Abstract: - In this paper, we propose a new method for a three dimensional fuzzy modeling of pilot's performance in the dogfight. The dogfight is a short-distance air combat with a gun. A sixth order nonlinear point mass vehicle model is considered for an aircraft's flight dynamics. The desired value of the velocity, the flight path and the heading angles are obtained from some rule bases developed in the paper. The physical control parameters are computed through a mean square error scheme. To model pilot's performance and generate a complicated offensive maneuver in an air combat, we need to mimic expert's decisions. This has been done by introducing a fuzzy model consisting of three distinct blocks that are fully discussed in the paper. The model still has satisfactory performance when two aircraft go out of sight. The capture rate of the proposed model is higher than that of the ones given in [4, 5, 7] because this model employs an optimal time combination of classic maneuvers in special cases when needed. Furthermore, The proposed model is robust against noise.

Key words: Fuzzy modeling – Pilot's performance – linguistic model – Dogfight – Maneuvering target – pursuit-evasion game.

1 Introduction
In the past, pilots have to obtain all necessary data and information only through their senses. Technology improvements cause to change the figure of the air combat. Nowadays, electronics and control systems decrease role of pilots in an air combat task. A pilot assistance system used in associated systems offers data and information to pilots [1]. An air combat between many aircraft has been presented in [2]. In [3], A guidance law is developed against very high-speed targets. A fuzzy guidance law has been proposed for midcourse phase based on human decision-making. A fuzzy guidance law for 2-D modeling offensive maneuvers is considered for an air combat and a two-phase pursuit law is presented as fuzzy "if...then..." rules in [4]. A three-dimensional fuzzy guidance law for generating a complex offensive maneuver against a maneuverable evader is presented in [5]. Reference [6] proposed a non-fuzzy based approach that introduces a way to produce a preference optimal flight path in one-on-one air combat in which the pilot's sequential maneuvering decision is modeled by a multistage influence diagram.

Theory of the optimal control has also been employed to model an air combat task. For example, [7] employed concepts of differential game theory for developing an optimal pursuit-evasion problem between two aircraft. Another application of differential game theory to pursuit-evasion problem is an air combat between two aircraft for a long-range missile duel that has been reported in [8]. An optimal guidance law was derived for a vehicle pursuing a maneuvering evader with the assumption that a complete knowledge of the evader's motions is
available to the pursuer in [9]. The methods that use the optimal theory for modeling an air combat are in nature different from those developed based on human expert's decision making.

In this paper, we propose a new method for three-dimensional fuzzy modeling of pilot's performance in the dogfight. A set of fuzzy "if ...then" rules that models pilot decisions are developed. The desired value of the velocity, the flight path and heading angles are obtained from some rule bases. These rule bases are fully discussed in the paper. The physical control parameters are computed through a mean square error Scheme. The computer simulations provide an encouraging validation of the proposed model.

2 Problem Statement
Here, we consider the situation given in figure 1. Two aircraft (P and E) are engaged in a pursuer and evader combat task that the P aircraft is in the offensive mode and the other is in the defensive situation. The P aircraft has an energy advantage and further can maneuver in the three dimensional space. The flight dynamics of both aircraft are governed by the sixth order nonlinear point mass equations given in [10]. Being more realistic, we consider here the two aircraft both have thrust and velocity limitations. Both fighters have turn radius limitations so that the fighters cannot freely change their heading angles. The problem is to find a model of pilot's performance that puts the E aircraft in its rear hemisphere. Then, the P aircraft tries to put target in its effective gun envelope. The effective gun envelope shall lie between a certain minimum and maximum range. The maximum range is because of velocity of bullet, the rate fire and bullet aerodynamics. The minimum range is to avoid of collision with the target or the target debris. The proposed model must act in a manner that P aircraft does not lose any energy. In addition to reaching a suitable position, the maneuver shall keep the offensive fighter out from placing in a bad position where the defensive fighter gains an offensive mode. The P aircraft must avoid dangerous overshoots. An overshoot is a cut-off flight path by the other aircraft. If this cut-off flight path is in front of the other fighter, the overshoot is dangerous.

3 Nonlinear Model for Aircraft
The point mass equations of motion for an aircraft for a flat non-rotating earth, thrust along the path and a quiescent atmosphere are given by the following equations [10].

\[
\begin{align*}
V &= \frac{T - D}{m} - g \sin \gamma \\
\dot{\gamma} &= \frac{L \sin \phi}{m V \cos \gamma} \\
\dot{\gamma} &= \frac{g (L \cos \phi)}{mg} - \cos \gamma \\
\dot{x} &= V \cos \gamma \cos \chi \\
\dot{y} &= V \cos \gamma \sin \chi \\
\dot{z} &= V \sin \gamma
\end{align*}
\]

In the above, \( V \) is the airspeed, \( T \) is the thrust, \( D \) is the vehicle drag, \( m \) denotes the vehicle mass, \( g \) is the acceleration due to gravity, \( \gamma \) is the flight pass angle, \( \chi \) is the heading angle and \( \phi \) represents the bank angle.

The aerodynamics drag and lift are calculated as:

\[
\begin{align*}
D &= \frac{T}{\rho} V^2 SC_D \\
L &= \frac{1}{2} \rho V^2 SC_L
\end{align*}
\]
In the equation (2) and (3), $\rho$ is the atmosphere density, $s$ is the reference area of wing, $C_d$ is the drag coefficient, $C_l$ is the lift coefficient, $D$ is the drag and $L$ denotes the lift. The motion equations of the evader are in the same form as those of the pursuer given in (1). The total "energy state" is sum of the kinetic energy and the potential energy. Eliminating the weight of aircraft from energy state calculation causes the "specific energy" defined in (4).

$$E_s = H + \frac{V^2}{2g}$$

Where, $H$ and $V$ stand for the potential and the kinetic energy, respectively.

4 Linguistic Models of The Pursuer and The Evader
In this section, we present a suitable linguistic model for both P and E aircraft. To do so, we are required to comply with the assumptions made in section (2) as well as expert decisions and [11,12,13]. After some investigations a linguistic model are proposed as follow.

The P aircraft attempt to do out of plane maneuvers to put the target in its effective gun envelope. In this air combat, the pursuer must avoid of both loss of energy and dangerous overshoots.

The evader defends by turning in a plane severely to escape from the pursuer or gain any angular advantage. The velocity of evader in this turn is the corner velocity. The corner velocity is an airspeed at which aircraft has a tightest turn radius.

In an air combat, pilots usually determine the flight path angle as well as the heading angle. Below, the linguistic model of the pilot of the pursuer is summarized for above cases.

4.1 The flight path angle in climbing
In this case, the change of flight path angle is positive. The change of the angle decreases when the horizontal distance increases while it increases when the vertical distance increases.

4.2 The flight path angle in descending
In this case, the change of flight path angle is negative. Therefore, the change of the flight path angle decreases when the horizontal distance increases while it increases when the vertical distance increases.

4.3 The heading angle
The P aircraft should place his nose at an angle behind the target but near the target, the P aircraft holds its nose directly on the target.

4.4 The switching from climbing to descending
This block will change the positive sign of the flight path angle into the negative sign. When the horizontal and the vertical distance between two aircraft are suitable and when both the velocity ratio and the closure ratio decrease, the switching from climbing to descending occur. According to the acquired data from [11,12,13] and pilots, the change of the flight path angle and the heading angle do not have same importance in an air combat. This relative importance of the angles is in the form given below.

4.5 The relative importance of the angles in climbing
If either of the following 1- The vertical distance of two aircraft is big 2- The flight path angle of the P aircraft is big 3- The velocity ratio of two aircraft is small is satisfied then the change of the heading angle becomes important. When the dangerous overshoot may be occurred, the change of the both angles has a
same importance. When the distance between two aircraft is small, the change of the heading angle is importance.

4.6 The relative importance of the angles in descending
If either of the following 1- The vertical distance of two aircraft is big 2- the flight path angle of the P aircraft is big 3- the velocity ratio of two aircraft is small is satisfied then the change of the flight path angle becomes important. When the dangerous overshoot may be occurred, the change of the angles has a same importance. When the distance between two aircraft is small, the change of the angles has a same importance.

5 The fuzzy modeling of the pursuer
According to the information that has been mention in section (4), we propose in this section the fuzzy model as depicted in figure (2).
As observed in figure (2), The proposed model of the pilot is contained three distinct blocks. The blocks are Input generator, Fuzzy guidance law (FGL), and optimal converter. Each block is discussed below.

5.1 The input generator
This block is for generating the inputs of the FGL block. For further information refer to [14].

5.2 The fuzzy guidance law block
This block is for generating maneuvers and guiding aircraft. This block includes five sub-blocks: the calculation of the heading angle, the calculation of the flight path angle, the switching block, the calculation of the weighting parameters and velocity prediction block. The proposed blocks are shown in figure (3). In the following sections, the term set vs, pb, s, ze, p, la, are stand for very small, positive big, small, zero, positive, and large.

5.2.1 The flight path angle in climbing
According to the linguistic model mentioned in (4.1), The inputs to this block are the horizontal and the vertical distance and the velocity ratio. A sample of the rules is shown below.
If (horizontal_distance is vs) and (vertical_distance is pb) and (velocity_ratio is s) then (gamma is ze)

The climbing rule base contains 75 rules.

5.2.2 The flight path angle in descending

According to the linguistic model given in (4.2), the flight path angle in descending can be calculated as:

\[ \gamma_{des} = \tan^{-1}\left(\frac{\text{vertical distance}}{\text{horizontal distance} + M}\right) \]  

(6)

\( \gamma_{des} \) is the desired value of \( \gamma \) in descending. \( M \) determines lead angle for the flight path angle. \( M \) is calculated from a rule base developed in this paper. Inputs to this rule base are the distance and the vertical distance. A sample of the rules is shown below.

if (distance is ze) and (vertical_distance is not ze) then (M is p)

The descending rule base contains 10 rules.

5.2.3 The calculation of the heading angle

According to the linguistic model described in (4.3), the heading angle in descending can be calculated as:

\[ \chi_{des} = \tan^{-1}\left(\frac{\text{distance along y axis}}{\text{distance along x axis} + N}\right) + \chi_c \]  

(7)

\( \chi_{des} \) is the desired value of \( \chi \) in the air combat. \( N \) is determined lead angle for the heading angle. \( N \) is calculated from the rule base developed in this paper. Whose inputs to this rule base are the distance and the horizontal distance. A sample of the rules is shown below.

if (distance is ze) and (horizontal_distance is not ze) then (N is p)

The heading angle rule base contains 10 rules.

5.2.4 The weighting parameter in climbing

According to the linguistic model discussed in (4.5), the inputs to this block are the horizontal and the vertical distance, the velocity ratio, the lead angle, the change of the lead angle and the flight path angle of the P aircraft. A sample of the rules is shown below.

If (horizontal_distance is not s) or (gamma is not s) or (velocity_ratio is not la) then (W is teta)

“W is teta” in above rule, means that \( W \) is important in this situation. The climb weighting parameter rule base consists of 17 rules.

5.2.5 The weighting parameter in descending

According to the linguistic model discussed in (4.6), inputs to this block are the horizontal and the vertical distance, the velocity ratio, the lead angle, the change of the lead angle and the flight path angle of the P aircraft. A sample of the rules is shown below.

If (horizontal_distance is not s) or (gamma is not s) or (velocity_ratio is not la) then (W is gama)

“W is gama” in above rule, means that \( W \) is important in this situation. The climb weighting parameter rule base consists of 17 rules.

5.2.6 The switching block between climbing and descending

According to the linguistic model mentioned in (4.4), inputs to this block are the distance, horizontal distance, velocity ratio, and off angle. A sample of the rules is shown below.

If (horizontal_distance is not la) or (distance is not b) or (velocity_ratio is s) and (off_angle is not ze) and (closure is n) then (switch is ok)

“Switch is ok” in above rule, means that switching is occurred in this situation. The switching rule base includes 9 rules.

5.2.7 Velocity prediction

This block is for the prediction of the velocity. For further information refer to [14].
5.3 The optimal converter block
This block is for converting the desired values of the state variables to the physical inputs of the aircraft dynamics such as the thrust, the lift, and the bank angle. For more information refer to [14].

6 Simulation Results
In this section, we wish to consider the performance of the proposed model. The P and E aircraft parameters are given below.

\[ T_{\text{max}} = 10^5 N; k = 0.179; V_{\rho \theta} = 200 \text{ m/s}; m = 10^4 \text{ kg} \]

\[ S = 26 \text{ m}^2; V_{es} = 120 \text{ m/s}; C_{es} = 0.0169; \rho = 0.8 \]

\[ V_{\text{plate}} = 60 \text{ m/s} \]

The simulation results easily approve the high capability of the proposed model for capturing the E aircraft in all cases. Table 1 compares the performance of the proposed model with the other model for 15000 initial conditions.

<table>
<thead>
<tr>
<th>Method</th>
<th>Lead Angle</th>
<th>Capture Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model In [5]</td>
<td>[90,180] ( \cup ) [180,270]</td>
<td>60%</td>
</tr>
<tr>
<td>Model In [5]</td>
<td>[0,90] ( \cup ) [-90,0]</td>
<td>75%</td>
</tr>
<tr>
<td>Proposed Model</td>
<td>[90,180] ( \cup ) [180,270]</td>
<td>85%</td>
</tr>
<tr>
<td>Proposed Model</td>
<td>[0,90] ( \cup ) [-90,0]</td>
<td>95%</td>
</tr>
</tbody>
</table>

The simulation results show that the thrust and lift are approximately Bang-Bang and our method is near optimal.

Case 1, In order to check how good our model performs, we next consider the following cases. The P and E aircraft are initially located in (0,400,1000) and (1000,0,1000) respectively initial lead angle is 10°. Figure (4) shows the capability of the model in generating a classic maneuver such as lag roll done by pilots.

Case 2, the P and E aircraft initially located in (0,400,1000) and (1000,0,1000) respectively and the initial lead angle is 180°. Figure (5) plots the performance of the proposed model in generating an optimal time combination of the classic maneuver. In this case, we easily see that the P aircraft initially uses out of plane lag turn and then uses a classic maneuver to gain angular advantage. To see how robust our method is against noise, we consider the same case whose result shown in figure (5) but the inputs of the P aircraft are contaminated by noise. The signal to noise ratio is 90%. Figure (6) shows the robustness of our model against noise. Figure (7) shows the thrust and the lift and the bank angle are bang–bang and robust against noise.
Fig. 7 Thrust, Lift and bank angle variation of aircraft P during an air combat

7 Conclusion
In this paper, an intelligent model is proposed for the three-dimensional modeling of the pilot’s performance. This model is based on fuzzy set theory that allows us to model uncertainty and imprecision of the expert’s knowledge. It is suitable for generating complex maneuvers in an air combat task. The proposed model is robust against noise. This model employs a near time optimal combination of classic maneuvers in special cases when needed. The computer simulations provide an encouraging validation of the proposed model.

Reference