A new amplifier placement scheme to reduce noise in WDM networks

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Abstract: In this paper, we introduce a new placement scheme for Erbium Doped Fiber Amplifiers in Wavelength Division Multiplexing (WDM) Local/Metropolitan Area Networks (LAN/MAN). This method can be used when the number of amplifiers and the total gain to be supplied per link are known. The aim is to place amplifiers so that the noise at the receivers is minimized. A comparison with previous placement schemes is performed to show that our method obtains a higher noise reduction.

Key-Words: - Wavelength Division Multiplexing (WDM), Optical Metro Networks, Erbium-Doped Fiber Amplifiers (EDFA), Placement Schemes, Gain Splitting, Amplified Spontaneous Emission (ASE) Noise.

1 Introduction

The continuous increase on the demand of telecommunication services makes Wavelength Division Multiplexing (WDM) networks to be one of the preferred techniques to upgrade present architectures and to support high-bandwidth services. The development of optical amplifiers has made WDM a feasible technology by means of increasing transmission distances in optical links. There are several types of optical amplifiers, but the most widespread-used ones are the Erbium-Doped Fiber Amplifiers (EDFA) [1,2].

Despite their advantages, such as high gain, high bandwidth and low noise figure, EDFAs have some disadvantages. They are very expensive devices and they need maintenance [1-5]. These reasons make network designers to develop techniques to minimize the number of amplifiers needed. Several algorithms have been proposed to find an optimal solution to this problem [3-7]. These algorithms can be classified into two groups: the link-by-link algorithms [3] and the global optimization algorithms [4-7]. The first group obtains the number of amplifiers in the network by analyzing link by link. The second group analyzes the whole network. Better results are obtained with these global algorithms because more information is used [4-7].

Once the number of amplifiers required is known, the problem is to find their exact location in the network. This is very important, because it can reduce noise at reception. If signal levels are maintained at the receivers, and the noise power is reduced, the result is an increase in the Signal-to-Noise Ratio (SNR) at those receivers. In this paper, we propose a method to place optical amplifiers reducing noise power at reception.

The rest of the paper is organized as follows. In section 2, some considerations about the EDFA model employed are made. Section 3 shows previous amplifier placement schemes. In section 4, we describe our placement scheme. Finally, in section 5 results are given to prove that our method reduces ASE noise at the receivers when compared to previous schemes.

2 EDFA model

We use the amplifier model described in [4-8]. It is defined by the following expression:

$$\frac{P_{in}}{P_{sat}} = \frac{1}{(G-1)}\ln\left(\frac{G_0}{G}\right) \tag{1}$$

where P_{in} is the total input signal power to the amplifier, P_{sat} is the internal saturation power, G_0 the small-signal gain and G the gain, (in absolute scale, not dB). Two more constraints are applied to the model. If $G_{\max}$ is the maximum small-signal gain and $P_{\max}$ is the maximum output power an amplifier can supply, the model must verify:

$$G_0 \leq G_{\max} \tag{2}$$

$$GP_{in} \leq P_{\max} \tag{3}$$

The amplifier model with these constraints is shown in Fig. 1. It is assumed that the amplifier has a flat gain over the bandwidth of interest.

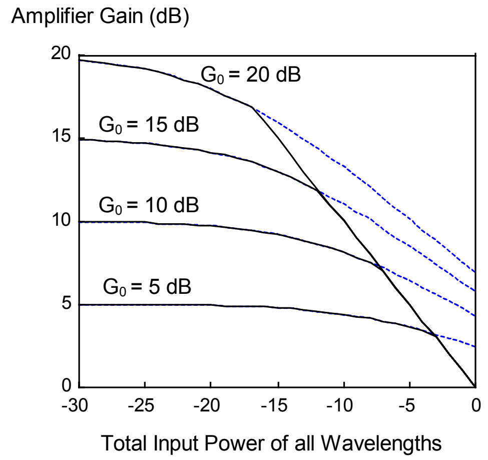

Fig.1: *Amplifier model. Dashed lines show the model of equation (1). The constraint (3) restricts the model to the solid lines.*

The dominant noise contribution in EDFAs is the Amplified Spontaneous Emission (ASE) noise [1,2]. The output noise power generated by an EDFA of gain G can be calculated as follows [9]:

$$P_{ASE} = 2n_{sp}hf_c(G-1)B_O \tag{4}$$

where the parameters, their meanings and the values used in this paper are shown in Table 1. The gain is in natural units.

When propagating through the link, noise is attenuated. Let us suppose that we have a link of length L and we put an amplifier l_0 km downstream. The noise power at the end of the link will be [8]:

$$P_{ASE} = 2n_{sp}hf_c(G-1)B_O\exp(-\alpha[L-l_0]) \tag{5}$$

Symbol	Parameter	Value used
G_{max}	Maximum small-signal Gain of the EDFA	100 (20 dB)
P_{sat}	Internal saturation power of the EDFA	1.298 mW
P_{max}	Maximum output power of an amplifier	1 mW (0 dBm)
α_{dB}	Fiber attenuation (dB/km)	0.2 dB/km
α	Fiber attenuation (natural units)	0.04605
n_{sp}	Spontaneous emission factor	1.4
h	Planck's constant	$6.625\cdot10^{-34}$ J·s
f_c	Central optical carrier frequency	193.41 THz
B_O	Optical channel bandwidth	50 GHz

Table 1: *Parameters and values used in this study.*

When we have a cascade of N EDFAs, the noise at the end of the link is [8]:

$$P_{ASE} = 2n_{sp}hf_cB_OG_N\exp(-\alpha l_N)\sum_{i=1}^{N-1}T_i + 2n_{sp}hf_cB_O(G_N-1)\exp(-\alpha l_N) \tag{6}$$

$$T_i = \frac{(G_i-1)}{G_i}\prod_{j=1}^{N-1}G_j\exp(-\alpha l_j) \tag{7}$$

where G_i is the gain of the i-th amplifier of the cascade, and l_j is the distance between the j-th amplifier and the end of the link. These equations will be used to obtain the ASE power later.

3 Previous work.

Several placement schemes have been proposed by different authors. Our start point is the work by Ramamurthy *et al.* [4-6], which focuses on WDM Local/Metropolitan Area Networks (LAN/MAN). They propose two mathematical formulations to minimize the number of amplifiers required in a network. When attempting to minimize this number, it is necessary to set upper and lower limits to the optical power propagating through the link. The upper limit is set by the maximum output power of an amplifier (P_{max}) or by the maximum power such that the nonlinear effects in the fiber are despicable. We assume the tighter bound is P_{max}. The lower bound is due to the sensitivity of network devices, and in this study it is set to –30 dBm per channel. The outputs of these formulations are the number of amplifiers per link and the aggregate gain they must supply. Then, their exact location is determined by using one of the algorithms also proposed in [4-6]: As Late As Possible (ALAP) or As Soon As Possible (ASAP).

ALAP places the amplifiers as follows: Each link is traversed downstream, and each of the N-1 first amplifiers is placed when the power level of the signal have reached its minimum acceptable value. These N-1 first amplifiers operate at their maximum possible gain. The last one gives the remaining gain needed in the link, and it is placed in the same way as previous amplifiers. The ASAP scheme is similar to ALAP, but instead of being the last amplifier the one with lowest gain, it is the first one. Differences between the two methods can be seen in Figs. 7 and 8 of [6].

A small variation of the ALAP method is the Last Amplifier as Soon As Possible (LASAP) scheme [8]. In this method, the N-1 first amplifiers are placed in the same way as in the ALAP scheme, but the last one is placed as soon as it is possible for it to give the remaining gain.

To illustrate how these methods place the amplifiers, let us consider a 150 km link with two amplifiers, where the total gain needed is 35 dB. There are 10 channels propagating through the link (for simplicity, we will assume that they have the same power levels) and the power at the beginning of the link is set to –20 dBm per channel. The results of the different placements are shown in Figs. 2 to 4.

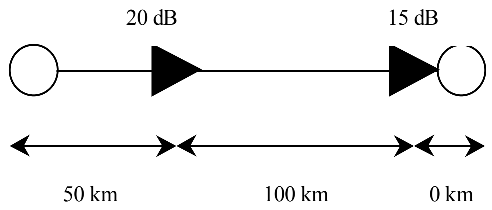

Fig.2: *Amplifier placement using the ALAP method.*

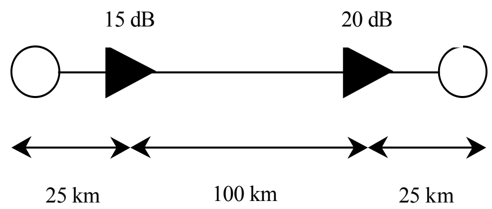

Fig.3: *Amplifier placement using the ASAP method.*

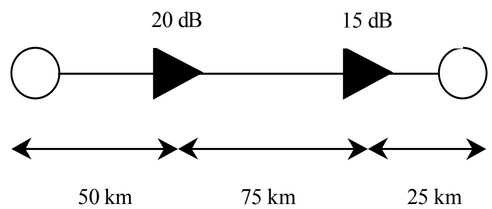

Fig.4: *Amplifier placement using the LASAP method.*

We are looking for a way to place amplifiers that obtains the lowest ASE power at the end of the links. To obtain this noise power, equation (6) must be applied. We can also calculate the noise reduction (*NR*) obtained with the different methods when they are compared to ALAP using the following equation:

$$NR\ (\%) \quad = \quad \left(1 - \frac{P_{ASE}^{scheme}}{P_{ASE}^{ALAP}}\right) \cdot 100 \qquad (8)$$

For the example shown, LASAP and ASAP methods obtain the best noise performance at the end of the link, 33.8% better than ALAP scheme.

These three methods assume that the number of optical amplifiers (*N*) and the total gain (SG) needed are known for any link of the network. When these restrictions are imposed, other placement schemes, such as the ones shown in [10,11], cannot be applied.

4 A heuristic scheme

In this section, we propose a heuristic method that reduces noise when compared with previous schemes. To prove this, numerical results are provided in section 5.

Previous studies show that if an amplifier providing a gain level is replaced with more amplifiers providing jointly the same gain as the original one, the ASE power is reduced [10]. Besides, [11] demonstrates that if the total gain needed by the link is equally distributed among all amplifiers, the Signal-to-Noise Ratio at the end of the link can be increased. We are going to adapt these considerations to our case, where we have a fixed number of amplifiers and the total gain required by the network links is known.

Let us equally distribute the total gain required by the link among the amplifiers, and place them as soon as the amplifier can provide the maximum output power ($P_{\max}$). For the example link of section 3, the placement result is shown in Fig. 5.

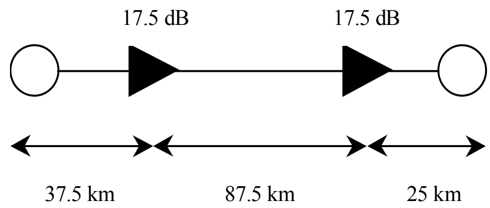

Fig.5: *Amplifier placement using the new method.*

Using equation (8), the noise reduction at the end of the link when compared to ALAP scheme is 43.6%.

In some cases where the first amplifier is located at the beginning of the link, it may not provide simultaneously $P_{\max}$ and the required gain. To show this, let us consider a 100 km link with two amplifiers, where the total gain needed is 42.74 dB. There are 20 equally-powered channels propagating through the link and the input power of the link is -29.3449 dBm per channel (aggregate power of -16.334 dBm). The results of the placement for ALAP, ASAP, LASAP and the equally distributed scheme are shown in Figs. 6 to 9. With the equally distributed method, the total output power of the first amplifier is -1.089 dBm, which is lower than $P_{\max}$.

In this example, the best noise performance is obtained when LASAP method is used, 26.4% better that ALAP applying equation (8). The equally distributed method obtains a noise reduction of 23.4% when compared with ALAP.

In order to improve the behavior of the method proposed, let us increase the gain of the first amplifier so that P_{max} can be obtained at its output without changing its location. The other amplifier provides the remaining gain. The new placement is shown in Fig. 10. Comparing ASE power at the end of the link between ALAP and this new placement, we can see that the result is 30.8% better than ALAP result (and therefore also better than LASAP).

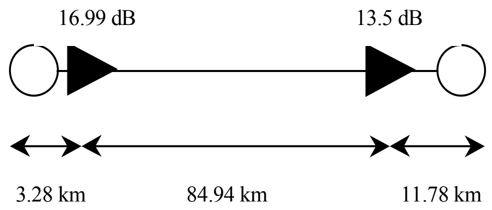

Fig.6: *Amplifier placement using the ALAP method for the new link.*

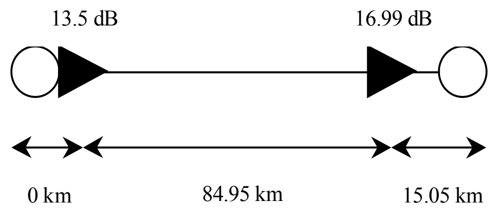

Fig.7: *Amplifier placement using the ASAP method for the new link.*

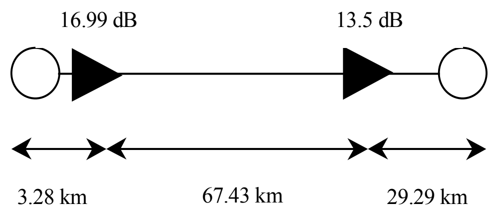

Fig.8: *Amplifier placement using the LASAP method for the new link.*

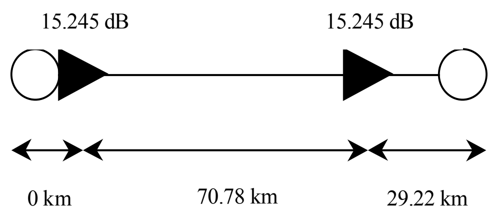

Fig.9: *Amplifier placement using the equally-distributed method for the new link.*

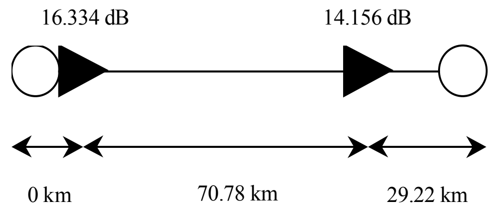

Fig.10: *Corrections to the placement.*

Hence, we propose a heuristic method, called DASAP (Distributed As Soon As Possible) which is described by the following rules:

- If a link has only one amplifier, it is placed as soon as it can provide an output power equal to P_{max}, while supplying the gain required by the link.
- If a link has more than one amplifier, the total gain required by the link is equally distributed among all amplifiers, and they are placed as soon as an output power equal to P_{max} can be provided.
- If an amplifier has to be placed at the beginning of the link, its gain has to be modified so that it provides an output power equal to P_{max}. The rest of the gain is equally distributed among the remaining amplifiers, and they are placed as soon as they can provide P_{max}.

5 Numerical results.

We have applied DASAP method to the sample networks shown in Figs. 11 and 12, to compare with the results obtained with ALAP, LASAP and ASAP methods. These networks are Passive Optical Networks (PON) with non-reflective Passive Star Couplers (PSC) [4,5,7]. The ASE power has been obtained using equation (6) with Table 1 parameters, and the results are shown in Tables 2 and 3. The tables also show the noise reduction achieved with the different placement schemes when compared with ALAP method.

In the worst case, our method has proved to obtain the same results as the other schemes. In links with only one amplifier, DASAP and LASAP obtain the same placement, but the noise results can be different because of the ASE power entering the link from previous links of the network.

More important than the fact of reducing ASE power is the increase experimented in the SNR, given that the signal levels at the end of the links maintain approximately constant (± 0.04%) for all schemes

6 Conclusion

In this paper we have proposed a heuristic method to reduce ASE power at the end of network links. It has been called DASAP (Distributed As Soon As Possible). Numerical results have shown that this method obtains better noise reductions that previous proposed placement schemes, and therefore improves SNR at reception.

References:

[1] G. P. Agrawal, *Fiber-Optic Communication Systems*, John Wiley and Sons, Inc, New York, 1992.

[2] S. Shimada, H. Ishido (ed), *Optical Amplifiers and their Applications*. John Wiley & Sons, Inc, West Sussex, England, 1994.

[3] C.-S. Li, F. F. Tong, C. J. Georgiou, M. Chen, "Gain Equalization in Metropolitan and Wide Area Optical Networks using Optical Amplifiers", *Proceedings, IEEE INFOCOM '94*, Toronto, Canada, pp. 130 – 137, June 1994.

[4] B. Ramamurthy, *"Efficient design of wavelength division multiplexing (WDM)-based optical networks"*, Ph.D. dissertation, Dep. Comput. Sci., Univ. California, Davis, CA, July 1998.

[5] B. Ramamurthy, J. Iness, B. Mukherjee, "Optimizing Amplifier Placements in a Multiwavelength Optical LAN/MAN: The Equally Powered-Wavelengths Case", *Journal of Lightwave Technology, vol. 16, nº 9*, pp. 1560 – 1569, September 1998.

[6] B. Ramamurthy, J. Iness, B. Mukherjee, "Optimizing Amplifier Placements in a Multiwavelength Optical LAN/MAN: The Unequally Powered-Wavelengths Case", *IEEE/ACM Transactions on Networking, vol. 6, nº 6*, pp. 755 – 767, December 1998.

[7] J.Iness, *"Efficient Use of Optical Components in WDM-Based Optical Networks"*, Ph.D Dissertation, Universitiy of California, Davis, Department of Computer Science, 1997.

[8] I. de Miguel, J. C. Aguado, P. Fernández, R. M. Lorenzo, E. J. Abril, M. López, "A Simple Method to Place Amplifiers in a WDM LAN/MAN", *ELECO'99 International Conference on Electrical and Electronics Engineering*, pp. 413 – 417, 1999.

[9] R. Ramaswami, K. N. Sivarajan, *Optical Networks: A Practical Perspective.* Morgan Kaufmann Publishers, Inc. 1998

[10] P. E. Green, Jr., *"Fiber Optic Networks"*, Prentice Hall Inc, Englewood Cliffs, New Jersey, 1993.

[11] H.-D. Lin, *"Gain Splitting and Placement of Distributed Amplifiers"*, Technical report RC 16216 (#72010), IBM, October 1990.

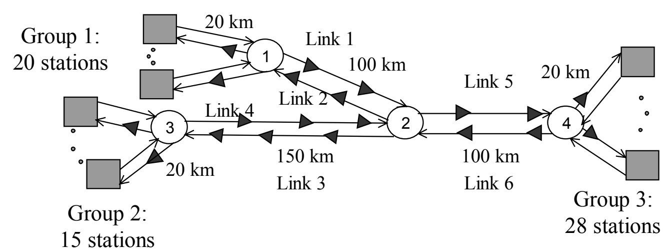

Fig.11: *Sample network 1 used in this study and in [4,5,8].*

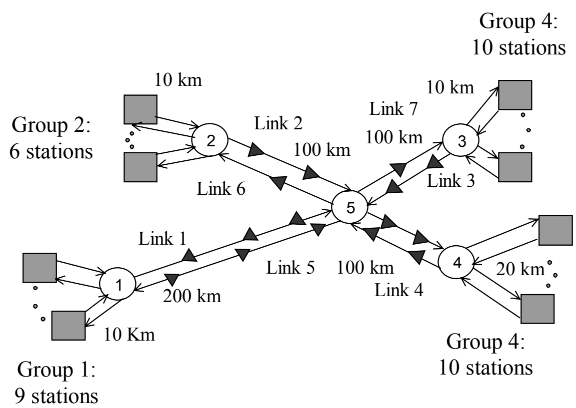

Fig.12: *Sample network 2 used in this study and in [6,7].*

Link	Gain (dB)			Distance (km)				***P**$_{ASE}$* ***(W)*** *(Reduction (%) when compared with ALAP)*			
	ALAP & LASAP	*ASAP*	*DASAP*	*ALAP*	*LASAP*	*ASAP*	*DASAP*	*ALAP*	*LASAP*	*ASAP*	***DASAP***
1	$G_1 = 16.99$ $G_2 = 13.50$	$G_1 = 13.50$ $G_2 = 16.99$	$G_1 = 16.33$ $G_2 = 14.15$	$l_0 = 3.28$ $l_1 = 84.94$	$l_0 = 3.28$ $l_1 = 67.43$	$l_0 = 0$ $l_1 = 84.95$	$l_0 = 0$ $l_1 = 70.78$	$9.038\cdot10^{-6}$	$6.657\cdot10^{-6}$ *(26.34 %)*	$1.255\cdot10^{-5}$ *(-38.85 %)*	**$6.249\cdot10^{-6}$** ***(30.85 %)***
2	$G_1 = 13.67$ $G_2 = 11.87$	$G_1 = 11.87$ $G_2 = 13.67$	$G_1 = 12.77$ $G_2 = 12.77$	$l_0 = 40.67$ $l_1 = 59.33$		$l_0 = 31.69$ $l_1 = 68.35$	$l_0 = 36.19$ $l_1 = 63.81$	$4.400\cdot10^{-5}$			**$3.771\cdot10^{-5}$** ***(14.29 %)***
3	$G_1 = 13.19$ $G_2 = 13.19$ $G_3 = 8.68$	$G_1 = 8.68$ $G_2 = 13.19$ $G_3 = 13.19$	$G_1 = 11.68$ $G_2 = 11.68$ $G_3 = 11.68$	$l_0 = 40.67$ $l_1 = 65.94$ $l_2 = 43.39$		$l_0 = 18.13$ $l_1 = 65.95$ $l_2 = 65.95$	$l_0 = 33.14$ $l_1 = 58.43$ $l_2 = 58.43$	$5.637\cdot10^{-5}$	$5.256\cdot10^{-5}$ *(6.76 %)*	$6.198\cdot10^{-5}$ *(-26.34 %)*	**$4.755\cdot10^{-5}$** ***(15.64 %)***
4	$G_1 = 17.47$ $G_2 = 17.47$ $G_3 = 4.77$	$G_1 = 4.77$ $G_2 = 17.47$ $G_3 = 17.47$	$G_1 = 16.04$ $G_2 = 16.04$ $G_3 = 16.04$	$l_0 = 7.13$ $l_1 = 87.36$ $l_2 = 55.51$		$l_0 = 0$ $l_1 = 27.13$ $l_2 = 87.35$	$l_0 = 0$ $l_1 = 59.16$ $l_2 = 59.16$	7.405×10^{-6}		$7.428\cdot10^{-6}$ *(-0.31 %)*	**$4.238\cdot10^{-6}$** ***(42.76 %)***
5	$G_1 = 14.56$ $G_2 = 11.87$	$G_1 = 11.87$ $G_2 = 14.56$	$G_1 = 13.21$ $G_2 = 13.21$	$l_0 = 40.67$ $l_1 = 59.33$		$l_0 = 27.22$ $l_1 = 72.80$	$l_0 = 33.93$ $l_1 = 66.07$	$6.249\cdot10^{-5}$	$5.726\cdot10^{-5}$ *(8.36 %)*	$7.088\cdot10^{-5}$ *(-13.42 %)*	**$4.802\cdot10^{-5}$** ***(23.15 %)***
6	$G_1 = 15.53$ $G_2 = 15.53$			$l_0 = 0.44$ $l_1 = 77.64$				1.37×10^{-6}			**1.37×10^{-6}** ***(0 %)***
Gr. 1	$G_1 = 3.35$			$l_0 = 3.28$	$l_0 = 0$			$2.309\cdot10^{-6}$	$2.251\cdot10^{-6}$ *(2.51 %)*		**$1.980\cdot10^{-6}$** ***(14.24 %)***
Gr. 2	$G_1 = 2.57$			$l_0 = 7.13$	$l_0 = 0$			$3.025\cdot10^{-5}$	$2.797\cdot10^{-6}$ *(7.53 %)*	$3.249\cdot10^{-6}$ *(-7.40 %)*	**$2.557\cdot10^{-6}$** ***(15.57 %)***
Gr. 3	$G_1 = 3.91$			$l_0 = 0.44$	$l_0 = 0$			$2.558\cdot10^{-6}$	$2.368\cdot10^{-6}$ *(7.42 %)*	$2.844\cdot10^{-6}$ *(-11.18 %)*	**$2.045\cdot10^{-6}$** ***(20.05 %)***

Table 2: *Results of the different methods for Fig.11 network. P_{ASE} has been calculated at the end of each link. l_i is the distance between amplifier i-1 and amplifier i. l_0 is the distance between the beginning of the link and the first amplifier.*

Link	Gain (dB)			Distance (km)				***P**$_{ASE}$* ***(W)*** *(Reduction (%) when compared with ALAP)*			
	ALAP & LASAP	*ASAP*	*DASAP*	*ALAP*	*LASAP*	*ASAP*	*DASAP*	*ALAP*	*LASAP*	*ASAP*	***DASAP***
1	$G_1 = 18.21$ $G_2 = 18.21$ $G_3 = 16.33$	$G_1 = 16.33$ $G_2 = 18.21$ $G_3 = 18.21$	$G_1 = 17.58$ $G_2 = 17.58$ $G_3 = 17.58$	$l_0 = 36.17$ $l_1 = 91.08$ $l_2 = 72.75$	$l_0 = 36.17$ $l_1 = 91.08$ $l_2 = 70.45$	$l_0 = 15.53$ $l_1 = 91.05$ $l_2 = 91.05$	$l_0 = 21.82$ $l_1 = 87.94$ $l_2 = 87.94$	$3.858\cdot10^{-5}$	$3.792\cdot10^{-5}$ *(1.68 %)*	$2.510\cdot10^{-5}$ *(34.94 %)*	**$2.455\cdot10^{-5}$** ***(36.36 %)***
2	$G_1 = 18.67$ $G_2 = 17.20$	$G_1 = 17.20$ $G_2 = 18.67$	$G_1 = 18.09$ $G_2 = 17.78$	$l_0 = 20.63$ $l_1 = 79.37$	$l_0 = 20.63$ $l_1 = 68.27$	$l_0 = 0$ $l_1 = 88.88$	$l_0 = 0$ $l_1 = 88.91$	$1.615\cdot10^{-5}$	$1.394\cdot10^{-5}$ *(13.68 %)*	$8.764\cdot10^{-6}$ *(45.73 %)*	**$7.908\cdot10^{-6}$** ***(51.03 %)***
3	$G_1 = 18.84$ $G_2 = 16.31$	$G_1 = 16.31$ $G_2 = 18.84$	$G_1 = 18.17$ $G_2 = 16.99$	$l_0 = 24.20$ $l_1 = 75.80$	$l_0 = 24.20$ $l_1 = 60.74$	$l_0 = 0$ $l_1 = 84.89$	$l_0 = 0$ $l_1 = 84.94$	$1.259\cdot10^{-5}$	$1.072\cdot10^{-5}$ *(14.85 %)*	$6.248\cdot10^{-6}$ *(50.37 %)*	**$5.095\cdot10^{-6}$** ***(59.53 %)***
4	$G_1 = 18.07$ $G_2 = 17.09$	$G_1 = 17.09$ $G_2 = 18.07$	$G_1 = 17.58$ $G_2 = 17.58$	$l_0 = 20$ $l_1 = 80$		$l_0 = 5.45$ $l_1 = 90.35$	$l_0 = 7.9$ $l_1 = 87.9$	$2.859\cdot10^{-5}$		$1.674\cdot10^{-5}$ *(41.44 %)*	**$1.664\cdot10^{-5}$** ***(41.80 %)***
5	$G_1 = 16.92$ $G_2 = 16.92$ $G_3 = 7.70$	$G_1 = 7.70$ $G_2 = 16.92$ $G_3 = 16.92$	$G_1 = 13.85$ $G_2 = 13.85$ $G_3 = 13.85$	$l_0 = 76.14$ $l_1 = 84.61$ $l_2 = 39.25$		$l_0 = 30.75$ $l_1 = 84.60$ $l_2 = 84.60$	$l_0 = 61.50$ $l_1 = 69.25$ $l_2 = 69.25$	$6.429\cdot10^{-5}$	$6.234\cdot10^{-5}$ *(3.03 %)*	$5.316\cdot10^{-5}$ *(17.31 %)*	**$4.308\cdot10^{-5}$** ***(32.97 %)***
6	$G_1 = 16.67$			$l_0 = 76.14$				$1.890\cdot10^{-5}$	$1.851\cdot10^{-5}$ *(2.03 %)*	$1.397\cdot10^{-5}$ *(26.08 %)*	**$1.370\cdot10^{-5}$** ***(27.51 %)***
7	$G_1 = 16.60$			$l_0 = 76.14$				$1.938\cdot10^{-5}$	$1.894\cdot10^{-5}$ *(2.23 %)*	$1.439\cdot10^{-5}$ *(25.74 %)*	**$1.416\cdot10^{-5}$** ***(26.93 %)***
8	$G_1 = 17.00$ $G_2 = 1.76$	$G_1 = 1.76$ $G_2 = 17.00$	$G_1 = 9.38$ $G_2 = 9.38$	$l_0 = 76.14$ $l_1 = 23.84$	$l_0 = 76.14$ $l_1 = 8.81$	$l_0 = 0$ $l_1 = 67.34$	$l_0 = 38.12$ $l_1 = 46.93$	$2.592\cdot10^{-5}$	$2.464\cdot10^{-5}$ *(4.85 %)*	$1.634\cdot10^{-5}$ *(36.95 %)*	**$1.220\cdot10^{-5}$** ***(52.92 %)***
Gr. 1	--			--				$4.507\cdot10^{-6}$	$4.370\cdot10^{-6}$ *(3.03 %)*	$3.726\cdot10^{-6}$ *(17.31 %)*	**$3.020\cdot10^{-6}$** ***(32.97 %)***
Gr. 2	--			--				$1.987\cdot10^{-6}$	$1.946\cdot10^{-6}$ *(2.03 %)*	$1.468\cdot10^{-6}$ *(26.08 %)*	**$1.440\cdot10^{-6}$** ***(27.51 %)***
Gr. 3	--			--				$2.445\cdot10^{-6}$	$2.390\cdot10^{-6}$ *(2.23 %)*	$1.815\cdot10^{-6}$ *(25.74 %)*	**$1.768\cdot10^{-6}$** ***(26.93 %)***
Gr. 4	--			--				$1.031\cdot10^{-6}$	$9.809\cdot10^{-7}$ *(4.85 %)*	$6.500\cdot10^{-7}$ *(36.95 %)*	**$4.856\cdot10^{-7}$** ***(52.92 %)***

Table 3: *Results of the different methods for Fig.12 network. P_{ASE} has been calculated at the end of each link. l_i is the distance between amplifier i-1 and amplifier i. l_0 is the distance between the beginning of the link and the first amplifier.*