A FDI Filter Based-on Inversion for Nonlinear Systems

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Abstract

The purpose of this paper is to show a new scheme to Fault Detection and Isolation filter design, based on nonlinear state observers. The first step of the design scheme is the computation of stable nonlinear state observers (detector) with the only additional requirement, that the system with the failure modes as inputs and output errors as outputs have to be irreversible. Then, the failure modes will be reconstructed (isolation) by inversion and, finally, the (possibly unstable) inverted error system is stabilized by post-filters. The method is illustrated on a diagnostic model derived from the torque controlled pendulum.

Key-Words: Fault Detection and Isolation, Nonlinear System Observers, System Inversion, State Elimination, Invertibility Condition.

1 Introduction

The Fault Detection and Isolation (FDI) filter design both the linear and nonlinear case is based on geometric condition for the failure signatures decoupling, which is known as output separability. The output separability in the linear case, can be characterized in terms of the minimal $C - A$ invariant subspaces generated by the failure signatures, see [6]. The same condition is a real geometric obstruction to fault isolation based on Luenberger’s observers. In [10], a new FDI filter design was proposed for linear diagnostic models based on Observers, Inversion, and Post-filters (OBIP). In the same paper an example is shown, where the classic methods does not work, however our OBIP method isolates successfully the two faults.

Now, the same scheme is analyzed in order to generalized to the widest class of nonlinear diagnostic systems as possible.

Each step is considered independently. First, the existence of the nonlinear observers with prescribed error dynamics and output is studied. This task can be treated in both the differential algebraic and differential geometric framework, see [4, 8, 11]. The other key step is the invertibility of control systems, which also has a rich literature, initiated by L.M. Silverman, [9], then followed by M. Flies, [3], R.M. Hirschorn, [5], and so on. Finally the stabilization of the inverted system by post-filters is studied.

The tasks of the followed steps of our design scheme intensively use the state elimination, see [2]. The differential algebraic setting and the using of the Diop’s state elimination algorithm give the general feature to our scheme with the very natural limitations coming from the computation complexity of the used algorithms, and the theoretic considerations of the detectability of systems. Instead of the output separability, roughly speaking, only the linear independence of the fault signatures of the observable quotient dynamic system is required.

2 Nonlinear FDI Scheme

The FDI design scheme proposed in [10] will be analyzed and extended to nonlinear diagnostic systems.

As the FDI fundamental problem may be divided into two stages: the first stage is the generation of residues. The second stage makes use of these residues for an adequate decision making with respect to the possible nature of the detected fault, then we also propose three steps for FDI filter design in nonlinear systems.

First step: Observer is designed with described error dynamics and error outputs. The dashed line traces the error input-output system, which depends on the control $u$ and the output $y$ of the plant:

The plant input $u$ and the measured plant output is supposed known variables. The error system input is the failure mode vector $v = (v_1, \ldots, v_J)^T$, and the output is the output error $\eta = (\eta_1, \eta_2, \ldots, \eta_J)^T$. It is sup-
posed that $J \geq I$.

Second step: The inversion of the error input-output system reconstructs the unknown input from the known output $\eta$. Unfortunately the simultaneous asymptotic (asymptotic and BIBO, etc.) stability of the error system and its inverse can not be guaranteed by appropriate choosing of the state observer. Hence, we need another step to correct the unstability of the inverse system $E(u, y)^{-1}$.

Third step: Asymptotically stable SISO post-filters design for all components $v_1, \ldots, v_I$, in order to cancel the unstabilities of the inverse system. The dashed block is the decoupling unit of the FDI filter.

Let us be summing up graphically the proposed scheme for FDI filter design in Figure 6.

3 Nonlinear observer design

The literature of the theory of nonlinear observers is very large. Principally, all algorithms are based on differential algebraic properties of the system dynamics and output. [8] is an excellent outline and improvement of previous results. Recently, in [11] a differential algebraic setting is presented to design nonlinear observers:

Let us consider the algebraic control system

$$\begin{align*}
\dot{x} &= f(x, u), \\
y &= g(x).
\end{align*}$$

(1)

The proposed design scheme is the following: The construction of the nonlinear observer starts from the equations

$$\begin{align*}
\dot{\hat{x}} &= f(x, u) + D(x - \hat{x}), \\
\hat{y} &= g(x) + d(x - \hat{x}).
\end{align*}$$

(2)

for the observer states $\hat{x}$, where $e \mapsto D(e), d(e)$ are algebraic functions, such that $D(0) = 0, d(0) = 0$, $\dot{e} = -D(e)$, is asymptotically stable algebraic differential equation. System (2) is considered as an observer, coupled with the plant states $x$.

Now, let us eliminate $x$ from the coupled system

$$\begin{align*}
\dot{x} &= f(x, u), \\
\dot{\hat{x}} &= f(x, u) + D(x - \hat{x}) \\
y &= g(x), \\
\hat{y} &= g(x) + d(x - \hat{x}).
\end{align*}$$

Hence, an implicit observer equation will be obtained

$$F(\hat{x}, \hat{x}, u, \hat{u}, \ldots, y, \hat{y}, \ldots, \hat{y}, \ldots) = 0,$$

with a non equality

$$G(\hat{x}, \hat{x}, u, \hat{u}, \ldots, y, \hat{y}, \ldots, \hat{y}, \ldots) \neq 0.$$

This approach works when the system (1) is algebraically observable. Thus, the plant state $x$ can be eliminated from (2) by the Diop’s algorithm, [2].

For example, let’s be the system following

$$\begin{align*}
\dot{x}_1 &= x_2^2, \\
\dot{x}_2 &= u, \\
y &= x_1.
\end{align*}$$

(3)

By output derivation, $\dot{y} = x_2^2$ and $\ddot{y} = 2ux_2$, hence the observer can be expressed by

$$\begin{align*}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix}
&= \begin{bmatrix}
x_2^2 \\
u
\end{bmatrix} + D \begin{bmatrix}
x_1 - \hat{x}_1 \\
x_2 - \hat{x}_2
\end{bmatrix} \\
\begin{bmatrix}
\hat{y} \\
u
\end{bmatrix}
&= \begin{bmatrix}
y - \hat{y} \\
u
\end{bmatrix} + D \begin{bmatrix}
y - \hat{y} \\
x_2 - \hat{x}_2
\end{bmatrix}
\end{align*}$$

The error equation

$$\begin{align*}
\begin{bmatrix}
\dot{e}_1 \\
\dot{e}_2
\end{bmatrix}
&= -D \begin{bmatrix}
e_1 \\
e_2
\end{bmatrix}
\end{align*}$$
can be chosen asymptotically stable by a stable matrix $-D$.

In the FDI case, let us consider a nonlinear diagnostic system

$$\dot{x} = f(x, u, \nu),$$
$$y = g(x, \nu),$$

where $x \in \mathbb{R}^n$ are the states, $u \in \mathbb{R}^m$ are the control, $\nu = (\nu_1, \nu_2, \ldots, \nu_l)$ are the failure modes, and $y \in \mathbb{R}^p$ are the measured outputs. In the no failure case $\nu = 0$.

Now, we can design a state observer to the no-failure case using, for example, the method presented in [11]. For just a moment, let us consider $(u, \nu)$ as a pair of inputs. Then, the observer design scheme of [11] can be applied to our problem, in the following manner.

The design starts from the (uncomputable) observer dynamics

$$\dot{x} = f(x, u, \nu) + D(x - \hat{x})$$
$$y = g(x, \nu) + d(x - \hat{x})$$

with desired error dynamics, such that

$$\dot{e} = D(e, u, \hat{u}, \ldots, y, \hat{y}, 0, 0, \ldots)$$

is asymptotically stable at $e = 0$. The corresponding output

$$\eta = d(e, u, \hat{u}, \ldots, y, \hat{y}, 0, 0, \ldots)$$

is also 0 at $e = 0$. This means, that

$$D(e, u, \hat{u}, \ldots, y, \hat{y}, 0, 0, \ldots) = 0$$
$$d(e, u, \hat{u}, \ldots, y, \hat{y}, 0, 0, \ldots) = 0$$

hold for all $(u, \hat{u}, \ldots, y, \hat{y})$ from their domain.

Applying the Diop's state elimination for the plant

$$\dot{x} = f(x, u, \nu),$$
$$y = g(x, \nu),$$

and coupled system (3), (4), input-output differential equations are obtained

$$F\left(\dot{x}, x, u, \hat{u}, \ldots, y, \hat{y}, 0, 0, \ldots\right) = 0,$$
$$G\left(\dot{x}, x, u, \hat{u}, \ldots, y, \hat{y}, 0, 0, \ldots\right) = 0,$$

which is computable for $\nu = 0$.

The implicit observer dynamics and outputs are the following

$$F\left(\dot{x}, \dot{x}, u, \hat{u}, \ldots, y, \hat{y}, 0, 0, \ldots\right) = 0,$$
$$G\left(\dot{x}, u, \hat{u}, \ldots, y, \hat{y}, 0, 0, \ldots\right) = 0.$$

The corresponding error equation with state error $e = x - \hat{x}$, (not available), and output error $\eta = y - \hat{y}$ (available), is

$$\dot{e} = D(e, u, \hat{u}, \ldots, y, \hat{y}, 0, \nu, 0, \ldots)$$
$$\eta = d(e, u, \hat{u}, \ldots, y, \hat{y}, 0, \nu, 0, \ldots).$$

For given, $u, y$, the dynamics and the output are time varying with $\nu$ as input and $\eta$ as output.

4 The inversion of the error dynamics

L.M. Silverman has considered the inversion of linear systems, in [9]. For non linear systems the invertibility is treated, for example, in [5], by R.M. Hirschorn, [3] by M. Fliess.

In differential algebraic setting, left invertibility (as our case) can be expressed in terms of the differential output rank of the system, see [3]. That is, if the differential output rank of the error equation is equal to $I$ (the number of possible failures), the error equation is invertible. This implies the condition $J \geq I$, that is, the number of outputs must be greater, or equal to the number of failures.

In [10], for linear systems, the invertibility of the error equation is guaranteed by the following conditions

a) The inequality $J \geq I$.

b) The orthogonal projection of the failure signatures into the observable subspace are linearly independent.

Let us consider a more particular diagnostic system, which is affine in the failure modes:

$$\dot{x} = f_0(x, u) + \sum_{i=1}^{I} f_i(x, u)u_i,$$
$$y = g_0(x, u) + \sum_{i=1}^{I} g_i(x, u)u_i.$$

If diagnostic system (5), (6) is factorized with respect to an asymptotically stable zero-dynamics, see [1], and the factorized failure signatures $f_i$ are linearly independent, $J \geq I$, then an invertible asymptotically stable error system can be achieved by a nonlinear observer.
5 Post-filter design

In [10], stabilizing post-filter was designed by pole-zero cancellation. However, pole-zero cancellation can not be applied for time varying or non linear systems. In order to generalize the concept of “pole-zero” cancellation, a linear system, in observable canonical form, will be considered:

\[
\begin{align*}
\dot{x}_1 &= a_1 x_n + b_1 u, \\
\dot{x}_2 &= x_1 + a_2 x_n + b_2 u, \\
&\vdots \\
\dot{x}_n &= x_{n-1} + a_n x_n + b_n u, \\
y &= x_n.
\end{align*}
\]  
(7)

Let us study the effect of a first order stable post-filter of form

\[
\begin{align*}
\dot{x}_{n+1} &= -x_{n+1} + y, \\
z &= cx_{n+1} + y,
\end{align*}
\]

where \(c\) is a design parameter, for choose. Instead of the transfer function description of the systems, state elimination is used in order to eliminate the instabilities from the coupled system (7) with

\[
\dot{x}_{n+1} = -x_{n+1} + x_n,
\]  
(9)

equipped with the output equation

\[z = cx_{n+1} + x_n.\]

It is natural to eliminate \(x_n\) : \(x_n = z - cx_{n+1}\). Then, the obtained system (not in Kalman’s state space form) is

\[
\begin{align*}
\dot{x}_1 &= -a_1 cx_{n+1} + a_1 z + b_1 u, \\
\dot{x}_2 &= x_1 - a_2 cx_{n+1} + a_2 z + b_2 u, \\
&\vdots \\
\dot{x}_{n-1} &= x_{n-2} - a_{n-1} cx_{n+1} + a_{n-1} z + b_{n-1} u, \\
\dot{x}_{n+1} &= -(1 + c) x_{n+1} + z,
\end{align*}
\]

with the new output equation,

\[z = (c + a_n) z - x_{n-1} - c(1 + c + a_n) x_{n+1} + b_n u.\]

If \(\lambda\) is the unique unstable pole of the original system, then, choosing \(c = -(1 + \lambda)\), the post-filter cancels the unstable pole \(\lambda\) and replaces with \(-1\).

If the original system has several unstable poles, then the same procedure can be applied with a cascade of first order post-filters in order to achieve pole-zero cancellation, by state elimination.

For nonlinear systems the same post-filters are proposed with nonlinear outputs

\[
\begin{align*}
\dot{x}_{n+1} &= -x_{n+1} + y, \\
z &= c(x, u) x_{n+1} + y,
\end{align*}
\]

(or several similar post-filters in cascade).

Applying the same state elimination procedure, \(c(x, u)\), (or \(c_1(x, u), c_2(x, u), \ldots, c_p(x, u)\)), as functions of \(x, u\), can be chosen such that the obtained system is asymptotically stable.

6 Example

Let us consider the controlled pendulum with observation

\[
\begin{align*}
\frac{d(m\ddot{x})}{dx} &= -c\sin x + u, \\
y_1 &= x, \\
y_2 &= \dot{x}.
\end{align*}
\]

The nominal values of the mass and “length” are, respectively, \(m_0\) and \(c_0\).

The purpose is the detection and isolation of smooth changes of \(c\) and \(m\). In order to constitute a diagnostic model, new variables will be introduced:

\[
\begin{align*}
x_1 &= c\sin x, \\
x_2 &= mx.
\end{align*}
\]

Then, if \(\nu_1 = \frac{c - c_0}{c_0}\sin x\) and \(\nu_2 = \left(\frac{m_0 - m}{m_0}\right)\dot{x}\), the new output equations are

\[
\begin{align*}
\sin y_1 &= \frac{x_1}{c_0} + \nu_1, \\
y_2 &= \frac{x_2}{m_0} + \nu_2.
\end{align*}
\]

The terms \(\nu_1, \nu_2\) are interpreted as failure modes, which are 0 if and only if \(c_0 = c(t), m_0 = m(t)\).

A third variable will also be introduced by

\[x_3 = \cos x.\]

Then, the new dynamics is

\[
\begin{align*}
\dot{x}_1 &= c_0 \left(\frac{x_2}{m_0} + \nu_2\right) x_3 + c_0 \dot{\nu}_1, \\
\dot{x}_2 &= -x_1 + u, \\
\dot{x}_3 &= -\left(\frac{x_2}{m_0} + \nu_2\right) \left(\frac{x_1}{c_0} + \nu_1\right).
\end{align*}
\]

The obtained diagnostic model is also non linear in the failure modes, moreover, failures also appear in the output.

The above mentioned observer design can also be interpreted as linearization by output injection, which is a particular case of the state elimination, proposed in
our paper. Let us define the observer by

\[ \dot{x}_1 = c_0 y_2 \cos y_1 + d_1 \left( \sin y_1 - \frac{x_1}{c_0} \right), \]
\[ \dot{x}_2 = -x_1 + u + d_2 \left( \frac{y_2}{m_0} - \frac{x_2}{m_0} \right), \]
\[ \dot{x}_3 = -y_2 \sin y_1 + d_3 \left( \cos y_1 - x_3 \right), \]

with observer outputs

\[ \sin y_1 = \frac{x_1}{c_0}, \]
\[ \dot{y}_2 = \frac{x_2}{m_0}. \]

Then, the error equations are

\[ \dot{e}_1 = -\frac{d_1}{c_0} e_1 - d_1 v_1 + c_0 \nu_1, \]
\[ \dot{e}_2 = -e_1 - \frac{d_2}{m_0} e_2 - d_2 v_2, \]
\[ \dot{e}_3 = -d_3 v_3, \]

with error outputs

\[ \eta_1 = \sin y_1 - \sin \tilde{y}_1 = \frac{e_1}{c_0} + v_1, \]
\[ \eta_2 = \tilde{y}_2 - \tilde{y}_2 = \frac{e_2}{m_0} + v_2. \]

The physical parameters \( c_0, \ m_0, \) are positive therefore if the observer gains \( d_1, \ d_2, \ d_3 \) are positive, then the error system is asymptotically stable.

The inverted error equations

\[ \nu_1 = \frac{s + \frac{d_1}{c_0}}{2s} \eta_1, \]
\[ \nu_2 = \frac{c_0 s + d_1}{2m_0 s^2} \eta_1 + \frac{m_0 s + d_2}{m_0 s} \eta_2. \]

are unstable, hence, post-filters are required in order to cancel the simple and the double poles in 0.

The first post-filter is

\[ \dot{x} = -x + \nu_1, \]
\[ x_1 = -x + \nu_1, \]

and the second one is

\[ \dot{x}_1 = -x_1 + \nu_2, \]
\[ \dot{x}_2 = x_1 - x_2, \]
\[ z_2 = -2x_1 + x_2 + \nu_2. \]

The simulation of the FDI filters with \( c_0 = 5, \ m_0 = 2, \ u = 0 \) are shown in the Figure 7, 8.

**7 Conclusions**

A new scheme to FDI filter design, based on nonlinear state observers has been presented. The method is structured in three steps: the one, in order to the fault detection, is the design of stable nonlinear state observers, via state elimination. The system with the failure modes as inputs and output errors as outputs have to be invertible. Second step, in order to the fault isolation, is the reconstruction of the failure modes by inversion, where, roughly speaking, only the linear independence of the fault signatures is required. The last step is the stabilization of the inverted error systems by stable post-filters.

The detection and tracking of the fault signals with simulations are shown for a nonlinear example. In the Figure 8, the detection and failure reconstruction of a \(|\sin(\omega t)|\) like failure mode is well presented. We notice that the second order post-filter attenuates the failure mode more than the first order one applied for \( \nu_1 \). Using post-filters of much higher order, deteriorates the detection capability.
References


