Abstract- In this paper, a powerful method for sensorless control of permanent magnet brushless motors (PMBM) is explained. The method doesn't include any modification to the classical construction of such motors and can be easily integrated within the drive electronics. It is applicable to both motor types, i.e. those with sinusoidal or trapezoidal back EMF. The method is independent of excitation current profile. It is also virtually independent of system parameters. The back EMF waveform is extracted on-line using simple and integration-free algebraic expression through a newly proposed measurement setup. Appropriate simple digital filters to make it directly usable for calculating rotor position and speed are then used to filter the extracted waveforms. This paper discusses the method application on the PMBM whose back EMF waveforms are sinusoidal; its application on the trapezoidal back EMF machine is left for a separate paper. System performance using the proposed method is tested using simulation and experimentation. Results obtained in both cases ensure the effectiveness of the proposed technique.

I- INTRODUCTION

Permanent magnet brushless motor (PMBM) is widely used as a basic element in high performance servo drive systems due to its high efficiency and torque density. Recently, a lot of research has been focusing on the sensorless control of PMB motors, which normally operates using position and speed sensors. Two different types of surface-mounted (SM) PMBM are available, the sinusoidal back EMF type and the trapezoidal back EMF type. The first type is frequently referred to as permanent magnet synchronous motor (PMSM) and the second type is frequently referred to as brushless DC motor (BDCM). Sensorless control is an attractive solution to mechanical sensor problems, which include extra cost, wiring complexity, reduced reliability and demand for additional mounting space. The authors of this study believe that space limitations [1], incompatibility with some harsh working environment (e.g. inside refrigerant compressors) [2] and weight reduction considerations (e.g. aviation industry) represent the most apparent motives behind sensorless control research efforts. This is particularly true if we consider one of the potential fields of application of such motors, i.e. disk drives where miniaturization and reliability come as top design priority [3]. Sensorless control schemes for surface-mounted (SM) PMBM are mostly depending on direct or indirect back EMF detection since the other schemes employing inductance changes with rotor position are applicable only to interior magnet (IM) PMBM [3].

Previous works developed sensorless control schemes for BDCM based on direct EMF detection include [3],[4],[5] and [6] where EMF zero crossings provide position and speed information six times the number of pole-pairs every rotor cycle. The major drawback of such technique is the availability of speed information once every one sixth of the electrical cycle. Trying to provide continuous position and speed information, Hamid A. Toliyat et al suggested a novel scheme for indirect back EMF detection [7], [8]. The scheme involves some modification to the classical structure of such motors and it is generally inconvenient for automatic winders; Serving the same target, a comparative study [10] have proven that substantial improvement in BDCM performance can be realized by adopting an extended Kalman filter (EKF) algorithm utilizing the open phase voltage measurement in its correction mechanism [9]. Unfortunately the implementation is dedicated to BDCM only and can’t be extended to PMSM. Silverio Blognani, et al [11] introduced a high-performance sensorless PMSM drive based on EKF. However EKF methods are characterized by intensive math manipulations and are some how difficult to tune.
In this study we try to make benefits from the special construction of PMBM as is, thus avoiding any structural modifications. A unified and general approach for position and speed estimation capable of handling both PMSM and BDCM is introduced. The proposed technique is also independent of the excitation current profile (sinusoidal or quasi-square). The proposed technique is based on a newly proposed method for indirect back EMF extraction. Part I through VI of this paper has the theory and part VII has the simulation and results while part XX introduces the experimental results.

II- SM PMBM MODEL

The effective air gap of SM PMBM is virtually large and uniform due to the considerable magnet depth in the radial direction and due to the fact that magnet materials has permeability close to that of free air [12]. The large air gap helps maintain the magnetic circuit nearly constant and independent of rotor position and excitation current. Current control loop can further support our assumption regarding the linearity of magnetic circuit by enforcing current limits that maintain magnetic circuit’s linearity. A quite general voltage equation of both surface-mounted PMBM motor types can be written as:

\[
\begin{bmatrix}
    v_a \\
    v_b \\
    v_c \\
\end{bmatrix} =
\begin{bmatrix}
    r_s & 0 & 0 \\
    0 & r_s & 0 \\
    0 & 0 & r_s \\
\end{bmatrix}
\begin{bmatrix}
    i_a \\
    i_b \\
    i_c \\
\end{bmatrix} +
\begin{bmatrix}
    L-M & 0 & 0 \\
    0 & L-M & 0 \\
    0 & 0 & L-M \\
\end{bmatrix}
\begin{bmatrix}
    p_b \\
    p_c \\
    p_b + p_c \\
\end{bmatrix} +
\begin{bmatrix}
    e_a \\
    e_b \\
    e_c \\
\end{bmatrix} \tag{1}
\]

where:
- \(v_a, v_b, v_c\): stator terminal voltage of phases a, b and c respectively in Volt.
- \(i_a, i_b, i_c\): currents of phases a, b and c in Amp.
- \(P\): differential operator.
- \(L\): self-inductance per phase in Henry.
- \(M\): mutual inductance in Henry.
- \(r_s\): stator resistance per phase in Ohm.
- \(e_a, e_b, e_c\): back EMF of phases a, b and c respectively in Volt, they are given by

\[
\begin{bmatrix}
    e_a \\
    e_b \\
    e_c \\
\end{bmatrix} =
\begin{bmatrix}
    \frac{d\theta_r}{dt} \\
    \frac{d\theta_r}{dt} \\
    \frac{d\theta_r}{dt} \\
\end{bmatrix}
\begin{bmatrix}
    \cos \theta_r \\
    \cos (\theta_r - \frac{\pi}{3}) \\
    \cos (\theta_r + \frac{\pi}{3}) \\
\end{bmatrix} \tag{2}
\]

Where:
- \(\lambda_m\): the EMF constant in volt.s/elect. rad.
- \(\theta_r\): Electrical angular displacement in rad.
- \(\frac{d\theta_r}{dt}\): Derivative of displacement rad/s.

Electromagnetic torque \(T_e\) is given in N.m by:

\[
T_e = \lambda_m \left( \frac{d\theta_r}{dt} \right) \left( i_a \cos \theta_r + \frac{\sqrt{3}}{2} i_b - i_c \sin \theta_r \right) \tag{3}
\]

where:
- \(P\): Number of poles.

The mechanical system dynamics is given by:

\[
\begin{align*}
T_e + T_L & = J \frac{d\omega}{dt} + \omega M \frac{d\omega}{dt} & \tag{4}
\end{align*}
\]

where:
- \(T_e\): electromagnetic developed torque and load torque in N.m respectively.
- \(J\): moment of inertia in N.m.s^2/rad
- \(B\): damping coefficient in N.m.s/rad.
- \(\omega_m\): rotor’s mechanical speed

\[
\omega_m = P \frac{d\theta_r}{dt}.
\]

\(J\): moment of inertia in N.m.s^2/rad

Equation (1), which is valid under the condition of isolated neutral point, shows that the three motor phases seem as if they were three independent coils of equivalent inductance (L-M). The three windings subject to the flux of a rotating magnet which induces a back EMF of value \(e_{ph}\).

III- BACK EMF DETECTION

III-1 Theory

![Fig. 1: External small inductor connected to each phase of PMBM](image)

In the circuit shown in Fig.1, let the circuit part from P to N represent one of these three independent coils of the PMBM, a small external coil is placed between points I (inverter’s feeding point) and P (motor terminal). Writing the voltage drop equations for the two parts:

\[
\begin{align*}
V_{PN} & = R_{ph} i + L_{ph} \frac{di}{dt} + e \tag{5} \\
V_{ip} & = R_s i + L_s \frac{di}{dt} \tag{6}
\end{align*}
\]

Where:
- \(V_{PN}\): voltage drop from P to N in Volt.
- \(V_{ip}\): voltage drop from P to I in Volt.
- \(e\): back EMF from P to I in Volt.
- \(L_{ph}\): equivalent inductance (L-M) of the phase winding in Henry.
- \(R_{ph}\): resistance \(r_s\) of the phase winding in Ohm.
- \(L_s, R_s\): inductance and resistance of the external inductor respectively.
- \(i\): phase current in Amp.

Solving the two previous equation for \(e\) yields:

\[
e = \frac{L_{ph}}{L_s} V_{ip} + \left( \frac{R_{ph}}{L_s} - R_{ph} \right) i + V_{PN} \tag{7}
\]
The previous equation can be applied for each motor phase independently. For PMSM we need a total of four voltage measurements since the EMFs of the three phases ideally sum to zero.

**III-2 Conditions of validity**

Equation (7) can be used on line to calculate the instantaneous value of the back EMF waveform to a reasonable degree of accuracy if the following presumptions could be satisfied.

- The motor magnetic circuit is operated in the linear region.
- Inductive voltage drops across windings’ inductance are too large compared with the resistive voltage drops; this is true due to high frequency switching process.
- Differential voltage measurement (and associated processing) at the selected points in Fig. 1 is simultaneous and fast enough relative to the current control switching frequency.
- The external inductors are to be made identical to each other with low-loss core, with constant inductance, minimum resistance and should have no-mutual coupling.

**IV. NOISE ANALYSIS AND PROPOSED FILTER**

In our system, noise is expected as:

- **white noise** due to surrounding environment.
- **impulse noise** due to high frequency current switching.

While simple finite impulse response (FIR) digital filters can be used to effectively filter white noise, impulsive noise can’t be filtered by FIR without introducing large phase shift and amplitude attenuation. Median filters are more suitable to deal with impulsive noise since they introduce very little phase shift and ideally no magnitude attenuation. Also it’s particularly useful in preserving the flat top of the back EMF waveform in BDCM types.

**IV.1 Digital FIR Filter**

The mathematical formula describing this filter (Fig. 2) is given by the following equation:

\[ Y = \sum_{i=0}^{N} a_i X(n-i) \]  \hspace{1cm} (8)

Where \( Y \) is the current filter output, the \( X(n-i) \) are current or previous filter inputs. The \( a_i \)'s are the filter’s feed forward coefficients corresponding to the filter’s zeros. If this filter is excited with an impulse, the output is present for only a finite (N) number of computational cycles. Due to its all zero structure (no poles at all), the FIR filter has a linear phase response in most standard filter applications.

![Fig. (2): FIR Filter implementation.](image)

**IV.2 Median Filter**

For an input signal \( V \), the vector \( U \) is an n-element vector whose elements are taken from the elements of \( V \) by a window sliding over the input signal \( V \) where \( n \) is the window size of the median filter. Normally \( n \) is chosen to be an odd number, then if \( Y = \text{median}(U) \) and \( S \) is the sorting of the elements of \( U \) by value. The output \( Y \) is the central element of the sorted vector \( S \) as follows

\[ Y = S((n+1)/2) \hspace{1cm} \cdots \hspace{1cm} \]  \hspace{1cm} (9)

For on-line filtering this implies that the last \( n \) samples of a noisy signal should be always kept in memory and refreshed in a first-in first-out dropping style.

**V DIRECT EMF-BASED POSITION AND SPEED CALCULATION**

The transformation of the extracted three phase EMF from stationary frame to the two phase stationary frame results normally in a vector whose magnitude represents the rotor electrical speed and whose angle represents the electrical rotor position.

**V.1 Related Problems**

Since both vector’s magnitude and phase would be affected by filtering, we’ve got two problems.

- A speed calculation error due to amplitude attenuation caused by filter.
- A position calculation error due to phase lag introduced by filter.

Another problem is invited if the drive operation requirements involve speed-reversing. As the speed gets closer to zero, position estimation error gets higher and higher due to poor signal to noise ratio, and eventually we may
loose any trustable information about the real rotor position. Therefore a modified estimation technique should be provided for position and speed in the vicinity of zero speed under speed reversal conditions. A similar situation at drive’s startup is fixed by open loop motor acceleration.

V.2 Problems Fixing

• Speed Error Compensation

The first problem can be resolved by considering a hypothetical value of the permanent magnet flux coefficient \( \lambda_m \). This hypothetical coefficient’s value is initially assumed equal to the machine EMF constant upon drive startup. Then it can be tuned on line by properly differentiating the position signal over integral electrical cycles to get the actual speed under steady state conditions. The hypothetical magnet flux coefficient is then refreshed using a fresh value of the motor actual speed and a fresh value of the EMF’s vector magnitude in the \( \alpha-\beta \) frame after filtering.

• Position Error Compensation

Through off-line simulation, the relation between phase shift introduced by any given filter combination with known parameters and speed can be predicted starting from standstill and up to rated motor speed. It has been shown by simulation that it has a linear change profile, which makes on-line compensation easier. Assuming that the extracted back EMF vector mapped in the two-phase stationary frame after applying digital filtering is \( E_{a^\prime b^\prime} \). The vector \( E_{a^\prime b^\prime} \) has a space angle \( \theta' \) with respect to the axis \( \alpha^\prime \) of the coordinate system \( \alpha^\prime-\beta^\prime \). According to the estimated speed and the linear lag coefficient \( E_{a^\prime b^\prime} \) is lagging the actual back EMF vector of the machine \( E_{\alpha\beta} \) when the later is mapped in the same reference frame by an angle \( \delta \).

We can imagine that there is some new reference frame or coordinate system \( \alpha-\beta \), obtained by rotating the original stationary coordinate system \( \alpha^\prime-\beta^\prime \) against the sense of motor rotation with an angle \( \delta \) (advance) such that the projections of the vector \( E_{a^\prime b^\prime} \) on its \( \alpha-\beta \) axis would give a resultant vector angle \( \theta \) where \( \theta = \theta' - \delta \). In this way, the phase shift introduced by the filtering process can be corrected. The projections of the vector \( E_{a^\prime b^\prime} \) on the axes of the new coordinate system can be deduced from the equation of Figure (3) and they are given by equation (10):

\[
\begin{bmatrix}
E_a \\
E_{\beta}
\end{bmatrix} = \begin{bmatrix}
\cos \delta & \sin \delta \\
-\sin \delta & \cos \delta
\end{bmatrix} \begin{bmatrix}
E'_{a} \\
E'_{\beta}
\end{bmatrix}
\]

• Estimation at speed’s zero-crossing

Fortunately, the motor speed changes linearly with time-under almost all loading conditions in the vicinity of zero speed during speed reversal. The rate of change can be captured on-line; extrapolation can be used to predict the motor speed once the estimated speed drops under a predefined threshold. The rotor position can be estimated by simple integration process (e.g. trapezoidal rule) applied on the predicted speed.

VI SIMULATION RESULTS

Simulation results of PMSM with sinusoidal-type excitations are shown in Figure (4)-a through Figure (4)-e. At the neighborhood of zero speed under speed reversal conditions, the alternative estimation technique is used. It is generally noticed from multiple simulation sessions that the higher the noise level, the higher the estimation error. However, the error limits reached under worst conditions are well tolerable. Simulation results for the case of light noise level are presented as a representative case, under practical conditions a comparable performance have been already obtained.
VII EXPERIMENTAL RESULTS

VII.1 On-line EMF detection

On-line extraction of the back EMF is verified through oscilloscope traces in this section. In Figure (5) the upper yellow colored trace shows the differential voltage measurement across the external inductor connected to phase "A", the high frequency voltage component is due to the inductor reaction to the fast switching process of the inverter while the lower frequency voltage changes is due to the inductor reaction with the fundamental component of the winding current. The next blue colored trace shows the terminal voltage measured at the motor terminal of phase A. The trace resembles portions broken out of a sinusoid wave and impaired by some offset and high frequency noise. The third pink colored trace shows the extracted EMF resulting from applying the algebraic filtering formula given in equation (7), the improvement established in the waveform is evident since it gets closer to a sinusoid without introducing any phase shift or magnitude attenuation. The fourth green colored waveform shows the EMF resulting after applying simple Finite Impulse Response (FIR) digital filtering, it is clear that some phase shift is now introduced due to this additional filtering. The filter’s math is executed on line upon acquiring each new fresh sample. Figure (6) further illustrates phase lag due to additional filtering. Consequently, the shift is reproduced in the estimated position as shown in Figure (7), where the green colored trace represents the measured position, the blue colored trace represents the position estimated from the filtered EMF and the yellow colored upper trace represents the algebraic difference between them. The apparently high error pulses near the $2\pi$ transitions actually have a low value.
(by dropping $2\pi$ representing the complete cycle).

**VII.2 Open Loop Acceleration**

Although the proposed technique is capable of starting the drive in sensorless mode from zero speed, the resulting sense of rotation may conform to the commanded one or not based on the initial rotor position, which is random and unknown.

Therefore, the first step in any running session is to provide an open loop acceleration period to serve two purposes:

- Guiding the sense of rotation to ensure conformance with the commanded one.
- Reaching a minimum speed to make the EMF detection more accurate and hence trustable.

Figure (8) illustrates the time profile of open loop acceleration through the rotor position information, the upper trace shows the actual rotor position, while the lower trace shows the rotor position considered within the open loop acceleration routine which is used to provide the excitation voltage to the motor. When the acceleration rate is well tuned to the motor and load inertia, the error between the assumed position and the actual position decreases and the expected transient due to switching from open loop to closed loop sensorless acceleration becomes a minimum and gets more tolerable.

**VII.3 Normal Sensorless Operation**

Once the motor acquires a reasonable speed value, the estimated speed and position can be trusted and directly used for closing the current and the speed control loops. Figure (9) gives a comparative vision of the actual (measured) rotor position and the estimated one under motor loading conditions when the motor is running in sensorless closed loop control mode with phase shift compensator enabled. The high frequency ripples superimposed on the yellow trace (measured rotor position) is merely a picked up noise.
Fig. (9): Rotor position during normal sensorless operation

The third trace represents the error between the actual and the estimated position. The picked up noise is reproduced in the error signal accordingly, the coincidence between the two signals are otherwise excellent.

Figure (10) illustrates the speed step response under loading conditions, the yellow is the speed command, and the blue is the measured speed response. The open loop acceleration is clear at the early beginning where the motor builds up some speed, during which the estimated position gets more trusted and the sense of rotation is defined (clockwise rotation is selected here).

Control is then transferred to closed loop mode, first the speed reference was step changed from zero to 59.6% of rated speed. Then a periodic change of speed reference from 59.27% to 24.6 % of rated speed and from the lower to the higher again is enforced. The motor takes longer to decelerate to the lower speed reference because it is left for free retardation during this period.

VIII- CONCLUSION

The advantages obtained by using simple digital filtering along with effective extraction of EMF from excited windings for PMSM sensorless control is discussed in this paper. Based on the concept presented here, power cable only is needed to connect the motor to the drive electronics; extra taps or winding modifications are not needed. Effective fixing procedures for canceling position and speed errors due to filtering are also proposed. These procedures try to avoid static dependence on motor parameters by on-line refreshing of those parameters affecting the estimated speed. The only noticeable drawback is the extra voltage amount that should be supplied by the inverter to compensate for voltage drop across the external inductors; selecting inductors of proper values can minimize this drawback. Very small inductor value would result in a low measurement accuracy due to poor signal to noise ratio. A compromise between reducing additional voltage and measurement accuracy improvement is best deciding the external inductor value. In our simulation a value of 10% of motor equivalent inductance per phase for external inductor was used. The on-line math power requirement can be satisfied by a suitable cheap microcontroller solution.

IX- APPENDIX

The parameters of a 400-watt SM PMSM motor used in simulation are listed below:

- R (Resistance per phase) = 0.560 Ω
- L’=L_s-L_M = 5.945e-4 H
- \( \lambda_m \) (Back EMF const) = 0.073 V/rad/s
- J (Inertia) = 831e-7 Kgm²
- B (friction coeff.) = 1.139e-4 N.m.s/rad
- P (Number of poles) = 2 pole
- Rated continuous torque = 660e-3 N.m
- Rated Speed = 5800 R.P.M.
- Rated power = 400 W
- Rated Current = 4.51 A

IX- REFERENCES