), the equalizer does not take into consideration the time varying nature of the channel. Finally the BER performance for different values of \(K\) is shown in Figure 5 for two equalizers: a) the proposed blind CBSE and b) a supervised CBSE which repeats the training procedure every \(K\) data. Clearly the performance of the blind equalizer is close to the performance of its supervised counterpart. Moreover, when \(K\) is large then the BER of both equalizers saturates to a worse case point. This is equivalent to a static equalizer where there is no consideration of the channel variation. Concluding, the performance of proposed blind equalizer is enhanced when the length of the subsequences is reduced.

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6. REFERENCES


Figure 1: a. Clusters of time varying channel with $H(z) = 1.0 + (0.2 + 0.0015t)z^{-1}$, b. Clusters of stationary channel with $H(z) = 1 + 0.2z^{-1}$.

Figure 2: The time variation of the channel coefficients.

Figure 3: Cluster representatives tracking, '-' K=40, ':' K=100, '-' K=1000.

Figure 4: Mean Square Error, '-' K=40, ':' K=100, '-' K=1000.

Figure 5: BER versus K, '-' blind CBSE, '-' supervised CBSE.