# A Near Optimal Fuzzy Modeling of pursuit-evasion in an Air Combat 

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#### Abstract

We propose a new method for a near optimal three-dimensional fuzzy modeling of pursuit-evasion in an air combat in this paper. A sixth order nonlinear point mass vehicle model is considered for an aircraft's flight dynamics. The desired value of the velocity, the flight path and the heading angles are obtained from some rule bases. The physical control parameters are computed through a mean square error scheme. We need to imitate expert's decisions for generating the rules. The capture rate of the proposed model is very high because this model employs a time optimal combination of classic maneuvers in special cases when needed. In addition, the proposed model is noise robustness. The computer simulations provide an encouraging validation of the proposed model.


Key words: Real Time Model- Near Optimal Fuzzy modeling - Air Combat - Maneuvering target - pursuit-evasion game.

## 1 Introduction

Technology, Electronics and control system improvements cause the decrease of the role of the pilots and the change of the figure of an air combat.
A differential game theory used for the modeling of the pursuit evasion problem in an air combat [1]. Another application of the differential game theory is a modeling of a long-range missile duel between two aircraft [2]. Reference [3] derives an optimal guidance law for capturing a maneuvering evader.
The optimal method is not suitable for modeling of an air combat because the performance of the model is different from the pilot performance.
Reference [4] derives a model of an air combat between many aircraft. Reference [5] presents a fuzzy guidance law against very high-speed targets. A fuzzy guidance law for 2-D modeling offensive maneuvers is
considered for an air combat and a two-phase pursuit law is presented as fuzzy "if...then..." rules in [6]. A 3-D fuzzy guidance law for generating an offensive maneuver against a maneuvering evader is presented in [7]. We propose a new 3-D fuzzy modeling of pursuit-evasion in an air combat. This model employs the pilot decisions for generating a set of fuzzy "if...then..." rules. The desired value of the velocity, the flight path and heading angles are obtained from some rule bases. The physical control parameters are computed through a mean square error Scheme. The computer simulations provide an encouraging validation of the proposed model.

## 2 Problem Statement

Here, we consider the situation given in figure 1. Two aircraft ( P and E ) are engaged in a pursuer and evader combat task that the P aircraft is in the
offensive mode and the other is in the defensive situation. The P aircraft has an energy advantage and further can maneuver in the three dimensional space. The flight dynamics of both aircraft are governed by the sixth order nonlinear point mass equations given in [10]. We consider here the two aircraft both have thrust, velocity, turn radius and heading angle limitations. The problem is to find a model of pilot's performance that puts the E aircraft in its effective gun envelope. The effective gun envelope shall lie between a certain minimum and maximum range. The maximum range is because of velocity of bullet, the rate fire and bullet aerodynamics. The minimum range is to avoid of collision with the target or the target debris. The proposed model must act in a manner that $P$ aircraft does not lose any energy. In addition to reaching a suitable position, the maneuver shall keep the offensive fighter out from placing in a bad position where the defensive fighter gains an offensive mode. The P aircraft must avoid dangerous overshoots. A dangerous overshoot is a cut-off flight path in front of the other fighter.


Fig. 1 air combat geometry The point mass equations of motion for an aircraft are given by the following equations [10].

$$
\begin{align*}
\dot{V} & =\frac{T-D}{m}-g \sin \gamma \\
\dot{\theta} & =\frac{L \sin \phi}{m V \cos \gamma}  \tag{1}\\
\dot{\gamma} & =\frac{g}{V}\left(\frac{L \cos \phi}{m g}-\cos \gamma\right) \\
\dot{x} & =V \cos \gamma \cos \theta \\
\dot{y} & =V \cos \gamma \sin \theta \\
\dot{z} & =V \sin \gamma \\
D & =\frac{1}{2} \rho V^{2} S C_{d} \\
L & =\frac{1}{2} \rho V^{2} S C_{L}
\end{align*}
$$

In the above, V is the airspeed, T the thrust, D the vehicle drag, m the vehicle mass, $g$ the acceleration due to gravity, $\gamma$ the flight pass angle, $\chi$ the heading angle, $\varphi$ the bank angle, $\rho$ the atmosphere density, $s$ the reference area of wing, $C_{d}$ the drag coefficient, $C_{l}$ the lift coefficient, $D$ the drag and $L$ the lift.
The motion equations of the evader are in the same form as those of the pursuer given in (1). The "specific energy" defined in (2).

$$
\begin{equation*}
E_{s}=H+\frac{V^{2}}{2 g} \tag{2}
\end{equation*}
$$

Where, H and V stand for the potential and the kinetic energy, respectively.

## 3 The fuzzy modeling of the pursuer

The $P$ aircraft attempt to do out of plane maneuvers to put the target in its effective gun envelope. In this air combat, the pursuer must avoid of both loss of energy and dangerous overshoots.
The evader defends by turning in a plane severely to escape from the pursuer or gain any angular advantage.
In an air combat, pilots usually determine the flight path angle as well as the heading angle. Below, we summarize the pilot performance of the P aircraft.
The change of flight path angle is positive in climbing and negative in


Fig. 2 3-D fuzzy modeling of pilot's performance
descending. The change of the angle decreases when the horizontal distance increases while it increases when the vertical distance increases.
For heading angle the P aircraft should place his nose at an angle behind the target but near the target, the P aircraft holds its nose directly on the target.
When the horizontal and the vertical distance between two aircraft are suitable and when both the velocity ratio and the closure ratio decrease, the switching from climbing to descending occur.
According to the acquired data from [ $9,10,11]$ and pilots, the change of the flight path angle and the heading angle do not have same importance in an air combat. This relative importance of the angles is in the form given below.
In climbing, if the vertical distance of two aircraft or the flight path angle of the P aircraft is big or the velocity ratio of two aircraft is small or the distance between two aircraft is small then the change of the heading angle becomes important.
In descending, if the vertical distance of two aircraft or the flight path angle of the P aircraft is big or the velocity ratio of two aircraft is small then the change of the flight path angle becomes important. When the distance between two aircraft is small, the change of the angles has a same importance.
When the dangerous overshoot may be occurred, the change of the both angles has a same importance

According to the information that has been mention in above, we propose in this section the fuzzy model as depicted in figure (2).
As observed in figure (2), The proposed model of the pilot is contained two distinct blocks. The blocks are Fuzzy guidance law (FGL), and optimal converter. Each block is discussed below.

### 3.1 The fuzzy guidance law block

This block is for generating maneuvers and guiding aircraft. This block includes five sub-blocks: the calculation of the heading angle, the calculation of the flight path angle, the switching block, the calculation of the weighting parameters and velocity prediction block. The proposed blocks are shown in figure (3).
In the following sections, the term set $v s, p b, s, z e, p, l a$, are stand for very small, positive big, small, zero, positive, and large.

### 3.1.1 The flight path angle in climbing

The inputs to this block are the horizontal and the vertical distance, the velocity ratio, the lead angle and the


Fig. 3 the proposed block of the flight path angle in climbing
change of the lead angle. Because of increment of the inputs, we propose the following model for this block.
The first inputs include the vertical and the horizontal distance and the velocity ratio. The second inputs are the distance, the lead angle and the change of the lead angle.
A sample of the rules in the first rule base is shown below.

If (horizontal_distance is
$v s)$ and(vertical_distance is
pb)and(velocity_ratio is s) then(gama is ze)
A sample of the rules in the second rule base is shown below.

> If (lead_angle is
$v s)$ and (delta_lead_angle is
$p b$ ) and(distance is not $v b$ )
then(delta_gama is ze)
The first rule base contains 75 rules.
The second rule base contains 10 rules

### 3.1.2 The flight path angle in descending

This block is for calculation of the flight path angle in descending. The inputs of this block the vertical and the horizontal distance and the distance. The proposed block is like figure 4.


Fig. 4 The proposed block of the flight path angle in descending
The first and second inputs are the vertical and the horizontal distance and the third inputs are the distance and the vertical distance. The second rule base is for calculation of the lag in the flight path angle. A sample of the rules in the first rule base is shown below.
if(horizontal_distance is
pb)and(vertical_distance is
ze)then(gama is ze)
A sample of the rules in the second rule base is shown below.

## if(distance is ze)and(vertical_distance

 is not ze)then(lag is $\bar{p}$ )The first rule base contains 30 rules and the second one contains 3 rules.

### 3.1.3 The heading angle

This block is for calculation of the heading angle. The inputs of this block the distance along x -axis and y -axis, the horizontal distance and the distance. The proposed block is like figure 5.


Fig. 5 The proposed block of the heading angle The first and second inputs are the distance along x -axis and y -axis and the third inputs are the distance and the vertical distance. The second rule base is for calculation of the lag in the flight path angle. A sample of the rules in the first rule base is shown below.

> if( $x \_$distance is $\left.p b\right)$ and $\left(y \_d i s t a n c e ~ i s ~\right.$ ze)then(teta is ze)

A sample of the rules in the second rule base is shown below.
if(distance is
ze) and(horizontal_distance is not ze)then(lag is $p$ )
The first rule base contains 20 rules and the second one contains 3 rules.

### 3.1.4 The weighting parameter

This block is for calculation of weighting parameter. The inputs to this block are the horizontal and the vertical distance, the velocity ratio, the lead angle, the change of the lead angle, the flight path angle of the P aircraft and its change. A sample of the rules is shown below.

If(change_gama is
p) and(horizontal_distance is not s) or_(gama is not s)or(velocity_ratio is not la)then( $W$ is teta)
" $W$ is teta" in above rule, means that $\chi$ is important in this situation.
The climb weighting parameter rule base consists of 30 rules.

### 3.1.5 The switching block between climbing and descending

This block employs for assignment of time between climbing and descending. The inputs to this block are the distance, the horizontal distance, the velocity ratio, and the off angle. A sample of the rules is shown below,

If(horizontal_distance is not la)or(distance is not
b) or(velocity_ratio is s) and(off_angle is not ze) and(closure is $n$ ) then(switch is ok)
"Switch is ok" in above rule, means that switching is occurred in this situation. The switching rule base includes 9 rules.
The membership functions are arbitrarily chosen as triangular and trapezoidal. The membership function parameters have been derived through the information acquired from pilots and [8,9,10,11].

### 3.1.6 The velocity prediction

According to constancy of energy in the $P$ aircraft, we take the derivative of (2) to obtain (3).

$$
\begin{array}{r}
\dot{V}=-g \sin \gamma  \tag{3}\\
T=D
\end{array}
$$

We must consider velocity limitation here as a follow.
If the velocity given in (3) is smaller than $V_{p M i n}$, we must use the following equations.

$$
\begin{equation*}
\dot{V}=0 \tag{4}
\end{equation*}
$$

### 3.2 Optimal converter block

Knowing that the outputs of previous rule base are desired values of the state variables. It is necessary to convert these desired values to the physical inputs of the aircraft dynamics such as
the thrust, the lift, and the bank angle. The Thrust, the lift, and the bank angle can be derived as:

$$
\begin{align*}
& \operatorname{Min}\left\{W\left(\gamma(k+1)-\gamma_{\text {des }}\right)^{2}+\left(\chi(k+1)-\chi_{\operatorname{des}}\right)^{2}\right\} \\
& \quad L \leq L_{M a x},-90 \leq \varphi \leq 90 \tag{4}
\end{align*}
$$

The condition $L \leq L_{\text {max }}$ states that the P aircraft will not lose any energy

## 4 Simulation Results

In this section, we wish to consider the performance of the proposed model. The P and E aircraft parameters are given below.
$T_{\text {Max }}=10^{5} \mathrm{~N} ; k=0.179 ; V_{p 0}=200 \mathrm{~m} / \mathrm{s} ; m=10^{4} \mathrm{~kg}$
$S=26 \mathrm{~m}^{2} ; V_{e 0}=120 \mathrm{~m} / \mathrm{s} ; C_{d 0}=0.0169 ; \rho=0.8$
$V_{\text {pMin }}=60 \mathrm{~m} / \mathrm{s}$
The computer simulations provide an encouraging validation of the proposed model.
The simulation results show that the thrust and lift are approximately BangBang and our method is near optimal. In order to check how good our model performs, we next consider the following cases. The P and E aircraft are initially located in $(0,400,1000)$ and $(1000,0,1000)$ respectively initial lead angle is $10^{\circ}$. Figure (6) shows the capability of the model in generating a classic maneuver such as high yo-yo done by pilots.
Consider the situation like figure (6) with the initial lead angle is $150^{\circ}$. Figure (7) plots the performance of the proposed model in generating an optimal time combination of the classic maneuver. In this case, we easily see that the $P$ aircraft initially uses out of plane lag turn and then uses a classic maneuver to gain angular advantage. To see how robust our method is against noise, we consider the same case whose result shown in figure (7) but the inputs of the P aircraft are contaminated by noise. The signal to noise ratio is $90 \%$. Figure (8) shows the thrust and the lift and the bank
angle are bang -bang and robust against noise.
The maximum time dissipation for calculation of the model with Pentium 4 computer is about $16-\mathrm{ms}$. this means that the model can employ in real time mode.


Fig. 6 the path produced by proposed model


Fig. 7 the path produced by proposed model


Fig. 8 Thrust, Lift and bank angle variation of aircraft $P$ during a maneuver

## 5 Conclusion

In this paper, a fuzzy modeling is proposed for the 3-D modeling of the pilot's performance. This method allows us to model uncertainty and imprecision of the expert's knowledge.

It is suitable for generating complex maneuvers in an air combat task. The proposed model is noise robustness. This model employs a near time optimal combination of classic maneuvers in special cases. The simulation results easily approve the high capability of the proposed model for capturing the evader aircraft in all cases.

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