A general model of vector hysteresis

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Abstract We formulate novel general models of vector hysteresis. For our model with two elementary output states, we establish the novel property of maximal reversal points (analogue of the wiping-out property of scalar models), and algorithms for identification of the weight function and the switching hypersurfaces.

Key-Words: Vector hysteresis, maximal reversal points, identification.

1 Introduction

The phenomenon of hysteresis is arises in many areas of Physics, Engineering, Economics, Biology, etc. Originally, hysteresis was most extensively studied in the context of magnetic hysteresis. It is well known that there is, in adition to magnetic hysteresis, also optical hysteresis, superconducting hysteresis, ecnomic hysteresis, hysteresis in biology (e.g. cell expresion), mechanical hysteresis (models of friction, rheological models, etc.), and other manifestations of hysteresis.
Generally, hysteresis is characterized by phenomena of non-local memory, of a nature that cannot be modelled by Volterra integral equations or Wiener-Volterra series due to lack of adequate smoothness and scale-invariance (to be explained below).
The problem of modelling hysteresis when the input is a scalar-valued signal was initiated in the paper of Preisach and continued in the works of Everett, Neel, and others. A big step in the study of hysteris has been the work of Krasnosel'skii and Pokrovskii [4] who provided (among other things) a firm rigorous foundation for the Preisach model as a strictly mathematical operator regardless of the physical interpretation of each manifestation of hysteresis.
In many applications, it is necessary to use a hysteresis model which accepts vector-valued input signals. Among many applications of vector hysteresis, we mention the new technology of magnetoresistive random-access memory (MRAM), where programming is achieved by a vector-valued input signal (time-varying electric currents along the two programming line directions), and particularly the Savtchenko switching invention. Vector hysteresis has also many other applications, for example, mechanical systems that contain elements that are rheologically modelled as hysteretic elements (such as reinforced concrete structures) under the effects of complex loading induced, e.g., by seismic excitation.
The extension of the classical Preisach model to the vector-valued case is not a straightforward matter. There exist many models of vector hysteresis, many of which are summarized in [5]. Other recent works dealing with system-theoretic and control-theoretic aspects of hysteresis include [1, 2, 3].

In this paper, we define and study a novel generalized model of vector hysteresis that preserves as many of the features of the scalar Preisach model as possible.

2 Brief outline of some properties of the Preisach model

The standard Preisach model is described in terms of the elementary transducers (non-ideal relays) and a Borel measure for the superposition of elementary transducers. For each (α,β), $\alpha>\beta$, the operator $V_{\alpha\beta}$, acting on continuous functions u(t), has two output states, which we may conventionally designate as 0 and 1. A switching from 0 to 1 takes place when u exits from $(-\infty,\alpha)$, and the reverse switching, from 1 to 0, takes place when u exits from $(\beta,+\infty)$. The hysteresis operator H is defined by

$$(Hu)(t) := \iint_{\alpha>\beta} (V_{\alpha\beta}u)(t)\mu(d\alpha,d\beta) \qquad (1)$$

In case μ is absolutely continuous relative to the Lebesgue measure in the $\alpha\beta$-space, the hysteresis operator becomes

$$(Hu)(t) = \iint_{\alpha>\beta} (V_{\alpha\beta}u)(t)w(\alpha,\beta)d\alpha d\beta \qquad (2)$$

As established in [5], the Preisach operator has, among other important properties, the following two:

(1) the Preisach operator stores only information about dominant local extrema of the input signal u(t), and every other information about u(t) is wiped out;

(2) the weight function $w(\alpha,\beta)$ can be identified (if it is unknown) on the basis of using special types of input signals and obtaining the first-order reversal curves.

In the sequel, we shall be interested in, among other things, establishing analogues of these two properties for general vector hysteresis models that are introduced in this paper.

3 Generalized vector model with two elementary output states

Every model that preserves some essential analogies with the Preisach scalar model has to represent the hysteretic output as a superposition of elementary transducers:

$$(Hu)(t) = \int_{\Omega} (V_{\gamma}u)(t)\mu(d\gamma) \qquad (3)$$

where $\{V_{\gamma}:\gamma\in\Omega\}$ is the family of elementary output operators parameterized by a (generally vector-valued) parameter γ, and μ is a finite Borel measure on the space Ω of parameter values.

The classical Preisach model has two output states for each elementary transducer (non-ideal relay). For the generalized vector model of this section, we take also two output states for the elementary transducers; we label these states as 0 and 1. We consider an open subset G of IR^{n} and 2 families of hypersurfaces (manifolds of co-dimension 1) $S^{+}(c^{+}), S^{-}(c^{-})$ in G, parameterized by m-dimensional vector-valued parameters c^{+}, c^{-} taking values in the sets C^{+}, C^{-}, respectively. We postulate that each hypersurface $S^{\pm}(c)$ divides $IR^{n}\setminus S^{\pm}(c)$ into two open subsets $R_{j}^{\pm}(c), j=0$ or 1. A pair (c^{+},c^{-}) of parameters will be called <u>admissible</u> if the following two conditions are satisfied:

$$\begin{aligned} &R_{1}^{\pm}(c^{+})\cup S^{+}(c^{+})\subseteq R_{1}^{-}(c^{-});\\ &R_{0}^{-}(c^{-})\cup S^{-}(c^{-})\subseteq R_{0}^{+}(c^{+}) \end{aligned} \qquad (4)$$

The set of all admissible pairs (c^{+},c^{-}) will be denoted by Ω; clearly, $\Omega\subseteq C^{+}\times C^{-}$. The set $A = A(c^{+},c^{-};u,t)$ of <u>admissible output states</u> of the elementary transducer V_{γ} $(\gamma := (c^{+},c^{-}))$ is defined as follows:

if $u(t) \in R_1^+(c^+)$, then A:={1};

if $u(t) \in R_0^-(c^-)$, then A:={0};

if $u(t) \in R_0^+(c^+) \cap R_1^-(c^-)$, then A:={0, 1}

(5)

The transducer V_γ is defined in terms of 4 variables: the $IR^n -$ valued signal u, the initial time t_0, the current time t, and the initial output state $v_0 \in A(c^+, c^-; u, t_0)$. The time-evolution of the transducer V_γ is described in terms of the exit times τ_0 or τ_1 of the signal u from the sets $R_0^-(c^-)$ or $R_1^+(c^+)$. The exit time $\tau \equiv \tau(u, S, t)$ of a signal u that satisfies $u(t) \in S$ where S is an open set in IR^n is the first time, after time t, that the signal hits the boundary of S, specifically

$$\tau(u, S, t) := \inf\{t' > t: u(t') \notin S\} \qquad (6)$$

The switching rule for V_γ is:

if $(V_\gamma u)(t^+) = 0$, and if $\tau_0 = \tau(u, R_0^+(c^+), t)$, then $(V_\gamma u)(t') = 0$ for $t < t' < \tau_0$, $(V_\gamma u)(\tau_0^+) = 1$;

if $(V_\gamma u)(t^+) = 1$, and if $\tau_1 = \tau(u, R_1^-(c^-), t)$, then $(V_\gamma u)(t') = 1 \text{ for } t < t' < \tau_0$, $(V_\gamma u)(\tau_1^+) = 0$

(7)

Under the conditions specified in this section, and if u a continuous function of t, then each transducer V_γ is well-defined. Admissible initial states for $(V_\gamma u)(t_0)$ have to be chosen so that the resulting function of γ is measurable.

We remark that this model can be extended to include a vector output, by making the modification of using two linearly independent vectors w_0, w_1, instead of the numbers 0 and 1, as the possible output states of elementary transducers.

4 Generalized vector model with an arbitrary number of output states

We define a vector hysteresis model with an arbitrary (finite) set of possible output states for the elementary transducers. Let A be the set of all possible output states. Otherwise the terminology and notation are the same as in section 2 above.
The model is defined through the following ingredients (for each γ in Ω):

(i) a collection $\{C^\alpha : \alpha \in A\}$ of open subsets of G such that $\bigcup_{\alpha \in A} C^\alpha = G$;

(ii) a partition of each boundary ∂C^α into mutually disjoint sets $S^{\alpha\beta}$, $\beta \in A \setminus \{\alpha\}$, with the property $S^{\alpha\beta} \subseteq C^\beta$.

The admissible output states, when the value of the input signal is u_0, comprise those α for which $u_0 \in C^\alpha$. The switching rule is:

if $(V_\gamma u)(t^+) = \alpha$ and $u(\tau_\alpha^-) \in S^{\alpha\beta}$ $(\tau_\alpha := \tau(u, C^\alpha, t))$, then $(V_\gamma u)(\tau_\alpha^+) = \beta$

(8)

The superposition of the elementary output operators V_γ is carried out as in section 3.

5 Some properties of generalized models of vector hysteresis

We shall examine some properties of themodel of section 3.

The first question we examine is: what is the analogue of the wiping-out property of the Preisach model?

The role of local extrema of the input signal is played by the reversal points of the input signal in the vector case. A reversal point of the vector signal u is a point where (i) u(t) belongs to the boundary of a set $R_j^{\pm}(c)$ (j=0 or 1) for some c, and (ii) there exists an ε>0 such that, for all t' that satisfy $0 \neq |t' - t| < \varepsilon$, we have $u(t') \in R_j^{\pm}(c)$. We also define a reversal interval to be an interval $[t_1, t_2]$ such that $u(t) \in \partial R_j^{\pm}(c^{\pm})\ \forall t \in [t_1, t_2]$ and, for some ε>0 we have

$$u(t) \in R_j^{\pm}(c^{\pm}) \quad \forall t \in (t_1 - \varepsilon, t_1) \cup (t_2, t_2 + \varepsilon) \tag{9}$$

The role of dominant local extrema is played by maximal reversal points, which we define now. We introduce a partial order on the sets C^+, C^- by the following conditions:

we say that $c_1^+ \le c_2^+$ if $R_1^+(c_1^+) \supseteq R_1^+(c_2^+)$;

we say that $c_1^- \le c_2^-$ if $R_0^-(c_1^-) \supseteq R_0^-(c_2^-)$

(10)

When a signal has a non-trivial interval (i.e. an interval $[t_1, t_2]$ with $t_1 < t_2$) of reversal points, then we call the elements of that interval lingering points. The properties and the identification methods described below apply to signals without lingering points. However, with some modifications, it is possible to include cases of lingering points.

A reversal point is said to be maximal if the corresponding value of $c^{\pm}$ is maximal relative to the partial order defined above.

The analogue of the wiping-out property for the model introduced in this paper is: if the signal u has no lingering part, then the hysteresis operator of section 3 of this paper stores information only about the maximal reversal points, and all other information is wiped out.

The second question we wish to address is the issue of identification. For the vector hysteresis model of section 3, there are two identification problems:

(I) identification of the measure (distribution) μ if the hypersurfaces $S^{\pm}(c^{\pm})$ are known;

(II) identification of the hypersurfaces $S^{\pm}(c^{\pm})$ if the parameter sets C^+, C^- are known (or conventionally defined) but the hypersurfaces themselves, as well as the measure μ, are unknown.

The solution of the first identification problem is based on the solution of the corresponding problem for the Preisach model. If there exists a curve (k) in G that intersects all surfaces $S^{\pm}(c^{\pm})$ transversally, and intersects each such hypersurface at one single point. The restriction of the operator V_γ to signals that take values in (k) is, under certain conditions to be stated below, an operator of the Preisach type. To show this, we observe that the curve (k) intersects the two families of hypersurfaces at points which we conventionally denote by α (for the family $S^+(c^+), c^+ \in C^+$) and β (for the family $S^-(c^-), c^- \in C^-$). Now we need an assumption:

(A) The parameters α, β defined above, corresponding to admissible values of (c^+, c^-), have no crossover, i.e. each admissible α is on the same side (relative to an orientation on the curve (k)) of the corresponding β.

In that case, each admissible α divides (k)\{α} into two relatively open parts (in the relative topology of (k) inherited from the set G) and the corresponding β lies in one of those parts; denote that part by (k_α). Similarly, each β divides (k)\{α} into two relatively open subsets, and we denote by (k_β) that part that contains α. In this way, a switching from 0 to 1 occurs when u exits from (k_α), and, symmetrically, a switching from 1 to 0 occurs when u exits from (k_β). But this is precisely the definition of a scalar KMPP operator. Consequently, the standard identification methods for scalar Preisach hysteresis, developed in [5], can applied.
We remark here that, without the assumption of no crossover, the identification method of [5] can still be modified to identify μ.
The second identification problem can be solved if, in addition to the previous assumptions, we postulate the existence of two families of curves $(k^i_\sigma), i = 1,2$ parameterized by a vector-valued parameter σ, so that each $(k^i_\sigma), i = 1,2$ intersects the two (now presumed unknown) families of hypersurfaces $S^\pm(c^\pm)$ transversally, and also each $(k^1_{\sigma_1})$ intersects each $(k^2_{\sigma_2})$ transversally. Further, we postulate that each of the two families $(k^i_\sigma), i = 1,2$ fill up the entire set G. Then each point in G can be represented as an intersection of some $(k^1_{\sigma_1})$ with some $(k^2_{\sigma_2})$. On each curve of each of the two families $(k^i_\sigma), i = 1,2$, we label all its points as $\alpha(\sigma_1), \beta(\sigma_1); \alpha(\sigma_2), \beta(\sigma_2)$.
On each curve of the first family, by using the identification method for a scalar Preisach model, we estimate the weight function $w_i(\sigma_i, \alpha, \beta), i = 1,2$ (assuming μ is absolutely continuous relative to the Lebesgue measure on the αβ-space). If the same point in G can be represented both as $(\sigma_1, \alpha, \beta)$ and $(\sigma_2, \alpha, \beta)$, then the values of w_1 and w_2 should be equal at that point.
The system of 2 equations

$$w_1(\sigma_1, \alpha, \beta) = w(\sigma_2, \alpha, \beta) = c \qquad (11)$$

can be solved, in principle, for each value of c, to give some solution

$$\alpha = g_1(\sigma_1, \sigma_2, c), \beta = g_2(\sigma_1, \sigma_2, c) \qquad (12)$$

The hypersurfaces described by these two families of equations are the wanted $S^\pm(c^\pm)$.

6 Conclusions

We have introduced a general model of vector hysteresis that possesses interesting analogues of the properties of the standard (scalar) Preisach model. We have established an anlogue of the wiping-out property, which in our model becomes the maximal reversal point property. The identification problem for our problem is qualitatively different from the case of the standard Preisach model, as it involves not only the identification of a measure (used in the superposition of elementary transducers) but also the identification of two families of hypersurfaces that determine the model. We have established methods for the solution of both identification problems. We have also extended the general vector hysteresis model to the case of an arbitrary number of elementary output states.

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