Application of passivity control theory to chaotic systems

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Abstract: - Nonlinear systems are difficult to control and may exhibit chaotic behavior under certain parameters selection. Typical example of these systems is the well known Lorenz attractor. In this paper passive theory is used to design controllers to a class of minimum phase nonlinear systems. The feedback controllers guarantee asymptotic stability of the closed loop system.
The method is applied to a permanent magnet synchronous motor. Simulation studies of test system are shown for both the open and closed loop system behavior show the effectiveness of the proposed controller.

Key-Words: - Lorenz chaotic systems, Passivity theory, Control of chaos, PM synchronous motor

1 Introduction

A key issue in the design of control systems is proving that the resulting closed-loop system is stable. Linear control theory give attractive control tools to linear or linearized plants. Unfortunately most practical applications involve nonlinear systems which are difficult or impossible to exploit linear theory. In the last decades nonlinear theory has been developed giving sometimes tools with surprisingly good results but these tools do not apply in general. These methods are adaptive control techniques, feedback linearization [8], Lyapunov approach and dissipative methods [1-2] providing alternative controller design and implementation methodologies guarantying global or local stability for classes of nonlinear systems [4].

In this paper passive control theory for nonlinear systems is reviewed and tested in practical applications. Passive systems, like linear systems containing only dampers or resistors, are stable. In this point is important to know when a nonlinear system is passive. Passive nonlinear systems enjoy the nonlinear analog of the minimum phase property. This property in nonlinear systems is the stability of zero dynamics. Moreover, the important issue in this analysis is to determine when a finite dimensional nonlinear system can be rendered passive via feedback. In other terms the design issue is to identify nonlinear systems that are feedback equivalent to passive systems. Results in [1] give the necessary condition to answer this question.

Here, there are studied systems having the mathematical form of Lorenz strange attractor [7]. This well known system exhibit chaotic behavior to certain range of parameters selection. Chaotic behavior is undesirable in practice. Furthermore, stability is the fundamental requirement. Conditions of Lorenz system feedback equivalence to a passive system are satisfied and the proposed controller is given

Permanent magnet (PM) synchronous motors are now very popular in a wide variety of industrial applications. When permanent magnets are buried inside the rotor core rather than bonded on the rotor surface, the motor not only provides mechanical robustness but also opens a possibility of increasing its torque capability. In a PM synchronous motor where inductances vary as a function of rotor angle, the 2 phase (d-q) equivalent circuit model is commonly used for simulation studies and control design [5-6].

The effectiveness of the controller is shown by means of numerical simulations. The paper is organized as follows: Section 2 states preliminary material of passive and chaotic systems. Section 3 presents the condition of feedback equivalence to passive systems and the proposed controller. In Section 4 the PM synchronous motor model is derived in closed and closed loop. Section 5 simulation studies are given and conclusion follows.

2 Preliminaries

Nonlinear chaotic systems ate presented and compared to passive systems. Feedback equivalence to passive systems is

2.1 Lorenz systems

Lorenz's system is a three-dimensional, nonlinear model. The three variables: x_1, x_2 and x_3 correspond to the location of a point in geometric *space*. Lorenz's system, although simple, is actually an insolvable problem except by numerical means and exhibits chaotic behavior for suitable selection of parameters. Lorenz system is described by the following equation set

$$\begin{aligned} \dot{x}_1 &= \sigma(x_2 - x_1) \\ \dot{x}_2 &= \rho x_1 - x_1 x_3 - x_2 \\ \dot{x}_3 &= -\beta x_3 + x_1 x_2 \end{aligned} \tag{1}$$

2.2 Passive systems

Let nonlinear systems described by the general form

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{2}$$

where $x \in R^n$, $y \in R^r$, $u \in R^m$ f,g are smooth (i.e. C^∞) vector fields and h is a smooth mapping. Suppose f has at least one equilibrium point.

Normal form of (2), used in next analysis, is the set of eq.

$$\begin{aligned} \dot{z} &= f(z) + p(z,u)u \\ \dot{y} &= b(z,y) + a(z,y)u \end{aligned} \tag{3}$$

Zero dynamics describe the dynamics of the system (3) where y=0. A system whose zero dynamics are asymptotical stable is called minimum phase system [1, 4].

Let a system states function *V(x)* called storage function for system (3) and *W(x)* called Lyapunov function.

Definition 1. [ ] System of the form (3) is called passive if

$$V(x) - V(x_0) \le \int_0^t y^T(\tau)u(\tau)d\tau \tag{4}$$

Definition 2. *W(x)* is a Lyapunov function if

$$\begin{aligned} W(x) &> 0 \\ W(0) &= 0 \\ \dot{W}(x) &\le 0 \end{aligned} \tag{5}$$

Definition 3. System has the KYP property [4] if

$$L_f V(x) = \nabla V(x) f(x) \le 0 \tag{6}$$

$$L_g V(x) = \nabla V(x) g(x) = h^T(x)$$

where L symbol denotes Lie derivative.

2.3 Feedback equivalence to passive systems

Based on the properties of passive systems essential conditions of feedback equivalence of nonlinear systems to passive are studied in next section. In particular state feedback controls of the form

$$u = k(x,v) \tag{7}$$

where *x* are the states and *v*, external input signal, are requested to result to a closed loop passive system.

3 Main result

Theorem 1.: If system (3) x=0 is ann equilibrium point, has relative degree {1,1, …1} and is weakly minimum phase then can be locally feedback equivalent to a passive system with proper storage function $V(x)$

Proof: see [1].

Corollary. Feedback control of form (8) to system (3) which satisfies the requirements of Theorem 1 is a closed loop dissipative system.

$$u = a(z,y)^{-1}\left(-b(z,y) - L_p W(z) + v\right) \tag{8}$$

Application of (8) results to following closed loop system.

$$\begin{aligned} \dot{z} &= f(z) + p(z,u)u \\ \dot{y} &= -L_p W(z) + v \end{aligned} \tag{9}$$

Further linear feedback control to the system improves its stability. Taylor approximation near equilibrium point gives the convergence condition where the Jacobian matrix should be Hurwitz [4].

4 PM synchronous motor model

The system can be stabilized if the linearised model is asymptotic stable (has all the eigenvalues with negative real part)

Application to synchronous motor

Consider a permanent-magnet synchronous motor model described as follows [6]

$$u_{sd} = R_s i_{sd} + L_{sd} \frac{d}{dt} i_{sd} - \omega_m L_{sq} i_{sq}$$

$$u_{sq} = R_s i_{sq} + L_{sq} \frac{d}{dt} i_{sq} + \omega_m (L_m i_{sd} + \Psi_F) \quad (10)$$

$$t_w = \frac{3}{2} p\left(\Psi_F i_{sq} + \left(L_{sd} - L_{sq}\right) i_{sq} i_{sd}\right)$$

$$\frac{d}{dt}\omega_m = \frac{1}{J}\left(\beta\omega_m + t_w - t_L\right)$$

where i_{sd}, i_{sq} are the d, q axes transformed currents, u_{sd}, u_{sq} the transformed input voltages ω_m the motor angular velocity , t_W and t_L are the electric and external load torques.

In Table 1. there is a list of PM synchronous motor parameters used in this analysis.

R_s	Stator resistance
L_{sd}	Direct axis inductance
L_{sq}	Quadrature axis inductance
L_m	Mutual stator, field infuctance
Ψ_F	Field flux
p	pole number
β	viscous friction constant

Table 1. List of PM synchronous motor parameters.

Since it is a PM synchronous motor, field flux is constant. We also consider symmetric construction so the inductances L_{sd} , L_{sq} are equal.

Converting the equations to state space representation we derive

$$\begin{aligned} \dot{i}_d &= -i_d + \omega i_q + u_d \\ \dot{i}_q &= -i_q - \omega i_d + a\omega + u_q \\ \dot{\omega} &= b(i_q - \omega) - T_L \end{aligned} \quad (11)$$

Considering the case where $u_d = u_q = T_L = 0$, $a = 4$ and b= 30 the system exhibits chaotic behaviour.

Selecting

$$\begin{aligned} u_d &= T_L = 0 \\ u_q &= (1-a)i_q - b\omega + v \end{aligned} \quad (12)$$

where v is external input signal

PM synchronous motor model results to

$$\begin{aligned} \dot{i}_d &= -i_d + \omega i_q \\ \dot{i}_q &= -i_q - \omega i_d + b\omega + u_q \\ \dot{\omega} &= 4(i_q - \omega) \end{aligned} \quad (13)$$

Selection of b so that Taylor approximation near equilibrium point be Hurwitz gives the convergence condition of the closed loop system.

5 Illustrative example

In Fig. 1 open loop response of PM synchronous motor response is shown with initial conditions [1.00 0.00 -1.00] and zero inputs. Parameters were selected to produce chaotic behavior which in this response is obvious.

In Fig. 2 feedback was applied and the system was simulated from the same initial point. Chaos was removed but long transients are present.

In Fig. 3 gain was changed to produce more acceptable response. Suitable parameter selection in the feedback controller improves closed loop system performance.

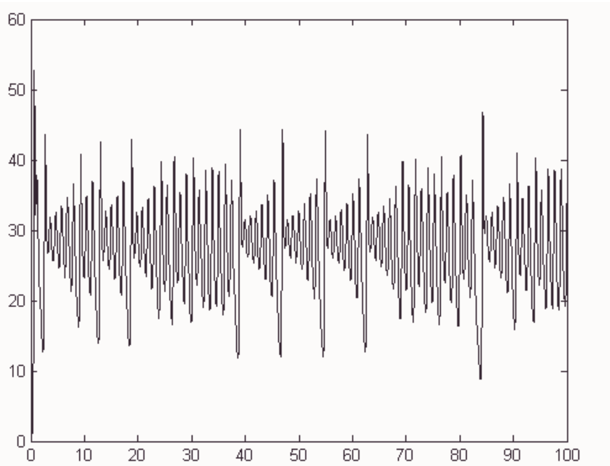

Fig. 1 Open loop response exhibits chaotic behavior (b=30).

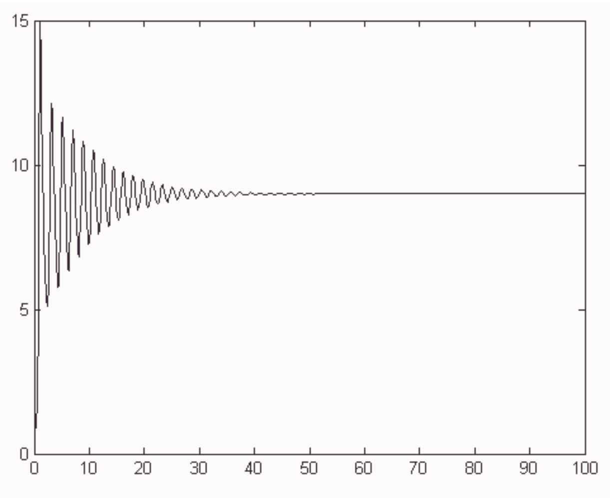

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Fig. 2. Stable closed loop output response with long transients (b=10).

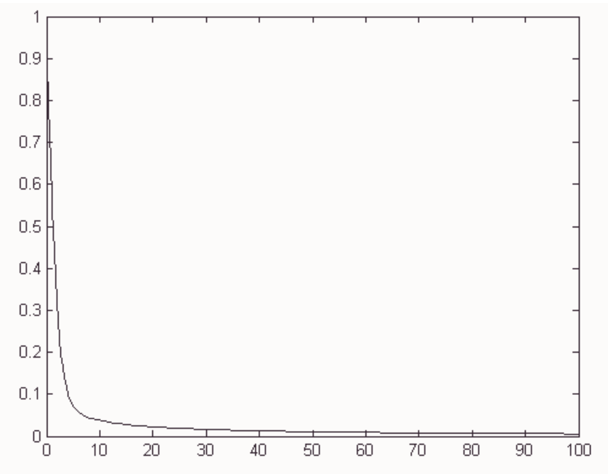

Fig. 3. Stable closed loop output response for another selection of b (b=1).

6 Conclusion

This paper has presented a design method to control nonlinear systems to avoid chaotic behavior and to guarantee asymptotic stability by feedback. Linear control methods can not apply here, so passivity theory was used.

The method is general and it was applied to PM synchronous motors. Illustrative examples showed the effectiveness of the method. These results are encouraging towards the applicability of the proposed control scheme.

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